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Eric Rasmusen, Erasmuse@indiana.edu. Web: Php.indiana.edu/∼erasmuse.

This is a section from Chapter 13 (Pricing) of *Games and Information.* that was dropped from the 3rd Edition.

Conjectural Variation

Conjectural variation, an equilibrium concept different in flavor from any that has yet appeared in this book, is a way to quantify the degree of cooperation between oligopolists. Let us continue to specify the strategies as quantities. In a Nash equilibrium, no player wants to deviate, and his beliefs about how the other players would behave are confirmed whatever nodes are reached. Under conjectural variation, a player believes, for reasons outside the model, that if he deviated, the other players would deviate in specified ways. This should seem quite an unnatural idea to anyone who has read this far in the book, since it violates the basic assumptions of Bayesian games and it is rather hazy about what is happening in this simultaneous-move game. The idea may be clearer in an example. Returning to the two-player model, we can use equation (13.**??**) to write Apex' self-perceived first order condition as

$$\frac{d\pi_a}{dq_a} = p + \left(\frac{dp}{dq}\right)\left(\frac{dq}{dq_a}\right) q_a - \frac{dc}{dq_a} = 0. \tag{1}$$

The difference between the first-order-conditions (13.**??**) and (13.1) is that (13.1) contains

$$\frac{dq}{dq_a} = 1 + \frac{dq_b}{dq_a}. \tag{2}$$

Equation (13.2) says that the expected effect on industry output of an increase in q_a by one unit has two components: a direct increase of one unit, and an indirect increase from Brydox increasing his output in response. The first-order condition (13.1) must be qualified by "self-perceived" because Apex might be mistaken in his beliefs about Brydox' response. The belief implicit in Nash equilibrium, that Apex' deviation is not followed by a response from Brydox, is the only belief that supports an equilibrium in which one player or the other is not mistaken. But if consistency of beliefs is not required, other beliefs are possible that lead to different behavior.

Firm i's **conjectural variation** *is the rate* $\frac{dq_{-i}}{dq_i}$ *at which he conjectures that the output of other firms would change if i's own output changed.*

$CV = 0$
In a Cournot-Nash equilibrium, Apex believes that if he deviated by producing more, Brydox would not deviate, so the conjectural variation equals 0.

$CV = -1$
If Apex believes that an increase in his output is matched by a decrease in Brydox' output, so the total industry output is left unchanged, the conjectural variation is -1. If both firms use this conjectural variation, the industry output is the competitive level; firms ignore the effect of their output in depressing the price. Of course, if both firms use a negative value, their beliefs are inconsistent.

$CV = 1$
If Apex believes that Brydox would exactly match his output changes, the conjectural variation is 1. With two firms, with identical cost curves, industry output is at the cartel level, though an n-player game would need $CV = n-1$ to achieve that level.

In Stackelberg equilibrium (Section 3.5), the conjectural variation of the Stackelberg follower is between 0 and 1, and takes the value given by a reaction function like equation (13.**??**).

In the world oil market, fringe producers like Britain face the OPEC cartel. If Britain's conjectural variation equals -1, Britain believes that producing more would make OPEC cut back an equal amount; if 0, that OPEC would ignore Britain; if 0.5, that OPEC would follow with a smaller increase; if 1, that OPEC would match every increase; and if 10, that OPEC would respond by flooding the market. Setting up equations with the appropriate value for the conjectural variations of all the players, we could solve for the equilibrium output. The idea is useful for organizing different models of duopoly and it is simple enough to be empirically estimated. Even without knowing the correct theory, an estimate could be made of how much OPEC actually does respond to Britain.

Notes:

The idea of conjectural variation is attributed to Bowley (1924) and is discussed in Jacquemin (1985) and Varian (1992, p. 302).

Do not confuse a conjectural variation of -1 with perfect competition, even though both may lead to the efficient output. In perfect competition, individuals do not believe that they affect the rest of the market, but if $CV = -1$, a firm believes that other firms will cut back when it produces more. Perfect competition is more like a game with players so small relative to the market that even though $CV = 0$, as in Nash equilibrium, each player correctly believes that his actions have a trivial effect on the market price.