3 October 2005. Revised June 25, 2022. Eric Rasmusen, Erasmuse@indiana.edu.
Http://www.rasmusen.org. http://www.rasmusen.org/GI/games/07_mhgame.pdf
This shows the idea of the game, but I never finished finding the optimal behaviors under the various contracts. I hope it may inspire others to do a better job, at least.

Moral Hazard: A Classroom Game for Chapter 7

Each student works as a salesman for Apex, Brydox, or neither firm, choosing anew each year. Each year you also pick your effort level, which is unobserved by the firms. Your sales equal

$$Q = 2 + e + u,$$

where u takes the values -2 and $+2$ with equal probability.

Your payoff is 600 if you work for neither firm and otherwise is a function of your wage and effort:

$$\pi = V(w) - e^2, \tag{1}$$

where $V(w)$ is shown in the following table:

w	<0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	≥ 14
$V(w)$	0	0	100	190	370	540	690	830	930	1000	1020	1030	1038	1044	1046	1046

You will have limited time to make your choices of contract and effort. If you do not hand in a card with your choice by the deadline, your default choices are to work for neither firm (if you fail to choose an employer) and $e = 0$ (if you don't choose your effort).

A **linear contract** takes the form,

$$w(Q) = \alpha + \beta Q, \tag{2}$$

where if the value of w from the equation is not an integer it is rounded up.

A **threshold contract** takes the form,

$$w(Q) = \alpha \; if \; Q \geq \beta; \;\; w = 0 \text{ otherwise} \tag{3}$$

A **monitoring contract** takes the form

$$w(Q) = \alpha \text{ unless you are caught with } e < \beta; \;\; w = 0 \text{ otherwise; probability of monitoring} = \gamma \tag{4}$$

Instructor's Notes

Bring a timer and notecards, and copies of the scoresheet to hand out.

Give each student a notecard.

In each year, write the Apex and Brydox contracts on the blackboard. Then give the students a fixed amount of time (using a buzzer) to hand in a card with their choices.

Then give them another fixed amount of time to choose an effort level.

Then flip a coin. Heads means the uncertainty variable u is +2, Tails means u is -2.

Then give them some time to calculate their payoffs, and go on to the next round.

You may choose contracts from the list below, or use your own.

(xxx in each case, what is the optimal effort level for each contract, and which contract yields the higher utility?)

Before you do Round 1, write the contracts up, using the overhead below that has blanks for the parameters, and fill in an entry in the scoresheet so the students understand what is going on.

Before each round, write the contract up on the board or on that overhead (erasing the old parameter numbers).

Round 1:
Apex, Linear: $\alpha = 6, \beta = 0$.
Brydox, Linear: $\alpha = 0, \beta = 1$.

A: $e = 0, \pi = 830 - 0 = 830$
B*: $e = 8, \pi = 958$
N: $\pi = 600$.

What happens in Round 1 is that under Apex's contract, the salesman can get output of 0 or 4, with a guaranteed wage of 6. Zero effort is optimal, because higher output doesn't give him any more pay. Under Brydox's contract, he keeps all the output. Compare his different payoffs: If $e = 7$, then $w = 7, 11$ and the payoff is $.5(930) + .5(1038) - 49 = 935$.
If $e = 8$, then $w = 8, 12$ and the payoff is $.5(1000) + .5(1044) - 64 = 958$.
If $e = 9$, then $w = 9, 13$ and the payoff is $.5(1020) + .5(1046) - 81 = 952$.

I did not finish in finding the optimal behavior for the remaining cases.

Round 2:
Apex, Linear: $\alpha = 4, \beta = 0$.
Brydox, Linear: $\alpha = -3, \beta = 2$.

A: $e = 0, \pi = 540 - 0 = 540$

B: $e =$ $, \pi =$
N: $\pi = 600$.

Round 3:
Apex, Threshold: $\alpha = 4, \beta = 6$.
Brydox, Threshold: $\alpha = 8, \beta = 8$.

A: $e =$ $, \pi =$
B: $e =$ $, \pi =$
N: $\pi = 600$.

Round 4:
Apex, Threshold: $\alpha = 6, \beta = 4$.
Brydox, Threshold: $\alpha = 12, \beta = 5$.

A: $e =$ $, \pi =$
B: $e =$ $, \pi =$
N: $\pi = 600$.

Round 5:
Apex, Monitoring: $\alpha = 8, \beta = 8, \gamma = .5$.
Brydox, Monitoring: $\alpha = 8, \beta = 9, \gamma = .1$.

A: $e =$ $, \pi =$
B: $e =$ $, \pi =$
N: $\pi = 600$.

Round 6:
Apex, Monitoring: $\alpha = 6, \beta = 7, \gamma = .5$.
Brydox, Monitoring: $\alpha = 8, \beta = 9, \gamma = 1$.

A: $e =$ $, \pi =$
B: $e =$ $, \pi =$
N: $\pi = 600$.

Round 7:
Apex, Monitoring: $\alpha = 6, \beta = 7, \gamma = .5$.
Brydox, Linear: $\alpha = -3, \beta = 2$.

A: $e =$ $, \pi =$
B: $e =$ $, \pi =$
N: $\pi = 600$.

Round 8:

Apex, Monitoring: $\alpha = 6, \beta = 7, \gamma = .5$.
Brydox, Threshold: $\alpha = 8, \beta = 8$.

A: $e =$ $\quad, \pi =$
B: $e =$ $\quad, \pi =$
N: $\pi = 600$.

You can then try combinations of these 12 contracts– a Linear contract vs. a Monitoring contract, and so forth.

Leave enough time at the end to ask the students to form into groups and figure out the efficient effort level (a) if $u = 0$ instead of being -2 or $+2$, and (b) if u takes the values -2 and $+2$ as in the game they just played. After they tell you, give them the answer:

No risk (u=0)
If $e = 6$, then $w = e + 2 + 0 = 8$ and the payoff is $V(w) - e^2 = 1000 - 36 = 964$.
If $e = 7$, then $w = e + 2 + 0 = 9$ and the payoff is $V(w) - e^2 = 1020 - 49 = 971$.
If $e = 8$, then $w = e + 2 + 0 = 10$ and the payoff is $V(w) - e^2 = 1030 - 64 = 966$.

Going from $e = 7$ to $e = 8$, the marginal cost of the effort is 15, but the marginal benefit of the output is 10. So $e^* = 7$.

Risk: (u = -2, +2)
Now the wage is either $e - 2$ or $e + 2$, with equal probability, if the salesman gets the entire return from his effort and the firm does not insure him. (If the firm can insure him, then effort should be 7 with a flat wage of $w = 9$).

If $e = 6$, then $w = 6, 10$ and the payoff is $EV(w) - e^2 = .5(830) + .5(1030) - 36 = 894$.
If $e = 7$, then $w = 7, 11$ and the payoff is $.5(930) + .5(1038) - 49 = 935$.
If $e = 8$, then $w = 8, 12$ and the payoff is $.5(1000) + .5(1044) - 64 = 958$.
If $e = 9$, then $w = 9, 13$ and the payoff is $.5(1020) + .5(1046) - 81 = 952$.

Going from $e = 7$ to $e = 8$, the marginal cost of the effort is 15 (=64-49), but the marginal benefit of the output is now 38, since the expected utility derived from output rises from 984 to 1022.

Going from $e = 8$ to $e = 9$, the marginal cost of the effort is 17 (= 81-64), but the marginal benefit of the output is just 11, since the expected utility derived from the output rises from 1022 to 1033. So $e^* = 8$.

Risk makes the efficient effort higher, so as to avoid getting a low outcome by chance. This is not like investing more in a risky project. Rather, by working harder, risk is *reduced*, because the marginal utility of wealth is made more equal across good and bad states. So working hard is like buying insurance against a bad state– that is the motivation, since the work is, ex post, mostly wasted if it is a good state because the marginal utility of money is then so small.

Then, if there is time. ask them to find a linear contract that elicits the efficient effort level.

$$\begin{aligned} Utility &= V(w(Q)) - e^2 \\ &= V(\alpha + \beta Q) - e^2 \\ &= V(\alpha + \beta[2 + e + u]) - e^2 \\ &= .5V(\alpha + \beta e) + .5V(\alpha + \beta[e + 4]) - e^2 \end{aligned} \tag{5}$$

xxx Find this optimal contract.

Extra rounds are provided on the scoresheet, so you can go up to 12 rounds if you want.

Lessons

(1) It is hard to quickly figure out which contract is best. That is one reason why employers like to use simple contracts. If they used complex contracts, employees are scared off. See Rasmusen (2001) on contract-reading costs.

(2) Risk can end up increasing optimal effort, rather than reducing it.

(3) There are various contracts that can achieve the first-best. A threshold contract or monitoring contract is simplest. Monitoring contracts have the disadvantage that monitoring is costly. What is the disadvantage of threshold contracts? (discussion question– answer: they create high risk when production is uncertain).

(4) When you don't have an equation for utility, and have to use a table, figuring out what is optimal becomes a much cruder process, because you can't use calculus. The principle of marginalism remains the same,though.

Scoresheet for "Moral Hazard: A Classroom Game for Chapter 7"

Name:

Year	Employer (A, B, None)	Contract	Effort e	Uncert. u	Output $Q = 2 + e + u$	Wage w	$V(w)$	Payoff
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
TOTAL	—	—	—	—	—	—	—	

$LINEAR :; \qquad w(Q) = \qquad + \qquad Q$

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$$
\begin{aligned}
THRESHOLD : \qquad w(Q) &= \qquad if\ Q \geq \\
&= 0 \qquad \text{otherwise}
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$$
\begin{aligned}
MON. : \qquad w(Q) &= \\
&\text{unless caught with } e < \\
&= 0 \qquad \text{otherwise} \\
&\text{monitoring prob.} =
\end{aligned}
$$

$$
\begin{aligned}
MON. : \qquad w(Q) &= \\
&\text{unless caught with } e < \\
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$$