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The Repeated Prisoner's Dilemma Under Incomplete Information: A Classroom Game for Chapter 6

Consider the Prisoner's Dilemma in Table 3, obtained by adding 8 to each payoff in Chapter 1's Table 2, and identical to Chapter 5's Table 10:

Table 3: The Prisoner's Dilemma

		Column		
		Silence		*Blame*
	Silence	7,7	→	-2, 8
Row		↓		↓
	Blame	8,-2	→	**0,0**

Payoffs to: (Row,Column)

This game will be repeated five times, and your objective is to get as high a summed, undiscounted, payoff as possible (*not* just to get a higher summed payoff than anybody else). Remember, too, that there are lots of pairing of Row and Column in the class, so to just beat your immediate opponent would not even be the right tournament strategy.

The instructor will form groups of three students each to represent *Row*, and groups of one student each to represent *Column*. Each *Row* group will play against multiple *Columns*.

The five-repetition games will be different in how *Column* behaves.

Game (i) Complete Information: Column will seek to maximize his payoff according to Table 3.

Game (ii) 80% Tit-for-Tat: With 20% probability, Column will seek to maximize his payoff according to Table 3. With 80% probability, Column is a "Tit-for-Tat Player" and must use the strategy of "Tit-for-Tat," starting with *Silence* in Round 1 and after that imitating what Row did in the previous round.

Game (iii) 10% Tit-for-Tat: With 90% probability, Column will seek to maximize his payoff according to Table 3. With 10% probability, Column is a "Tit-for-Tat Player" and must use the strategy of "Tit-for-Tat," starting with *Silence* in Round 1 and after that imitating what Row did in the previous round. The identities of the Game (ii).

The probabilities are independent, so although in Game(ii) the most likely outcome is that 8 of 10 Column players use tit-for-tat, it is possible that 7 or 9 do, or even (improbably) 0 or 10.

Instructor's Notes

This is very like the classroom game for Chapter 5, except that now (a) there are only 5 rounds, and (b) in the second and third plays, there is some probability that *Column* must use the tit-for-tat strategy.

Here are some assignments for player types. N represents a normal player and T represents tit-for-tat.

Game (ii): (T T T N T) (T N T N T) (8 of 10 exactly)

Game (iii): (N N N N N) (N N N N N) (0 of 10–that can happen with a 10% probability–in fact it has probability xxx of happening)

The Equilibrium

Let the probability that Column is a Tit-for-Tat player be θ. In the fifth round, Row will certainly choose Blame, and so will Column if Column is a Normal player. How much cooperation can we get up until the fifth round? There are two kinds of equilibrium outcomes, depending on θ.

Cooperation: High Probability θ of a TFT Column
Row: SSSSB
TFT Column SSSSS
Normal Column: SSSBB

Defection: Low Probability θ of a TFT Column
Row: BBBBB
TFT Column SBBBB
Normal Column: BBBBB

Let us see for what values of θ the first kind of equilibrium exists. Row's expected payoffs from various strategies would be:

$$\pi(SSSSB) = \theta(7+7+7+7+8) + (1-\theta)(7+7+7-2+0) = \theta(36) + (1-\theta)(19) \tag{1}$$

$$\pi(SSSBB) = \theta(7+7+7+8+0) + (1-\theta)(7+7+7+0+0) = \theta(29) + (1-\theta)(21) \tag{2}$$

$$\pi(SSBBB) = \theta(7+7+8+0+0) + (1-\theta)(7+7+8+0+0) = \theta(22) + (1-\theta)(22) \tag{3}$$

Thus, if θ is high, SSSSB is Row's best response, but if θ is low, it is not. $\pi(SSSSB) > \pi(SSSBB)$ if

$$\theta(36) + (1-\theta)(19) > \theta(29) + (1-\theta)(21), \tag{4}$$

that is, if $36\theta + 19 - 19\theta > 29\theta + 21 - 21\theta$ or $9\theta > 2$ or $\theta > 2/9$.

For even lower values of θ, though, Row thinks cooperation is likely enough to break down that he wants to pre-emptively break it, so at least he can get one period of the high (Blame, Silence) payoff, even though if Column is TFT he has lost one extra period of cooperation. $\pi(SSSBB) < \pi(SSBBB)$ if

$$\pi(SSSBB) = \theta(29) + (1-\theta)(21) < \pi(SSBBB) = \theta(22) + (1-\theta)(22) \tag{5}$$

so $29\theta + 21 + -21\theta < \pi(SSBBB) = 22$ or $\theta < 1/8$.

To summarize: the Cooperation equilibrium above will be an equilibrium only if $\theta \geq 2/9$, and otherwise Row would deviate to one or another strategy of early Blame.

What happens if $\theta < 2/9$? The strategy profile (SSSBB, SSSBB) is not an equilibrium either. There cannot be an equilibrium in which the Row and Normal Column players first chooses Blame in the same period (except for the first period). Either one would then deviate to choosing Blame one period earlier. But there cannot be an equilibrium in which the Normal Column player chooses Blame later than Row. And we have just ruled out, by making θ small, situations in which Row does not choose Blame till after the Normal Column. Thus, it must be that in equilibrium Row chooses Blame starting in the first period.

Name:

Scoresheet for "The Repeated Prisoner's Dilemma Under Incomplete Information: A Classroom Game for Chapter 6"

The instructor will assign each pairing, Row and Column, a pairing letter (A,B, C....)

Write the pairing letter and name of each player or group after "Row:" and "Column:" Each Row group will have one scoresheet for each of its members, so it will be able to keep track of its rounds against its three Column opponents.

For each round, write the per-round payoff, either -2, 0, 7, or 8. Add them up for the total.

Game (i): Complete Information

Round:	1	2	3	4	5	Total
Row:						
Column:						

Game (ii): 80% Chance of Tit-for-Tat

Round:	1	2	3	4	5	Total
Row:						
Column:						

Game (iii): 10% Chance of Tit-for-Tat

Round:	1	2	3	4	5	Total
Row:						
Column:						

GAME (i): COMPLETE INFORMATION

Pairing:	A	B	C	D	E	F	G	H	I
Row's Name:									
Row's Payoff									

Pairing:	A	B	C	D	E	F	G	H	I
Column's Name:									
Column's Payoff:									

Pairing:	J	K	L	M	N	O	P	Q	R
Row's Name:									
Row's Payoff:									

Pairing:	J	K	L	M	N	O	P	Q	R
Col's Name:									
Col's Payoff:									

GAME (ii): 80% TIT-FOR-TAT

Pairing:	A	B	C	D	E	F	G	H	I
Row's Name:									
Row's Payoff									

Pairing:	A	B	C	D	E	F	G	H	I
Column's Name:									
Column's Payoff:									

Pairing:	J	K	L	M	N	O	P	Q	R
Row's Name:									
Row's Payoff:									

Pairing:	J	K	L	M	N	O	P	Q	R
Col's Name:									
Col's Payoff:									

GAME (iii): 10% TIT-FOR-TAT

Pairing:	A	B	C	D	E	F	G	H	I
Row's Name:									
Row's Payoff									

Pairing:	A	B	C	D	E	F	G	H
Column's Name:								
Column's Payoff:								

Pairing:	J	K	L	M	N	O	P	Q	R
Row's Name:									
Row's Payoff:									

Pairing:	J	K	L	M	N	O	P	Q	R
Col's Name:									
Col's Payoff:									