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Comments welcomed.

Bayes Rule at the Bar: A Classroom Game for Chapter 2

I have wandered into a dangerous bar in Jersey City. There are six people in there. Based on past experience, I estimate that three are cold-blooded killers and three are cowardly bullies. I know that 2/3 of killers are aggressive and 1/3 reasonable; but 1/3 of cowards are aggressive and 2/3 are reasonable. Unfortunately, I spill my drink on a mean-looking rascal, who asks me if I want to die.

In crafting my response in the two seconds I have to think, I would like to know the probability I haveoffended a killer. Give me your estimate.

The story continues. A friend of the wet rascal comes in from the street outside the bar and learns what happened. He, too, turns aggressive. I know that the friend is just like the first rascal— a killer if the first one was a killer, a coward otherwise. Does this extra trouble change your estimate that the two of them are killers?

This game is a descendant of the game in Charles Holt & Lisa R. Anderson. "Classroom Games: Understanding Bayes Rule," *Journal of Economic Perspectives*, 10: 179-187 (Spring 1996), but I use a different heuristic for the rule and a barroom story instead of urns. Psychologists have found that people can solve logical puzzles better if the puzzles are associated with a story involving social interactions. See Chapter 7 of Robin Dunbar's *The Trouble with Science*, which explains experiments and ideas from Cosmides & Toobey (1993).

Instructor's Notes for Bayes Rule at the Bar

The instructor picks seven students for this classroom game. Six are given signs big enough to be read from the back of the classroom (8x11 cardstock is fine). 2 students are Aggressive Killers, 1 is a Reasonable Killer, 2 are Reasonable Bullies and 1 is an Aggressive Bully. The seventh is given no sign.

Before class, randomize using pieces of paper to select an Aggressive Killer with probability 2/3 or an Aggressive Bully with probability 1/3 to be the Rascal. This is so later if some smart student says he was trying to predict instructor psychology, you can tell him that was clever to try, but didn't work this time.

Tell the chosen Rascal that he should sit up front, and put the other five near him to represent the bar scene. When you spill a drink on him, he is to say to you,"Lick my shoes or get ready to run, bozo!" Make sure it is ok with this student that you spill some water on him.

Tell the seventh student, the Friend, to sit in the back of the classroom. When you give him a signal, he is to stumble drunkenly up to the front, ask the wet rascal what happened, and on finding out, tell the instructor, "Guys who mess with my friend don't deserve to live."

Bring a cup with a little water in it to class. If you prefer, you may have a student be the one who spills water on the rascal, rather than yourself, but that reduces your control of the situation a little.

Perform the first part of the drama, explaining who the six bar customers are, and spilling the water on the Rascal. Then tell the students to write down their estimates of the probability the Rascal is a Killer, together with their own identity, and pass them in. The students with labels can do it too, if they like– though they have more information, of course, and the Rascal knows exactly who he is.

You'll may see 0, 1/3, 1/2, 2/3, or 1 as estimates.

I suggest starting discussion by saying, "One estimate I have here is 1/3. Could whoever made that estimate explain his reasoning?" It will probably be "There is a 2/3 chance of a Killer being aggressive, and a 1/2 chance of

a Killer, and 2/3 times 1/2 is 1/3." You can answer that by saying, "Well, then what is the probability the Rascal is a Bully?"

Then ask about the 1/2 estimate, and teh reasoning behind it. If nobody picked 1/2, go to 2/3 first, but then return to 1/2 and ask why it isn't right, since there are equal numbers of Bullies and Killers in the bar.

Then ask about the 2/3 estimate, and the reasoning behind it.

Then ask about 0 or 1. One of these people is presumably the Rascal himself. Others might be either entirely confused or trying to outsmart the instructor by thinking how he would want to rig the game. To those who are trying to outsmart you, say that you randomized before the game expecting exactly that kind of guess. Say that it is a good guess, though, because Life is a Game, and if you weren't a game theory instructor, the student could probably have outguessed you.

(1) Then explain the 2/3 answer yourself. Start by having the six bar customers stand up and display their signs. Then say that you are only interested in the Aggressive ones, and have them stand together. Then point out that there are 2 Killers and 1 Bully, so if you spill a drink and the response is Aggressive, the probability that the person is a Killer is 2/3. (Of course, the students will discover the identity of the Rascal in the drama this way, so the probability of that person has changed to 0 or 1.)

(2) Next, do a picture version, as in Chapter 2 of Games and Information. Show that there is 1 unit of probability, split into 1/2 Killers and 1/2 Bullies, drawing two shorter lines for those. Then draw lines of length 1/6 and 2/6 for Reasonable and Aggressive Killers, and lines of length 2/6 and 1/6 for the Bullies. Then show that the proportion of the Aggressive lines that are Killers is 2/6 to 1/6, which is 2/3. Here, I should add a graph to this file.

(3) Third, do the formula for Bayes Rule:

$$Prob(Killer|Aggressive) = \frac{Prob(Killer)Prob(Aggressive|Killer)}{Prob(Aggressive)} \quad (1)$$

Explain that this is the number of Aggressive Killers, 2, which is Prob(Aggressive|Killer) Prob (Killer), divided by the number of Aggressive people in total, 3, which

is Prob(Aggressive).

Explain that Prob (Aggressive), in turn, is gotten by selecting out all the Aggressives from both types:

$$Prob(Aggressive|Killer)Prob(Killer)+Prob(Aggressive|Bully)Prob(Bully) \quad (2)$$

So now we think there is a 2/3 probability we drew a Killer.

It is worth discussing the paradox that actually the probability is either 0 or 1, not 2/3. The rascal is either a killer or he isn't. The resolution to the paradox is that for game theory we are usually interested in the subjective probabilities, and here we are interested in my best guess, not what is actually true but I can't know.

The Second Part of the Drama: The Friend

A friend of the drunk comes in the door. He is just like his friend– either a killer or a bully. He is aggressive to you. Do we think he is a killer?

Again have the students write down their estimates. This time, though, tell them to start by **not** using the formula, even though that is the easiest way. But tell them that they can draw pictures if they want.

You can discuss their various answers.

This is a much harder problem. The diagram approach works well. First draw the line of 1 unit of probability. Then draw the 2/3 probability line for Killers and 1/3 for Bullies. We can do that, because we already had decided in the first part that there was a 2/3 chance the first rascal was a bully.

Then draw lines for 2/3 of the 2/3 line, the 4/9 probability of Aggressive Killers, and lines for the 1/3 of the 1/3 line, the 1/9 probability of Aggressive Bullies. The ratio is 4/9 to 1/9 for Killers, given Aggressive behavior, so your posterior is 4/5. Here, I should add a graph to this file.

More simply: the probability of coming across two aggressive Killers is (2/3) (2/3) = 4/9, and the probability of coming across two aggressive bullies is (1/3)(1/3) = 1/9. That is a ratio of 4 to 1, or 4/5.

Then do the same thing plugging numbers into Bayes Rule. The difference is that now we start with a prior of a 2/3 probability of a Killer, not 1/2, and that also increases the $Prob(Aggressive)$.

$$Prob(Killer|Aggressive) = \frac{Prob(Killer)Prob(Aggressive|Killer)}{Prob(Aggressive)} = \frac{(2/3)(2/3)}{(2/3)(2/3) + (1/3)(1/3)} \tag{3}$$

This will take about 40 minutes.