June 22, 2008. Thanks to Warren Manners for finding these errors.

there is a surplus of 1 from the consumer's purchase of a car that costs 11 and yields 12 in utility. Both versions of the Repossession Game give all the bargaining power to the bank in the sense that where there is a surplus to be split, the bank gets 100 percent of it. But this does not help the bank in Repossession Game II, because the consumer can put himself in a position where the bank ends up a loser from the transaction despite its bargaining power.

*8.5 State-space Diagrams: Insurance Games I and II

An approach to principal–agent problems, especially useful when the strategy space is continuous, is to use diagrams. The term "moral hazard" comes from the insurance industry, where it refers to the idea that if a person is insured he will be less careful and the danger from accidents will rise. Suppose Smith (the agent) is considering buying theft insurance for a car with a value of 12. Figure 8.1, which illustrates his situation, is an example of a **state-space diagram**, a diagram whose axes measure the values of one variable in two different states of the world. Before Smith buys insurance, his dollar wealth is 0 if there is a theft and 12 otherwise, depicted as his endowment, $\omega = (12, 0)$. The point $(12, 0)$ indicates a wealth of 12 in one state and 0 in the other, while the point (6, 6) indicates a wealth of 6 in each state.

One cannot tell the probabilities of each state just by looking at the state-space diagram. Let us specify that if Smith is careful where he parks, the state *Theft* occurs with probability 0.5, but if he is careless the probability rises to 0.75. He is risk-averse, and, other things equal, he has a mild preference to be careless, a preference worth only some small amount ϵ to him. Other things are not equal, however, and he would choose to be careful were he uninsured, because of the high correlation of carelessness with carlessness.

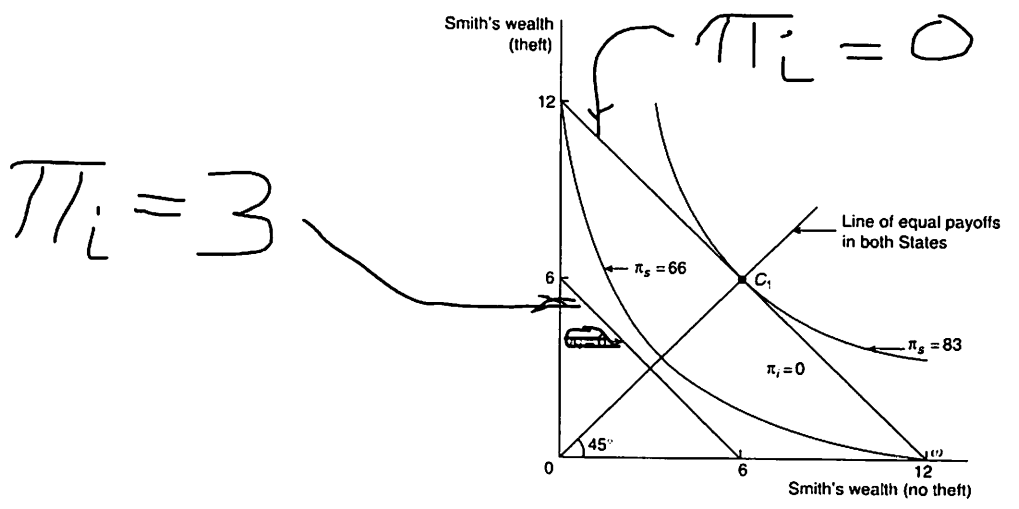

Figure 8.1 Insurance Game I.

The insurance company (the principal) is risk-neutral, perhaps because it is owned by diversified shareholders. We assume that no transaction costs are incurred in providing insurance and that the market is competitive, a switch from Production Game V, where the principal collected all the gains from trade. If the insurance company can require Smith to park carefully, it offers him insurance at a premium of 6, with a payout of 12 if theft occurs, leaving him with an allocation of $C_1 = (6, 6)$. This satisfies the competition constraint because it is the most attractive contract any company can offer without making losses. Smith, whose allocation is 6 no matter what happens, is **fully insured**. In state-space diagrams, allocations which like C_1 fully insure one player are on the 45° line through the origin, the line along which his allocations in the two states are equal.

The game is described below in a specification that includes two insurance companies to simulate a competitive market. For Smith, who is risk-averse, we must distinguish between dollar *allocations* such as (12, 0) and utility *payoffs* such as $0.5U(12) + 0.5U(0)$. The curves in figure 8.1 are labelled in units of utility for Smith and dollars for the insurance company.

Insurance Game I: Observable Care

PLAYERS
Smith and two insurance companies.

THE ORDER OF PLAY
1 Smith chooses to be either *Careful* or *Careless*, observed by the insurance company.
2 Insurance company 1 offers a contract (x, y), in which Smith pays premium x and receives compensation y if there is a theft.
3 Insurance company 2 also offers a contract of the form (x, y).
4 Smith picks a contract.
5 Nature chooses whether there is a theft, with probability 0.5 if Smith is *Careful* or 0.75 if Smith is *Careless*.

PAYOFFS
Smith is risk-averse and the insurance companies are risk-neutral. The insurance company not picked by Smith has a payoff of zero.

Smith's utility function U is such that $U' > 0$ and $U'' < 0$. If Smith picks contract (x, y), the payoffs are:

If Smith chooses *Careful*,

$$\pi_{Smith} = 0.5U(12 - x) + 0.5U(0 + y - x)$$
$$\pi_{company} = 0.5x + 0.5(x - y), \quad \text{for his insurer.}$$

If Smith chooses *Careless*,

$$\pi_{Smith} = 0.25U(12 - x) + 0.75U(0 + y - x) + \epsilon$$
$$\pi_{company} = 0.25x + 0.75(x - y), \quad \text{for his insurer.}$$

$\pi_i = 3$

$= 6 + .5(-6)$

In equilibrium, Smith chooses to be *Careful* because he foresees that otherwise his insurance will be more expensive. Figure 8.1 is the corner of an Edgeworth box which shows the indifference curves of Smith and his insurance company given that Smith's care keeps the probability of a theft down to 0.5. The company is risk-neutral, so its indifference curve, $\pi_i = 0$, is a straight line with slope $-1/1$. Its payoffs are higher on indifference curves such as $\pi_i = 6$ that are closer to the origin and thus have smaller expected payouts to Smith. The insurance company is indifferent between points ω and C_1, at both of which its profits are zero. Smith is risk-averse, so if he is *Careful* his indifference curves are closest to the origin on the 45° line, where his wealth in the two states is equal. Picking the numbers 66 and 83 for concreteness, I have labelled his original indifference curve $\pi_s = 66$ and drawn the preferred indifference curve $\pi_s = 83$ through the equilibrium contract C_1. The equilibrium contract is C_1, which satisfies the competition constraint by generating the highest expected utility for Smith that allows nonnegative profits to the company.

Insurance Game I is a game of symmetric information. Insurance Game II changes that. Suppose that

1 the company cannot observe Smith's action (care is *unobservable*); or
2 the state insurance commission does not allow contracts to require Smith to be careful (care is *noncontractible*); or
3 a contract requiring Smith to be careful is impossible to enforce because of the cost of proving carelessness (care is *nonverifiable* in a court of law).

In each case Smith's action is a noncontractible variable, so we model all three the same way, by putting Smith's move second. The new game is like Production Game V, with uncertainty, unobservability, and two levels of output, *Theft* and *No Theft*. The insurance company may not be able to directly observe Smith's action, but his dominant strategy is to be *Careless*, so the company knows the probability of a theft is 0.75. Insurance Game II is the same as Insurance Game I except for the following.

Insurance Game II: Unobservable Care

THE ORDER OF PLAY

1 Insurance company 1 offers a contract of form (x, y), under which Smith pays premium x and receives compensation y if there is a theft.
2 Insurance company 2 offers a contract of form (x, y).
3 Smith picks a contract.
4 Smith chooses either *Careful* or *Careless*.
5 Nature chooses whether there is a theft, with probability 0.5 if Smith is *Careful* or 0.75 if Smith is *Careless*.

Smith's dominant strategy is *Careless* in Insurance Game II, so in contrast to Insurance Game I the insurance company must offer a contract with a premium of 9 and a payout of 12 to prevent losses, which leaves Smith with an allocation $C_2 = (3, 3)$. Making thefts more probable reduces the slopes of both players' indifference curves, because it decreases the utility of points to the southeast of the 45° line and increases utility to the northwest.

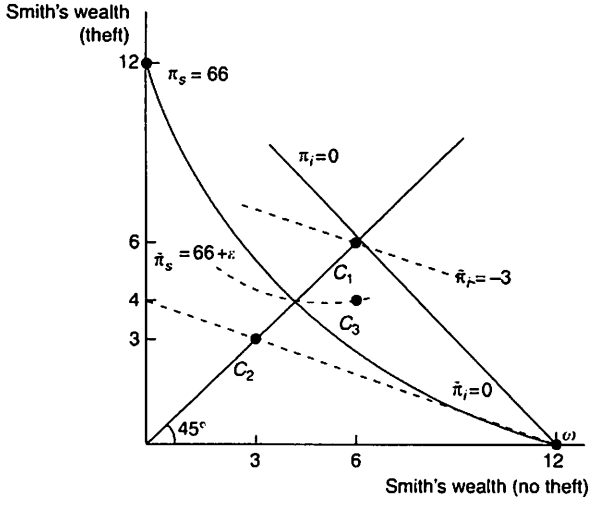

Figure 8.2 Insurance Game II with full and partial insurance.

In figure 8.2, the insurance company's isoprofit curve swivels from the solid line $\pi_i = 0$ to the dotted line $\tilde{\pi}_i = 0$. It swivels around ω because that is the point at which the company's profit is independent of how probable it is that Smith's car will be stolen (at point ω the company is not insuring him at all). Smith's indifference curve also swivels, from the solid curve $\pi_s = 66$ to the dotted curve $\tilde{\pi}_s = 66 + \epsilon$. It swivels around the intersection of the $\pi_s = 66$ curve with the 45° line, because on that line the probability of theft does not affect his payoff. The ϵ difference appears because Smith gets to choose the action *Careless*, which he slightly prefers.

Figure 8.2 shows that no full-insurance contract will be offered. The contract C_1 is acceptable to Smith, but not to the insurance company, because it earns negative profits, and the contract C_2 is acceptable to the insurance company, but not to Smith, who prefers ω. Smith would like to commit himself to being careful, but he cannot make his commitment credible. If the means existed to prove his honesty, he would use them even if they were costly. He might, for example, agree to buy off-street parking even though locking his car would be cheaper, if verifiable.

Although no full-insurance contract such as C_1 or C_2 is mutually agreeable, other contracts can be used. Consider the partial-insurance contract C_3 in figure 8.2, which has a premium of 6 and a payout of 8. Smith would prefer C_3 to his endowment of $\omega = (12, 0)$ whether he chooses *Careless* or *Careful*. We can think of C_3 in two ways:

1 Full insurance except for a **deductible** of ~~four~~ two. The insurance company pays for all losses in excess of four.
2 Insurance with a **coinsurance** rate of one-third. The insurance company pays two-thirds of all losses.

The outlook is bright because Smith chooses *Careful* if he only has partial insurance, as with contract C_3. The moral hazard is "small" in the sense that Smith barely prefers *Careless*.