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Quasilinearity

This elaborates a little on the discussion in Chapter 7 of *Games and Information.* Here is what Chapter 7 says in Sectin 7.2 around page 202:

Quasilinearity and Alternative Functional Forms for the Production Game

Consider the following three functional forms for utility:

$$\begin{aligned} U(e,w) &= log(w) - e^2 && (a) \\ U(e,w) &= w - e^2 && (b) \\ U(e,w) &= log(w - e^2) && (c) \end{aligned} \tag{1}$$

Utility function (a) is what we just used in Production Game I. Utility function (b) is an example of **quasilinear preferences**, because utility is separable in one good— money, here— and linear in that good. This kind of utility function is commonly used to avoid wealth effects that would otherwise occur in the interactions among the various goods in the utility function. Separability means that giving an agent a higher wage does not, for example, increase his marginal disutility of effort. Linearity means furthermore that giving an agent a higher wage does not change his tradeoff between money and effort, his marginal rate of substitution, as it would in function (a), where a richer agent is less willing to accept money for higher effort. In effort-wage diagrams, quasilinearity implies that the indifference curves are parallel along the effort axis (which they are *not* in Figure 2).

Quasilinear utility functions most often are chosen to look like (b), but my colleague Michael Rauh points out that what quasilinearity really requires is just linearity in the special good (w here) for some monotonic transformation of the utility function. Utility function (c) is a logarithmic transformation of (b), which is a monotonic transformation, so it too is quasilinear. That is because marginal rates of substitution, which is what matter here, are a feature of general utility functions, not the Von Neumann-Morgenstern functions we typically use. Thus, utility function (c) is also a quasi-linear function, because it is just a monotonic function of (b). This is worth keeping in mind because utility function (c) is concave in w, so it represents a risk-averse agent.

New Comments

Let's take some derivatives to help understand quasilinearity. First, utility function (a), which is

$$U(e,w) = log(w) - e^2 \tag{2}$$

$$\frac{\partial U}{\partial w} = 1/w \tag{3}$$

$$\frac{\partial U}{\partial e} = -2e \tag{4}$$

$$\frac{\partial U/\partial w}{\partial U/\partial e} = -\left(\frac{1}{2ew}\right) \tag{5}$$

Thus, the marginal rate of tradeoff between w and e depends on both e and w. That makes things complicated, because wealth matters to choices.

Let's try a monotonic transform of the utility function:

$$U(e, w) = exp^{log(w)-e^2} = w * exp^{-e^2} \tag{6}$$

Does this change the tradeoff?

$$\frac{\partial U}{\partial w} = exp^{-e^2} \tag{7}$$

$$\frac{\partial U}{\partial e} = -2ew * exp^{-e^2} \tag{8}$$

$$\frac{\partial U/\partial w}{\partial U/\partial e} = -\left(\frac{1}{2ew}\right) \tag{9}$$

Thus, a monotonic change in the utility function does not change the tradeoff between effort and money. That's what we should expect, since utility's units of measurements are arbitrary for static problems. If we were looking at questions of behavior under risk, we'd have to use von-Neumann Morgenstern utlity, where the cardinal values do matter. But risk tradeoffs are different from the effort-money tradeoff.

Now let's go on to quasilinear utility function (b):

$$U(e, w) = w - e^2 \tag{10}$$

$$\frac{\partial U}{\partial w} = 1 \tag{11}$$

$$\frac{\partial U}{\partial e} = -2e \tag{12}$$

$$\frac{\partial U/\partial w}{\partial U/\partial e} = -\left(\frac{1}{2e}\right) \tag{13}$$

The marginal rate of tradeoff between w and e depends only on e, not on w. Wealth does not matter to choice of effort.

Now let's go on to quasilinear utility function (c), which is a monotonic transform of utility function (b)

$$U(e, w) = log(w - e^2) \tag{14}$$

$$\frac{\partial U}{\partial w} = \left(\frac{1}{w - e^2}\right) \tag{15}$$

$$\frac{\partial U}{\partial e} = -2e\left(\frac{1}{w - e^2}\right) \tag{16}$$

$$\frac{\partial U/\partial w}{\partial U/\partial e} = -\left(\frac{1}{2e}\right) \tag{17}$$

As expected, it behaves the same function (b) the marginal rate of tradeoff between w and e depends only on e, not on w, and is exactly the same as in function (b). Wealth does not matter to choice of effort.

Note, however, that someone with utility function (b) is risk-neutral, but someone with utility function (c) is risk-averse. Thus, for looking at money gambles, or situations with uncertainty over pay, (b) and (c) are not equivalent.