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9.5 Other Equilibrium Concepts: Wilson Equilibrium and Reactive Equilibrium

In Insurance Game III , any pooling contract is vulnerable to a cream-skimming contract that draws away the $Safe$s, but this is a little strange, because it seems that after that happens the now unprofitable old pooling contract (which was soaking up the $Unsafe$s) would be withdrawn. The game tree does not reflect this, nor does the Nash equilibrium concept.

One way to obtain a pure strategy equilibrium is to redefine the equilibrium concept. C. Wilson (1980) suggests that the pooling equilibrium is legitimate because a principal (an uninformed player) who was thinking about introducing the new contract would realize that it would be unprofitable once the old contract was withdrawn.

A **Wilson equilibrium** *is a set of contracts such that when the agents (informed players) choose among them so as to maximize profits,*
(1) All contracts make nonnegative profits; and
(2) No new contract (or set of contracts) could be offered that would make positive profits even after all contracts that would make negative profits as a result of its entry were withdrawn.

The Wilson equilibrium is the same as the Nash separating equilibrium if that exists, and otherwise it is the pooling contract most preferred by the $Safe$s. In figure 9.6, the Wilson equilibrium is the same as the Nash equilibrium, the separating pair (C_3, C_5). In figure 9.7, where no Nash equilibrium exists, the Wilson equilibrium is the zero-profit pooling contract, F'. It is on the line $\omega F'$, so it satisfies part (a) of the definition. It provides the fullest insurance of any zero-profit pooling contract, so that no new pooling contract would be more attractive, and while some new separating contract might be profitable if the $Unsafe$s stayed with F', any such contract would cause F' to be withdrawn and would be unprofitable thereafter.

The idea of Wilson equilibrium can also be incorporated into the game by modifying the game tree instead of redefining the equilibrium concept, as suggested by Fernandez & Rasmusen (unpublished), to obtain the Wilson outcome as the perfect equilibrium of the modified game.

Wilson Equilibrium

(1)Principals simultaneously offer contracts, called "old contracts."

(2) Principals may simultaneously offer other contracts, called "new contracts."

(3) Principals may simultaneously withdraw any old contracts.

(4) Agents choose from among the remaining old and new contracts, and trading occurs.

In the perfect Bayesian equilibrium of this game, the principals offer the contracts that form a Wilson equilibrium in move (1). The approach of changing the game tree may seem more complicated than changing the equilibrium concept, but that is because it clearly delineates the somewhat vague intuition behind the equilibrium concept. Making use of the Wilson concept is not just a technical assumption: it is assuming that the market has a particular structure.

Riley (1979b) uses reasoning similar to Wilson's to justify his concept of "reactive equilibrium." Under this concept, an equilibrium is a set of contracts such that though some new contract might be profitable, that new contract would itself become unprofitable if a second new contract were introduced. More formally, following Engers & Fernandez (1987),

A **reactive equilibrium** *is a set of contracts S yielding nonnegative profits such that for any nonempty set of contracts S' (the defection), where $S \cup S'$ is closed, there exists a closed set of contracts S'' (the reaction) such that:*

(1) S' incurs losses when only these three sets are tendered, and

(2) S'' does not incur losses when these three sets are tendered, whether or not other contracts are also offered.

In both figure 9.6 and 9.7, the reactive equilibrium is the separating pair (C_3, C_5). That pair yields zero profits, and while in figure 9.7 there is a profitable deviation (C_6), that deviation would become unprofitable if a cream-skimming contract were added as a reaction. Moreover, condition (2) of the definition is met, because if the reactive cream-skimming contract is chosen carefully, no additional contracts can be added which make it unprofitable, given that C_6 continues to be offered.

A separating reactive equilibrium always exists because any pooling contract disrupting it could be reacted against: reactive equilibrium makes constructive use of the nonexistence of a pooling equilibrium. The Wilson concept is based on withdrawing contracts in response to deviation, whereas the reactive concept is based on adding them. As a result, when Nash equilibrium does not exist, the Wilson concept favors a pooling equilibrium, while the reactive concept favors a separating equilibrium.

Wilson Equilibrium and Reactive Equilibrium (from the Signalling Chapter 10)
As in Insurance Game III, it is possible to go beyond Nash equilibrium to find an equilibrium of some other kind for Education VII. Is it reasonable to say that a pooling equilibrium could always be broken by a contract which draws away the *High*'s? After the *High*s departed, the old pooling contract, which would still soak up all the *Low*s, would be unprofitable. In figure 10.2, if C_5 is withdrawn after C_6 is offered, the *Low*s prefer C_6 to the zero they obtain from unemployment, and C_6 becomes a pooling contract. This is irrelevant to the question of whether C_5 is a Nash equilibrium, but it might lead one to doubt the wisdom of the equilibrium concept.

Under the concept of the **Wilson equilibrium** from section 9.5, the pooling equilibrium is legitimate, because an employer thinking about introducing the new equilibrium-breaking contract would realize that the new contract would be unprofitable once the old contract was withdrawn. **Reactive equilibrium** can also be applied, and generates a separating equilibrium. Under its reasoning, the separating equilibrium cannot be broken by a pooling contract, because the pooling contract would in turn be broken by a second separating contract. The Wilson C_5 and the reactive (C_3, C_4) are the two clear candidates for equilibrium in figure 10.2. Alternately, we could restructure the model so that the worker moves first— the assumption in sections 10.1 and 10.2. While avoiding the existence problem, that introduces the need to think about out-of-equilibrium beliefs, and it really models a different situation, in which workers cannot change their education in response to employers' contracts.

N9.5 Other Equilibrium Concepts: Wilson and Reactive Equilibrium

- Engers & Fernandez (1987) show how to transform a simultaneous move game into a sequential move game such that the reactive equilibrium of the original game is one of the perfect Bayesian equilibria of the transformed game.

- In adverse-selection games it often matters whether the informed player or the uninformed player offers the contract. Wilson and reactive equilibrium are important when the uninformed player offers the contract, since it is only he who runs the risk of receiving something unexpected in the transaction and he might then wish to withdraw an offer. The issues involved are the same as in the difference between screening and signalling, which will be discussed at length in chapter 10.