G406: Business and Government

EXTRA PROBLEMS (Ch 5, 6)

CHAPTER 5: TIME AND LIFE

5.1 The EPA is considering a regulation that will require \$200 million in immediate costs and cost \$20 million per year in maintenance, but will save 3 lives per year, valued at \$10 million each. Ignoring OMB, they are using a discount rate of 5.1 percent. What is the present discounted value of the regulation?

The present value is** $-200 + \frac{30-20}{r}$**, which is a little below** $-200 + \frac{10}{.05}$**, which equals zero, so the value is slightly negative.

5.2 Someone is wondering what \$100/year in dividends would be worth if the risk is the same as that of large-company stocks from 1926 to 2007. What discount rate would you use, and what is the answer?

Use the real (not nominal) discount rate on large-company stocks from 1926 to 2007. This is like a perpetuity of \$100/year at a nominal discount rate of 9.2%, so it would be worth 100/.092, about \$1090. The reason to use the real rate rather than the nominal rate is that we want to know what the flow is worth in present-day dollars.

5.3 A company can comply with a new OSHA workplace safety rule by installing safety features equal to 30% of the cost of each new structure it builds or via worker training program that is estimated to cost 4% of the structure's cost each year.

(a) What discount rate would make both options equally attractive? To solve the equation you come up with, go to the free web program Wolfram Alpha at http://www.wolframalpha.com/. It has a box saying "Enter what you want to calculate or know about:" where you can enter your equation like this:
$32 + rX == (1/r)^(23)(1/x)$,
where the right-hand side means 1/r to the 23rd power times 1/x—which is ***not*** the correct equation, just an example. You need to use double equals signs to means "happens to equal" here as in many computer languages because a single equals sign means "is assigned the value of". The solution will be a bit down the webpage under "Real solutions:".

(b) Suppose an alternative is a more intensive training program that will cost 5% of the structure's cost each year for 10 years but can be discontinued at that point because the safe behavior will have become habitual. What discount rate would make the two training options equally attractive?

(a) The two options would be equally attractive if

$$.3C = \frac{.04C}{r}$$

This is solved at a discount rate r of .04/.3, which is about 13.33.

(b) *Let C be the building cost. The two options are equal if*

$$.3C = \sum_{t=1}^{10} \frac{.05C}{(1+r)^t}$$

where r is the discount rate.

This can also be expressed as

$$.3C = \left(\frac{.05C}{r}\right)\left(1 - \left(\frac{1}{1+r}\right)^{10}\right).$$

The discount rate r that solves this is about .16.

5.4 A company has to choose between two ways of complying with a pollution regulation. One way costs 5 million dollars immediately to construct a new building and then 2 million dollars at the end of the second year and each succeeding year. The other way costs nothing immediately, but costs 30 million dollars two years from today. Set up equations to show to show how the company should calculate which method to use. You do not need to solve the equations.

The first way has a present value cost of

$$5 + \frac{1}{1+r}\left(\frac{2}{r}\right)$$

The perpetual stream of payments is only discounted one year, because the perpetuity formula gives the value today of a payment one year from today, two years, three years, and so forth. Here, the stream starts in two years, so it is like an end-of-year perpetuity security worth 2/r that you receive at the end of 1 year.

The second way has a present value of

$$\left(\frac{1}{1+r}\right)^2(30)$$

The company should use the second method if

$$5+\frac{1}{1+r}\left(\frac{2}{r}\right) > \left(\frac{1}{1+r}\right)^2(30)$$

5.5 The government is considering two projects. One is a new computer that will save the Social Security Administration \$10 million/year. The other is a new sanitation regulation for chicken slaughter that will save \$10 million per year in health costs from salmonella outbreaks. Both have a one-time cost of \$180 million. All these amounts are in current dollars, adjusted for inflation. The nominal rate on treasury bonds is 4%. Explain why one, both, or neither project should be implemented.

Both regulations have present values of $10/r - 180$, but they should use different values of r. The new computer is supposed to save the government money, so it is a cost-effectiveness calculation and should use the government's cost of capital. That cost is 4% in nominal terms, but since all of our other numbers are constant-dollar, we need to subtract expected inflation. Inflation of 3.1% is a reasonable guess based on 20th-century inflation. That results in a discount rate of 0.9% and a present value of 1,111-180, which is positive. For the chicken regulation, it should be 7%, as OMB directs. The present value is 143-180, which is negative. So only the computer project should be undertaken.

5.6 The directors of Apex Corporation are worried about what will happen if the CEO, John Doe, dies suddenly. One director suggests they buy "key man insurance", which would pay the company a lump sum if he died. The directors expect Doe to retire in 15 years, and they estimate that his death would cost the company an immediate 5 million dollars in disruption plus 1 million per year from not having his special abilities. The company is currently borrowing at an interest rate of 10%. One director thinks that the inflation rate ought to enter their decision somehow.

(a) If zero inflation is expected, how much insurance should they buy?

(b) How would inflation matter for how much insurance they should buy?

(a) The company wants to cover the present value of John Doe to the company plus the disruption cost. The present value can be seen as a perpetual value of $1 million per year minus a perpetual value of $1 million that has to be given up 15 years from now, so it is:

$$5 + \frac{1}{.10}\left(1 - (\frac{1}{1+.10})^{15}\right) = 5 + 10(1 - .76) = 8.4$$

Thus, it should buy $8.4 million in coverage.

(b) The value of $8.4 million will diminish over time because of inflation, so by 15 years from now, their insurance coverage would be too low. Thus, they will want to buy more insurance, or make a deal with the insurance company to have the amount rise over time.

5.7 How does the forensic approach to the value of life differ from the value-of-a-statistical-life approach?

Under the forensic approach, the analyst estimates the present value of future earnings of the person. Under the statistical life approach he sees how much the person is willing to spend to avoid small risks and uses the probability of those small risks to scale up to the value of a life.

5.8 A study finds that 2 million people are paying in aggregate $12 million for the motorcycle warning light that several manufacturers offer that cut the number of deaths from accidents in that group from 7 to 4 for the lifetime of the motorcycle. What is the value of a statistical life for these people?

They are paying $12 million for a reduction of 3 deaths, so they value the warning light at least at $4 million, but if they valued it more, they would still only have to pay $4 million since manufacturers compete down the price and allow them some consumer surplus. Since there is either a light or not, it is not like looking at a continuous job or product characteristic, where people are buying a quantity where the price equals the marginal benefit of the last unit they buy.

5.9 The Bush Administration of 2001-2009 was accused of devaluing human life because it reduced the value of a statistical life used in pollution regulations. How could an honest "better economic study" cause the Environmental Protection Agency to reduce the value of a statistical life?

An honest study could cause the EPA to reduce the value of a statistical life is if a hedonic regression was run on different data than the original study and it happened to show that the wage premium for a dangerous job was lower than that estimated from the data in the original study. When doing empirical estimation, the data can change, and even if it does not, the same data can be interpreted different ways, to a certain extent. You can't honestly say the result is any number you want, but if two datasets show different results, honest men can disagree as to which is more reliable.

5.10 Explain why if every person in a group of 1,000 people had a value of a statistical life of 6.9 million dollars, it would make sense to use that finding for pollution regulations even though if you offered any one of them 6.9 million dollars in exchange for certainty of death, we know they all would refuse.

Someone might rationally accept $6,900 to do something that had a 1 in 1,000 risk of death but refuse $6.9 million to die for sure. Suppose he does not care about his heirs. Then he might want be willing to accept the $6,900 because he can spend it if he lives, but he will think that if he's dead the $6.9 million will be useless to him. Thus, it is quite rational to accept different prices per 1% chances of death for different levels of risk.

5.50 There are 5,000 fishermen working in various jobs on a fishing boat that vary in their dangerousness. Regression analysis shows that their wages can be predicted by the following equation:

wage = 24,000 + 1,000*Experience - 800*past injury + 500*deathrate

Experience is measured in years working in fishing. Past injury equals 1 if the worker had already been injured some time in the past and 0 if he was not. Death rate is in deaths per 10,000 workers per year. The government is thinking of investing $4.3 million per year in weather prediction technology that would reduces deaths by 1 per year. Should it do so? Note that not all the numbers here are needed to find the answer.

What is relevant here is how much a worker requires in extra pay for extra risk of death. The equation tells us that a fisherman

requires $500 extra pay for a 1/10,000 chance of death per year. Thus, his statistical value of life is $500 (10,000) = $5 million. Since the weather prediction only costs $4.3 million to save one life per year, it is worthwhile. Note that the other numbers in the question do not matter to the answer; the point was just to make you look for which numbers were relevant.

5.51 Under what circumstances would a company prefer immediate cash equal to 30% of its building costs instead of a tax break equal to 5% of its cost each year for 20 years? (you do not need to solve out for the values of the variables in the equation that is the answer)
Let C be the building cost. The cash is better if $.3C > \Sigma_{t=1}^{20} \frac{.05C}{(1+r)^t}$ ***where r is the discount rate. Or, you can use the perpetuity trick like this, which will equal the same number:***

$.3C > (.05C/r)(1 - 1/(1+r)^{20})$.

CHAPTER 6: EXTERNALITIES

6.1 Suppose the externality cost of sulfur dioxide pollution from steel production is \$2/ton, the supply curve is $P = 2 + Q/2$ if $P > 2$ and Q=0 for $P < 2$, and the demand curve is $P = 34 - Q$. What steel output level maximizes surplus?

The supply equation can be written as P= 2+ Q/2. The externality cost adds 2 to that, so marginal social cost is MSC= 4+ Q/2. The demand equation can be written as P = 34-Q. Equating the two yields 4+Q/2 = 34-Q, so 3Q/2 = 30 and Q=20.

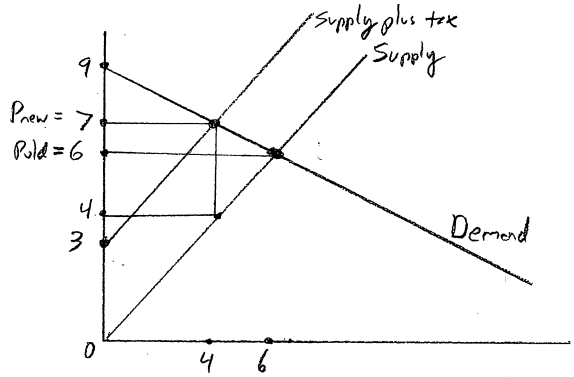
6.2 The supply curve for vodka is $Q = P$ and the demand curve is $Q = 18 - 2P$. Vodka has a negative externality of 3 per unit bought, and the government is thinking about its optimal policy.

(a) What is the producer surplus, consumer surplus, and third-party loss in equilibrium?

(b) Suppose the government imposes a tax on vodka, paid by the sellers, to maximize surplus. How much will the price rise, how much revenue will the government raise, and how much will total welfare rise?

(a) The equilibrium has $P = 18 - 2P$***, so*** $P = 6$***. In that case,*** $Q = 6$ ***from the supply curve. The consumer surplus is .5 (9-6)6 = 9. The producer surplus is .5 (6)(6) =18. The externality cost is 6(3) = 18.***

(b) The tax should equal the externality amount, 3. With the tax, the supply curve becomes $Q = 0$ *if the price is less than 3,* $Q = (P - 3)$ *otherwise. Thus, the new equilibrium is at* $P - 3 = 18 - 2P$*, so* $3P = 21$ *and **P** = 7. The quantity according to the demand curve is* $18 - 2P = 4$*. Producer surplus is .5 (7- 3) (4)= 8. The consumer surplus is .5 (9 - 7) 4= 4. The externality cost is 4(3) = 12. Tax revenue is* $3Q$*, which equals 12.*



6.3 Suppose the supply curve is $P^s = 2Q^s$, the demand curve is $P^d = 48 - 3Q_d$, and each transaction creates an externality with marginal cost $X = Q$.

(a) What is the equilibrium quantity without regulation?

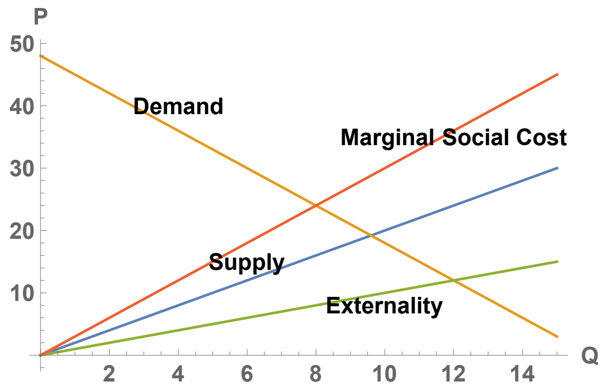
(b) Draw a diagram to show the shape of the supply curve, the marginal externality level, the marginal social cost, and the demand curve.

(c) What quantity yields the greatest possible social surplus?

(d) What pollution tax size would yield the greatest possible social surplus?

(a) Setting price on the demand curve to price on the supply curve, $2Q = 48 - 3Q$ *so* $Q = 48/5 = 9.6$.

(b)



(c) The marginal social cost is $MSC = 3Q$*. Equating this to the price on the demand curve yields* $3Q = 48 - 3Q$ *so* $Q = 8$*.*

(d) The pollution tax needs to be equal to the size of the externality when $Q = 8$*, so it should equal 8.*

6.4 Read "The SuperFreakonomics Global-Warming Fact Quiz."

(a) Why does Professor Levitt say that we should use climate engineering instead of a carbon tax?

(b) Levitt omits a major reason why global engineering is a more practical solution than carbon taxes. What is that? Discuss.

(c) Would a higher discount rate make Levitt's solution for global warming more attractive or less attractive relative to cap-and-trade? Discuss.

(a) He has two reasons. (1) A carbon tax will reduce emissions, but that will only delay warming, not stop it. Warming will continue for many years, and eventually reach the same high temperatures it would without the carbon tax. (2) Reducing emissions is very expensive—over a trillion dollars per year—whereas geoengineering is extremely cheap by comparison. Thus, if the goal is to maximize net benefit or reduce net cost, geoengineering is superior.

(b) He omits the difficulty of implementation because of the international free rider problem. Carbon taxes must be imposed worldwide, or some countries will keep on increasing their emissions. In fact, the non-taxing countries will increase emissions all the faster, because their industries will be advantaged and will export carbon-heavy goods to the high-tax countries. The non-taxing countries will see no need to tax if other countries are doing the taxing, and

no country can benefit by unilaterally imposing a carbon tax. In addition, some countries will prefer global warming to a carbon tax even if a carbon tax could be imposed worldwide—India and Russia, for example. Geoengineering, on the other hand, can be done unilaterally, and it is cheap enough that one medium-sized country could find it profitable to do it even if it had to bear the entire cost itself.

He also omits the advantage that geoengineering can be tried on a smaller scale, and later, so that if warming turns out not to be such a problem, large costs have not been incurred.

(c) With a high discount rate, geoengineering is more attractive because its cost could be delayed till many years from now, whereas to significantly delay global warming a cap-and-trade policy would have to start in the next fifty years. On the other hand, if discount rates are very low, cap-and-trade has the problem that without exorbitant cost it doesn't seem to be a permanent solution, and if the discount rate is low, the costs of warming further out than 100 years would matter more than with a high discount rate.

6.5 The market will supply zero to 10 units of cough syrup at a price of 8, 10 to 20 units at a price of 11, and any number of units at a price of 15. Cough syrup is sometimes bought by meth dealers to use as a raw ingredient, and this has a marginal externality equal to $2Q$ when Q units of cough syrup are sold. The demand curve for cough syrup, bought for legal or illegal uses, is $24 - Q$.

(a) Graph supply, demand, and the marginal externality.

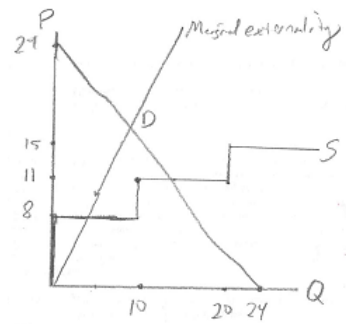
(b) What is the equilibrium price and quantity?

(c) What is the equilibrium total surplus?

(d) What quantity would maximize social surplus, and what would the total (not marginal) amount of externality cost be at that level of cough syrup?

(e) What tax on cough syrup would be optimal?

(a) The graph is:



(b) If P =11, the quantity demanded will be 13, and sellers are willing to supply that quantity at that price, so that is the equilibrium.

(c) At a price of 11 and quantity of 13, consumer surplus is .5(24-11)(13) = .5(169)= 84.5. Producer surplus is 10(11-8) + 3(11-11) = 30. The externality is the area under the externality curve from 0 to 5.33, which is .5(13)(26)=169. Another way to think of it is that since the marginal externality is $2Q$ *and it is the derivative of the total externality, the total externality must be* Q^2*, which is* 169 *(That idea is the same as integrating the marginal externality function— probably the only time you'd take an integral in undergraduate economics.) Thus, total surplus is 84.5+ 30- 169= -54.5.*

(d) The marginal social cost is $2Q + 8$ *for quantities less than 10,* $2Q + 11$ *for quantities between 10 and 20, and*$2Q + 15$ *for greater quantities. We want this to equal the price from the demand curve,* $P = 24 - Q.24 - Q = 2Q + 8$ *solves to* $16 = 3Q, Q = 16/3 \approx 5.33$*, which is less than 10, so it is in the appropriate region of **Q** for **2Q+8**. At* $Q = 5.33$*, the total dollar amount of externality is the area under the externality curve from* $Q = 0$ *to* $Q = 5.33$*, which is .5(16/3)(32/3) or* $256/9 \approx 28.4$*. If we tried setting* $2Q + 11 = 24 - Q$*, we would get* $3Q = 13$*, so* $Q \approx 4.33$*, which is inappropriate for* $2Q + 11$ *since that only applies for* Q *between 10 and 20.*

(e) Set the tax equal to the marginal cost of the externality at the optimal quantity of 5.33. The marginal cost of the externality is $2Q$*, so the tax should equal* $32/3 \approx 10.66$*.*

6.6 Contract law in Nowhereistan says that if the buyer breaches a contract by refusing to accept delivery of the goods, he must not only compensate the seller for lost profits, but be publicly flogged by the seller. Transactions costs are zero, so the Coase Theorem applies. What will be the effect of the flogging privilege on contract language, the amount of non-acceptance of goods, and social surplus?

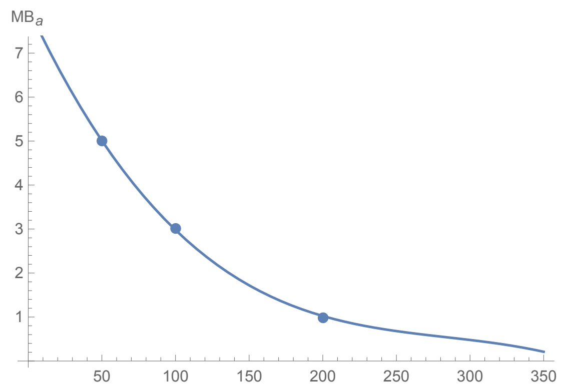
If there are no transaction costs, each time people make a contract they can easily include a clause to override the law and waive the right to flog. Good contract law matters because it reduces transaction costs. Or, if contracts do not include that clause, then the price for the goods will be lower, and when the buyer wants to refuse delivery, he will negotiate to pay money to the seller in place of the flogging, except perhaps for the occasional Shylock who likes flogging better than money (but a rational buyer wouldn't make a contract with such a seller). In the end, the amount of non-acceptance will be the same as without the flogging rule, so social surplus will be the same too.

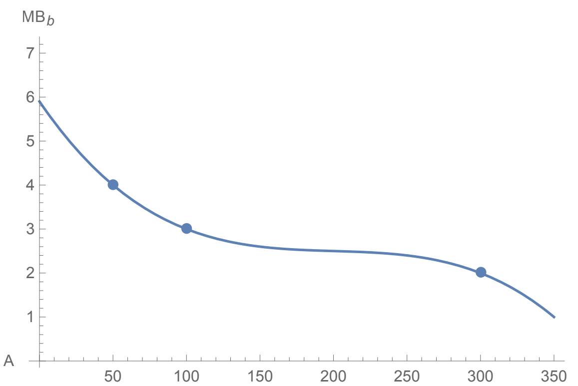
6.7 The Bigmountain Basin Pollution Authority currently is served by two power companies. Apex Power emits 200 tons of sulfur dioxide per year and Brydox Electric emits 300 tons. It would cost Apex 1 thousand dollars to reduce its emissions by one ton, and it would cost Brydox 2 thousand dollars. It would cost Apex 3 thousand dollars to reduce emissions by one ton if it had already reduced its emissions to 100 tons, and 5 thousand if it was down to 50 tons. It would cost Brydox 3 thousand dollars per ton to reduce emissions by one ton if it were down to 100 tons, and 4 thousand if it were down to 50 tons.

(a) Draw a pair of graphs to show the marginal benefit of emissions for Apex and Brydox.

(b) The Pollution Authority decides to make emissions allowances tradeable. As best as you can tell, how much will each company pollute in equilibrium, and what quantity will be traded at what price?

(a) You only have 3 points to work with, so there are various shapes the curves could have, but they're downward sloping.





(b) The initial situation is not an equilibrium, because Apex's marginal benefit is less than Brydox's. Thus, we know Apex would be willing to sell some allowances to Brydox at a price above $1 and below $2. We know that if Apex sold 100 allowances, its marginal benefit would be $3 and Brydox's would be less than $3 (because Brydox's marginal benefit only is as high as $3 if it has 100 or fewer allowance). Thus, we also know that less than 100 allowances will be sold. As Apex sells allowances to Brydox, Brydox's marginal benefit will fall below $2 and Apex's will rise above $1. Thus, the equilibrium price will be somewhere between those values.

6.8 Zoning regulations control what landowners can do with their property. A city zoning rule might say, for instance, that in a particular neighborhood you cannot operate a business or build a house on a lot less than 1/2 acre in size. What externality argument is used to justify zoning? Construct a numerical example to show how a zoning rule against businesses could increase property values in a neighborhood even if each homeowner would prefer that he be allowed to run a business from his home.

The argument for zoning is that restricting one person's right to use his property can avoid negative externalities for everyone else. The person who opens a gas station in a residential neighborhood, for example, would make much more from his land than if he build a house on it. The neighborhood would become less attractive for homes, though. If the gas station is placed next to other businesses, on the other hand, it creates less of a negative externality and might even create a positive externality if another business finds the car traffic bring him more customers.

As a numerical example, suppose that each of 100 residential homes is worth $200,000 if they are all used as homes, but the value drops to $100,000 if one or more is torn down and replaced with a gas station. If one person turns his home into a gas station, his gas station is worth $300,000, but if two more people do that, each gas station is only worth $90,000. With zoning, the total value of the properties is 100 $200,000 = $20 million. Without zoning, the first person to build a gas station will have $300,000, and each of the others would only have $90,000 if he, too, built a gas station, so he would keep his house. The total value would only be $300,000 + 99*$100,000 = $10.2 million. Thus, surplus is maximized by forbidding gas stations.*

You could also construct an example in which building gas stations made every *property owner worse off, because when there are more than one gas station, each is worth $110,000. In that case, everybody would build a gas station and the total value would be $11 million.*

Note that if the Coase Theorem applied, the neighbors would all sign a contract not to turn their property into a gas station, but with this many people, bargaining costs would be high.

6.9 Suppose that if we do not reduce the amount of carbon dioxide in the atmosphere by using a carbon tax, the cost will be $1,000 billion/year ($1 trillion) starting 50 years from now, but that the welfare loss from the tax would be $90 billion/year forever starting immediately. All these numbers are in constant dollars, zero inflation.

(a) If the discount rate is 7%, what is the present discounted value of the carbon tax policy? Some computations that might be relevant are: $\frac{90}{.07} \approx 1285$, $\frac{1000}{.07} \approx 14,285$, ...

(b) If you had $90/billion per year income and invested it at a return of 7%, what total sum would you have 50 years from now?

(c) If the discount rate is 1%, what is the present discounted value of the carbon tax policy?

(d) Which discount rate is more appropriate to use?

(e) Suppose that there is a 50% probability that increased carbon dioxide will turn out not to affect the world temperature enough to create clear costs or benefits, and we will discover that in 40 years. By that time, though, we would need to impose a bigger carbon tax, that would create $200 billion/year in triangle losses. Using the 1% discount rate, is it worth waiting?

(a) The value would be $-\frac{90}{.07} + \left(\frac{1}{1.07}\right)^{50} \frac{1000}{.07} \approx -90 * 14.28 + .0339 * 1000 * 14.28 \approx -1285.2 + 484.1 \approx -801.1$ *billion.*

(b) The present value of $90/billion at a 7% discount rate is $1285, computed in part (a). A dollar invested today at 7% would rise to $(1.07)^{50}$ ***in 50 years, which is about 29.46. Thus, the value in 50 years would be $37,843.***

(c) The value would be $-\frac{90}{.01} + \left(\frac{1}{1.01}\right)^{50} \frac{1000}{.01} \approx -90 * 100 + .608 * 1000 * 100 \approx -9000 + 60,800 = 51,800$ *billion.*

(d) The better discount rate to use is 7%. That is about the return on private investment. If we could invest the avoided $90 billion in tax triangle loss at 7%, then part (b) showed that the value would rise to $37,843 billion in 50 years, which is more than the present value of the warming would be at that time (which is $1000*14.28 billion from part (a)). In other words, if we kept investing the $37,843 billion, it would yield more than $1,000 billion/year

(e) Now the net benefit of starting to tax is $-\frac{90}{.01} + .5\left(\frac{1}{1.01}\right)^{50} \frac{1000}{.01} \approx -90 * 100 + (.5).608 * 1000 * 100 \approx -9000 + 30,400 = 21,400$ *billion.*

The net benefit of the bigger tax starting later is $.5\left(-\frac{200}{.01}\left(\frac{1}{1.01}\right)^{40} + \left(\frac{1}{1.01}\right)^{100} \frac{1000}{.01}\right) \approx -.5(20000) * .671 + (.5).608 * 1000 * 100 \approx -6710 + 30,400 = 23,690$ *billion.* ***Thus, the later tax is a better idea.***

6.10 Andrew and Betty have been going out for six months, and like each other, except that Betty really hates how Andrew always wears a baseball cap, and she doesn't even think Andrew likes the cap that much. Betty tells their mutual friend Cathy, and Cathy says, "I just learned in my economics class that the Coase Theorem says the surplus-maximizing outcome is always achieved. What I mean is, since you hate the cap and he doesn't care that much, you two ought to be able to make a deal where you pay him $10 to stop wearing it or agree to let him choose what movies you go to, and you'll both be happier."

Betty admits the logic of how they could both be better off, but she thinks it would be a bad idea to try.

(a) What's wrong with Cathy's idea? How does the Coase Theorem fail here?

(b) Is there another way that Ann could be helpful in curing this inefficient situation?

(a) The problem here is asymmetric information. If Betty suggests this to Andrew, he will think she's weird, since he doesn't know her all that well, and her main concern is not the cap: it's that Andrew think well of her. Betty may also be concerned that if she makes this suggestion, Andrew will take it as a criticism of his personal taste and interpret it as a sign she wants to break up. Or, he might think she thinks he values money a lot, or that her offering money means she doesn't want to accept favors from him, or that she thinks of life in terms of money. If they both knew each other very well, the deal could indeed go through.

(b) If Cathy approaches Andrew and tells him that Betty hates the cap but would never mention it, then what Andrew learns is that Betty has a friend who cares about her, which makes Betty look good. Often an intermediary (say, a lawyer) can make offers that would raise awkward questions coming directly from the offeror.

6.50 Companies A, B, and C emit amounts 10, 20, and 30 of air pollution (amounts E_a, E_b, E_c). The marginal benefit of pollution is $X_a = 20 - 2E_a$ for company A, $X_b = 30 - E_b$ for company B, and $X_c = 30 - E_c$ for company C. Each company is assigned the rights to pollute 10 units. If these rights can be bought and sold, how much will each company end up polluting?

The Coase Theorem says the companies will trade emission rights to get to the surplus-maximizing allocation. We need the marginal benefits to equal each other. Thus, we want $20 - 2E_a = 30 - E_b = 30 - E_c$*. Equating the marginal benefits that way implies that* $E_a = E_b/2 - 5$ *and* $E_c = E_b$*.*

We also know that emissions will total 30, since each company starts with the right to pollute 10, so $E_a + E_b + E_c = 30$*. Thus,* $E_b/2 - 5 + E_b + E_b = 30$*. Then* $2.5E_b = 35$*, so* $E_b = 14$*. As a result,* $E_c = 14$ *and* $E_a = 2$*.*