

## Beliefs about Beliefs

In class we talked about how to model these. Suppose the agent might be either a High or a Low ability type. The game could be like this:

### Game I

0. Nature chooses the agent to be High with probability 0.7 and Low with probability 0.3, observed by the agent but not by the principal.

1. The principal offers the agent a flat wage.
2. The agent accepts or rejects.

Payoffs: A High agent has reservation utility of 10, if he rejects the contract, and output of 40 and a Low agent has a reservation utility of 5 and output of 0. If an agent accepts, his utility is his wage. The principal's utility is the agent's output minus his wage.

Figure 1 illustrates the game.

In equilibrium, the principal will offer a wage of 10, a pooling equilibrium because both types of Agents do the same thing, Accept. The principal's payoff is then  $-10 + .7(40) = 28$ .

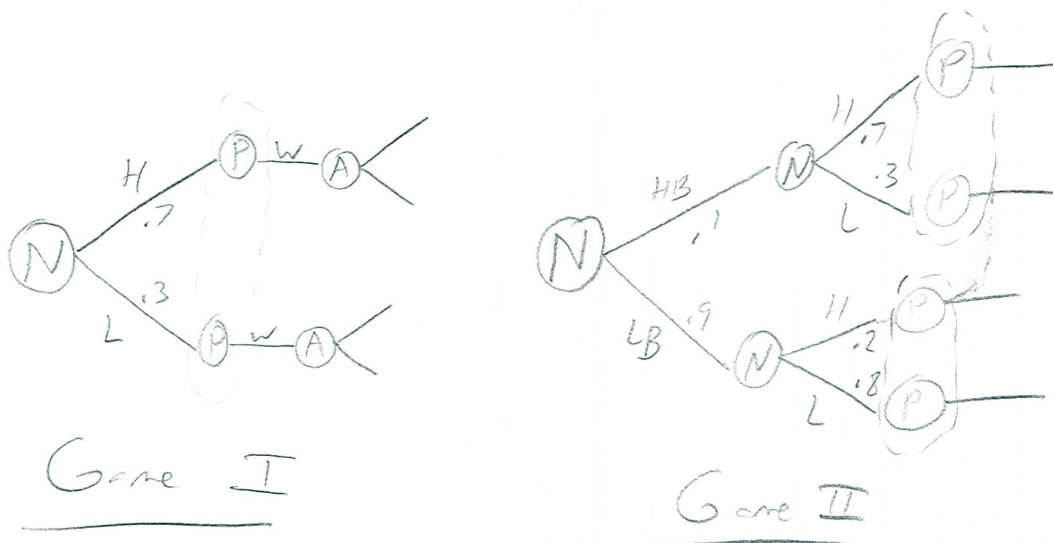


Figure 1: Information in Games I and II (III is Similar to II)

## Game II

-1. Nature chooses the Principal to be either High-Believing, with probability .1, or Low-Believing, with probability .9, observed by the Principal but not the Agent.

0a. If the Principal is High-Believing, Nature chooses the agent to be High with probability 0.7 and Low with probability 0.3, observed by the agent but not by the principal.

0b. If the Principal is Low-Believing, Nature chooses the agent to be High with probability 0.2 and Low with probability 0.8, observed by the agent but not by the principal.

1. The principal offers the agent a flat wage.
2. The agent accepts or rejects.

Payoffs: A High agent has reservation utility of 10, if he rejects the contract, and output of 40 and a Low agent has a reservation utility of 5 and output of 0. If an agent accepts, his utility is his wage. The principal's utility is the agent's output minus his wage.

In equilibrium, the Principal offers a wage of 10 if he is high-believing—as found in Game I. If he is low-believing, though, a wage of 10 would yield a payoff of  $-10 + .2(40) = -2$ , so he will offer a wage of less than 5 and hire neither type of agent.

The agent's lack of knowledge of the Principal's type does not matter in Game II, but I present the game to illustrate how you would model lack of knowledge.

## Game III

-1. Nature chooses the Principal to be either High-Believing, with probability .1, or Low-Believing, with probability .9, observed by the Principal but not the Agent.

0a. If the Principal is High-Believing, Nature chooses the agent to be High with probability 0.7 and Low with probability 0.3, observed by the agent but not by the principal.

0b. If the Principal is Low-Believing, Nature chooses the agent to be High with probability 0.2 and Low with probability 0.8, observed by the agent but not by the principal.

1. The Agent offers to work for the Principal for a flat wage.
2. The Principal accepts or rejects.

Payoffs: A High agent has reservation utility of 10, if he rejects the contract, and output of 40 and a Low agent has a reservation utility of 5 and output of 0. If an agent accepts, his utility is his wage. The principal's utility is the agent's output minus his wage.

This is a more difficult game. The High-Believing Principal would pay up to  $.7(40) + .30(0) = 28$  in a pooling equilibrium. The Low-Believing Principal would pay up to  $.2(40) = 8$ .

Low Agent: Offer to work for 28.

High Agent: Offer to work for 28.

Low-believing Principal: Accept only if the wage is less than 5.

High-believing Principal: Accept only if the wage is less than 28.

Out of equilibrium belief: Passive conjectures. If the agent offers to work for anything other than 28, the principal keeps his initial prior beliefs.

Here is another equilibrium:

Low Agent: Offer to work for 28.

High Agent: Offer to work for 50.

Low-believing Principal: Never accept.

High-believing Principal: Never accept.

Out of equilibrium belief: If the agent offers to work for anything other than 28 or 50 the principal believes the agent is Low with probability 1.