ODD

Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

PROBLEMS FOR CHAPTER 1

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This appendix contains answers to the odd-numbered problems in the gourth edition of *Games and Information* by Eric Rasmusen, which I am working on now and perhaps will come out in 2005. The answers to the even- numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I must ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

PROBLEMS FOR CHAPTER 1

1.1. Nash and Iterated Dominance (medium)

(a) Show that every iterated dominance equilibrium s^* is Nash.

<u>*Answer.*</u> Suppose that s^* is not Nash. This means that there existS some i and s_i' such that i could profitably deviate, i.e., $\pi_i(s^*) < \pi_i(s_i', s_{-i}^*)$. But that means that there is no point during the iterated deletion at which player i could have eliminated strategy s_i' as being even weakly dominated for him by s_i^*. Hence, iterated deletion could not possibly reach s^* and we have a contradiction; it must be that every iterated dominance equilibrium is Nash.

(b) Show by counterexample that not every Nash equilibrium can be generated by iterated dominance.

<u>*Answer.*</u> In Ranked Coordination (Table 7 of Chapter 1) no strategy can be eliminated by dominance, and the boldfaced strategies are Nash.

(c) Is every iterated dominance equilibrium made up of strategies that are not weakly dominated?

<u>*Answer.*</u> Yes. As defined in Chapter 1, strategy x is weakly dominated by strategy y only if y has a strictly higher payoff in some strategy profile and has a strictly lower payoff in no strategy profile. An iterated dominance equilibrium only exists if the iterative process results in a single strategy profile at the end.

In order for x to be in the final surviving profile, it would have to weakly dominate the second-to-last surviving strategy for that player (call it x_2). Thus, it is strictly better than x_2 as a response to some profile of strategies of the other players: $\pi_i(x, s_{-i}) > \pi_i(x_2, s_{-i})$ for some particular s_{-i} that has survived deletion so far. But for x_2 to have survived deletion so far means that x_2 must be at least a good a response to the profile s_{-i} as the third-to-last surviving strategy: $\pi_i(x_2, s_{-i}) \geq \pi_i(x_3, s_{-i})$, and in turn none of the earlier deleted x_i strategies could have done strictly better as a response to s_i or they would not have been weakly dominated. Thus, x must be a strictly better response in at least one strategy profile than all the previously deleted strategies for that player, and it cannot have been weakly dominated by any of them.

If we define things a bit differently, we would get different answers to this question. Consider this:

(i) Define "quasi-weakly dominates" as a strategy that is no worse than any other strategy.

A strategy that is in the equilibrium strategy profile might be a bad reply to some strategies that iterated deletion of quasi-dominated strategies removed from the original game. Consider the Iteration Path Game in Table A1.1. The strategy profile (r_1, c_1) is a iterated quasi-dominance equilibrium. Delete r_2, which is weakly dominated by r_1 (0,0,0 is beaten by 2,1,1). Then delete c_3, which is now quasi-weakly dominated by c_1 (12,11 is equal to 12,11). Then delete r_3, which is weakly dominated by r_1 (0,1 is beaten by 2,1). Then delete c_2, which is strongly dominated by c_1 (10 is beaten by 12).

		Column		
		c_1	c_2	c_3
	r_1	**2,12**	1,10	1,12
Row:	r_2	0,7	0,10	0,12
	r_3	0,11	1,10	0,11

Payoffs to: (Row, Column)

Table A1.1: The Iteration Path Game

1.3. Pareto Dominance (medium) (from notes by Jong-Shin Wei)

(a) If a strategy profile s^* is a dominant strategy equilibrium, does that mean it weakly pareto-dominates all other strategy profiles?

Answer. No— think of The Prisoner's Dilemma in Table 1 of Chapter 1. (*Confess, Confess*) is a dominant strategy equilibrium, but it does not weakly pareto-dominate (*Deny, Deny*)

(b) If a strategy profile s strongly pareto-dominates all other strategy profiles, does that mean it is a dominant strategy equilibrium?

Answer. No— think of Ranked Coordination in Table 7 of Chapter 1. (*Large, Large*) strongly pareto- dominates all other strategy profiles, but is not a dominant strategy equilibrium.

The Prisoner's Dilemma is not a good example for this problem, because *(Deny, Deny)* does not pareto-dominate *(Deny, Confess).*

(c) If s weakly pareto-dominates all other strategy profiles, then must it be a Nash equilibrium?

Answer. Yes. (i) If and only if s is weakly pareto-dominant, then $\pi_i(s) \geq \pi_i(s'), \forall s', \forall i$. (ii) If and only if s is Nash, $\pi_i(s) \geq \pi_i(s'_i, s_{-i}), \forall s'_i, \forall i$. Since $\{s'_i, s_{-i}\}$ is a subset of $\{s'\}$, if s satisfies condition (i) to be weakly pareto-dominant, it must also satisfy condition (ii) and be a Nash equilibrium.

1.5. Drawing Outcome Matrices (easy)

It can be surprisingly difficult to look at a game using new notation. In this exercise, redraw the outcome matrix in a different form than in the main text. In each case, read the description of the game and draw the outcome matrix as instructed. You will learn more if you do this from the description, without looking at the conventional outcome matrix.

(a) The Battle of the Sexes (Table 7). Put (*Prize Fight, Prize Fight*) in the northwest corner, but make the woman the row player.

<u>Answer.</u> See Table A1.2.

Table A1.2: Rearranged Battle of the Sexes I

		Man		
		Prize Fight		*Ballet*
	Prize Fight	**1,2**	←	- 5,- 5
Woman:		↑		↓
	Ballet	-1,-1	→	**2, 1**

Payoffs to: (Woman, Man).

(b) The Prisoner's Dilemma (Table 2). Put (*Confess, Confess*) in the northwest corner.

<u>Answer.</u> See Table A1.3.

Table A1.3: Rearranged Prisoner's Dilemma

		Column		
		Confess		*Deny*
	Confess	**-8,-8**	←	0,-10
Row:		↑		↑
	Deny	-10,0	←	-1,-1

Payoffs to: (Row, Column).

(c) The Battle of the Sexes (Table 7). Make the man the row player, but put (*Ballet, Prize Fight*) in the northwest corner.

<u>Answer.</u> See Table A1.4.

Table A1.4: Rearranged Battle of the Sexes II

		Woman		
		Prize Fight		*Ballet*
	Ballet	-5,-5	→	**1,2**
Man:		↓		↑
	Prize Fight	**2,1**	←	- 1,- 1

Payoffs to: (Man, Woman).

1.7. Finding More Nash Equilibria

Find the Nash equilibria of the game illustrated in Table 12. Can any of them be reached by iterated dominance?

Table 12: Flavor and Texture

		Brydox	
		Flavor	*Texture*
	Flavor	-2,0	0,1
Apex:			
	Texture	-1,-1	0,-2

Payoffs to: (Apex, Brydox).

Answer. $(Flavor, Texture)$ and $(Texture, Flavor)$ are Nash equilibria. $(Texture, Flavor)$ can be reached by iterated dominance.

1.9. Choosing Computers (easy)
The problem of deciding whether to adopt IBM or HP computers by two offices in a company is most like which game that we have seen? *Answer.* The Battle of the Sexes or Pure Coordination, depending on whether the offices differ in their preferences.

1.11. A Sequential Prisoner's Dilemma (hard)
Suppose Row moves first, then Column, in the Prisoner's Dilemma. What are the possible actions? What are the possible strategies? Construct a normal form, showing the relationship between strategy profiles and payoffs.

Hint: The normal form is *not* a two-by-two matrix here.

Answer. The possible actions are $Confess$ and $Deny$ for each player.

For Column, the strategy set is:

$$\left\{ \begin{array}{l} (C|C,\ C|D), \\ (C|C,\ D|D), \\ (D|C,\ D|D), \\ (D|C,\ C|D) \end{array} \right\}$$

For Row, the strategy set is simply $\{C, D\}$.

The normal form is:

		Column			
		$(C\|C,\ C\|D)$	$(C\|C,\ D\|D)$	$(D\|C,\ D\|D)$	$(D\|C,\ C\|D)$
Row	*Deny*	-10,0	-1, -1	-1, -1	-10,0
	Confess	-8,-8	-8, -8	0, -10	0, -10

Payoffs to: (Row, Column)

The question did not ask for the Nash equilibrium, but it is disappointing not to know it after all that work, so here it is:

Equilibrium	**Strategies**	**Outcome**
E_1	{*Confess, (C|C) (C|D)*}	Both pick $Confess$.