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ADDITION FOR SECTION 8.1

The Sender-Receiver Game of Crawford and Sobel: Coarse Information Transmission

Even if the informed and uninformed players have different incentives and they cannot commit to a mechanism, if their incentives are close enough it may still be possible for truthful messages to be sent in equilibrium. Let us call the informed player "the sender" and the uninformed player "the receiver" instead of agent and principal. Suppose the information is the sender's type, t, uniformly distributed on $[0, 10]$, the sender sends a message, m, and the receiver chooses an action, a, where a and m are also in $0, 10]$. At the extremes of payoff function similarity, it is clear what happens. Suppose the sender wants a to be as close to t as possible. If the sender also wants a to be close to t, then he will tell the truth; $m = t$. If he wants a to be as big as possible, on the other hand, because his ideal action is $a = 10$, then he will lie, and the receiver will ignore any message. But what happens if, say, the sender's ideal action is $t + 1$, so he doesn't want a to be too large, but he does want a to be bigger than a fully-informed receiver would choose? Crawford & Sobel (1982) discovered the answer in a famous article titled "Strategic Information Transmission." We will see what happens in the game below, which I adapted from Gibbons (1992, Section 4.3.A), a good place to start if you want to learn more.

The Crawford-Sobel Sender-Receiver Game

Players

The sender and the receiver.

The Order of Play

0 Nature chooses the sender's type to be $t \sim U[0, 10]$.
1 The sender chooses message $m \in [0, 10]$.
2 The receiver chooses action $a \in [0, 10]$.

Payoffs

The payoffs are quadratic loss functions in which each player has an ideal point and wants a to be close to that ideal point.

$$
\begin{aligned}
\pi_{sender} &= \alpha - (a - [t+1])^2 \\
\pi_{receiver} &= \alpha - (a - t)^2
\end{aligned}
\tag{1}
$$

First, let's see why perfect truthtelling cannot happen in equilibrium. Suppose the receiver believed that the sender always sent $m = t$ and so chooses $a = m$. Would the sender indeed be willing to tell the truth?

He would not. The sender would not always report $m = 10$, because his ideal point is $a = t + 1$, rather than a being as big as possible. If, however, the sender thinks the buyer will believe him, he will deviate to reporting $m = t + 1$, always exaggerating his type slightly.

What if the receiver adapts to this, and chooses $a = m - 1$? Then the sender would change too, and send $m = t + 2$, exaggerating more. This unravelling away from the truth (as opposed to the unravelling *toward* the truth when outright lying is forbidden) continues until the only message the sender reports is $m = 10$, regardless of his type and the receiver ignores it. This explanation is heuristic, and does not prove that there is no fully separating equilibrium, in which each type of sender reports a different message, but Crawford & Sobel (1982) prove that this is the case.

Thus, one equilibrium is the pooling equilibrium in which the sender's message is ignored and the receiver chooses $a = Et = 5$. This equilibrium could take either of two forms:

Pooling Equilibrium 1

Sender: Send $m = 10$ regardless of t.

Receiver: Choose $a = 5$ regardless of m.

Out-of-equilibrium belief: If the sender sends $m < 10$, the receiver uses passive conjectures and still believes that $t \sim U[0, 10]$.

Pooling Equilibrium 2

Sender: Send m using a mixed-strategy distribution independent of t that has the support $[0, 10]$ with positive density everywhere.

Receiver: Choose $a = 5$ regardless of m.

Out-of-equilibrium belief: Unnecessary, since any message might be observed in equilibrium.

In each of these two equilibria, the sender's action conveys no information and is ignored by the receiver. Although the sender is happy about this if it happens that $t = 4$ and the receiver is if $t = 5$, the expected payoffs are lower than if the sender could commit to truthtelling–something I will leave to the reader to calculate. There also, however, exists a partial pooling equilibrium in which the sender truthfully reports whether his type is in the low interval $[0, x]$ or the high interval $[x, 10]$, with $x = 3$.

Partial Pooling Equilibrium 3

Sender: Send $m = 0$ if $t \in [0, 3]$ or $m = 10$ if $t \in [3, 10]$.

Receiver: Choose $a = 1.5$ if $m < 3$ and $a = 6.5$ if $m \geq 3$

Out-of-equilibrium belief: If m is something other than 0 or 10, then $t \sim U[0, 3]$ if $m \in [0, 3)$ and $t \sim U[3, 10]$ if $a \in [3, 10]$.

In effect, the Sender has reduced his message space to two messages, LOW (=0) and HIGH (=10), in Equilibrium 3. Rather than just testing that this is an equilibrium, let us derive it, to show why the equilibrium interval-splitting type is $x = 3$.

First, note that the receiver's optimal strategy in a a partially pooling equilibrium is to choose his action to equal the expected value of the type in the interval the sender has chosen. Thus, if $m = 0$, the receiver will choose $a = x/2$ and if $m = 10$ he will choose $a = (x + 10)/2$.

The receiver's equilibrium response determines the sender's payoffs from his two messages. The payoffs between which he chooses are:

$$\begin{aligned} \pi_{sender,m=0} &= \alpha - \left([t+1] - \frac{x}{2}\right)^2 \\ \pi_{sender,m=10} &= \alpha - \left(\frac{10+x}{2} - [t+1]\right)^2 \end{aligned} \tag{2}$$

There exists a value x such that if $t = x$, the sender is indifferent between $m = 0$ and $m = 10$, but if t is lower he prefers $m = 0$ and if t is higher he prefers $m = 10$. To find x, equate the two payoffs in expression (**??**) and simplify to obtain

$$[t+1] - \frac{x}{2} = \frac{10+x}{2} - [t+1]\,. \tag{3}$$

We set $t = x$ at the point of indifference, and solving for x then yields $x = 3$.

Thus, the divergence in preferences of the sender and receiver coarsens the message space, in effect. The sender will not send a truthful precise message, but if expectations are right (so we have the partially pooling equilibrium) he will send a truthful coarse message. If the true value of t is small, the sender will report the fairly precise information that t lies in [0,3]. If t is larger, it is harder to induce a truthful report, since the sender has a tendency to exaggerate and report t larger than it is, but the message can at least rule out the interval [0,3).

If instead of wanting $(t+1)$ to be the action, the preferences of sender and receiver diverged more— say, to $(t+8)$— then there would only be the uninformative pooling equilibrium. If they divereged less— say, to $(t+0.1)$— then there would exist other partially pooling equilibria that had more than just two effective messages and would distinguish between three or more intervals instead of between just two.

In the Crawford-Sobel Sender-Receiver Game, the receiver cannot commit to the way he reacts to the message, so this is not a mechanism design problem. Nor is the sender punished for lying, so the unravelling argument for truthtelling does not apply. Nor do the players' payoffs depend directly on the message, which might permit the signalling we will study in Chapter 11 to operate. Instead, this is a **cheap-talk game**, so called because of these very absences: m does not affect the payoff directly, the players cannot commit to future actions, and lying brings no directly penalty. In Chapter 3 I alluded to how cheap talk might help select among equilibria in the Battle of the Sexes. In particular, knowing what the other player was thinking of doing would help to avoid the mixed-strategy equilibrium, with its low payoff. Our sender and receiver are in a similar situation here: their interests are similar but not identical, and they could both benefit from some transfer of information. If expectations are appropriate, they do so, in the partially pooling equilibrium. If they do not expect the cheap talk to be informative, however, it will not be, and coordination will fail.