

Table 7: Battle of the Sexes
Woman

		<i>Prize Fight</i>		<i>Ballet</i>
Man	<i>Prize Fight</i>	2,1	←	0, 0
	<i>Ballet</i>	0, 0	→	1,2

Payoffs to: (Man, Woman).

Let's do Battle of the Sexes with communication. Woman can say Ballet or Fight first. What is the man's strategy set?

(Ballet|ballet, Ballet|fight)

(Ballet|ballet, Fight|fight)

(Fight|ballet, Ballet|fight)

(Fight|ballet, Fight|fight)

What is the woman's strategy set?

Ballet, (Ballet|ballet, Ballet|fight)

Ballet, (Ballet |ballet, Fight|fight)

Ballet, (Fight|ballet, Ballet|fight)

Ballet, (Fight|ballet, Fight|fight)

Fight, (Ballet|ballet, Ballet|fight)

Fight, (Ballet |ballet, Fight|fight)

Fight, (Fight|ballet, Ballet|fight)

Fight, (Fight|ballet, Fight|fight)

We found the Woman's strategy set is:

Ballet, (Ballet|ballet, Ballet|fight)

Ballet, (Ballet |ballet, Fight|fight)

Ballet, (Fight|ballet, Ballet|fight)

Ballet, (Fight|ballet, Fight|fight)

Fight, (Ballet|ballet, Ballet|fight)

Fight, (Ballet |ballet, Fight|fight)

Fight, (Fight|ballet, Ballet|fight)

Fight, (Fight|ballet, Fight|fight)

Notice that the woman's strategies are overly complicated in that they include action plans for situations that are never reached unless the woman herself departs from her strategy. If we drop those, we have a "reduced strategy set":

Ballet, (Ballet|ballet)

Ballet, (Fight|ballet)

Fight, (Ballet|fight)

Fight, (Fight|fight)

There are lots of strategy profiles. How do you find an equilibrium?

Best response function (set-valued). Any best response function must have the players choosing the same equilibrium strategies in the event subgame. The man's strategy must choose what the woman's strategy tells her to do.

Off the equilibrium path, it doesn't matter. That narrows it down.

Let stars denote an equilibrium that leads to going to the ballet:

*Ballet, (Ballet|ballet) Man: Ballet|ballet, X|fight

Ballet, (Fight|ballet) Man: Fight|ballet, X|fight

*Fight, (Ballet|fight) Man: X|ballet, Ballet|fight

Fight, (Fight|fight) Man: X|ballet, Fight|fight

The woman won't want to pick an announcement that leads to Fight, unless the announcement doesn't matter to the man's action.

Thus, wherever strategy profile leads to Fight, we know it can't be an equilibrium unless the man's X is to choose Fight no matter what the announcement is:

*Ballet, (Ballet|ballet) Man: Ballet|ballet, X|fight
Ballet, (Fight|ballet) Man: Fight|ballet, Fight|fight

*Fight, (Ballet|fight) Man: X|ballet, Ballet|fight
Fight, (Fight|fight) Man: Fight|ballet, Fight|fight

These are the Nash equilibria. The ones that lead to Ballet clearly are. The ones that lead to Fight are equilibria because the man's strategy is unconditional, which means the woman's announcement doesn't matter.

This illustrates two features of Cheap Talk:

1. There is always a "babbling equilibrium", where the message is ignored.

2. The content of the message doesn't matter—"Fight" could lead to a Ballet equilibrium just as easily as "Ballet" could.

Or, rather, everything has to do with expectations. If people expect the word "Ballet" to mean we are going to the dance performance, then that is what it will mean.

It is hard to model communication, because it is specific to the context.

Table 8: Ranked Coordination

		Jones	
		<i>Large</i>	<i>Small</i>
Smith	<i>Large</i>	2,2 ←	-1, -1
	<i>Small</i>	-1, -1 →	1,1

Payoffs to: (Smith, Jones). Arrows show how a player can increase his payoff.

In this game, the payoffs in each profile are the same for each player.

Table 9: Dangerous Coordination

		Jones	
		<i>Large</i>	<i>Small</i>
Smith	<i>Large</i>	2,2	← −1000, −1
	<i>Small</i>	−1, −1	→ 1,1

Payoffs to: (Smith, Jones).

(Large, Large) is Nash and Pareto superior (payoff dominant) but not “risk dominant”: given a 50-50 chance of the other player choosing each strategy, each player would choose SMALL, as “safer.”

Stag Hunt (Assurance Game) (not in book)

		Jones		
		<i>Stag</i>		<i>Stag</i>
	<i>Hare</i>	2,2	←	1,0
Smith		↑		↓
	<i>Hare</i>	1,0	→	1,1

Payoffs to: (Smith, Jones).

In the Stag Hunt, each player can assure himself of a payoff of 1 by hunting for a hare, which doesn't require cooperation.

It is crucial whether expectations of which equilibrium is going to be played out match between players. A Nash equilibrium is really a shared set of expectations.

**Table 6: The Modeller's Dilemma
Column**

		<i>Silence</i>		<i>Blame</i>
	<i>Silence</i>	0, 0	\leftrightarrow	-10, 0
Row		\updownarrow		\downarrow
	<i>Blame</i>	0, -10	\rightarrow	-8 , -8

Payoffs to: (Row, Column) .

What are the equilibria?

What would you predict to happen?

Here, Blame Blame is still made up of weakly dominant strategies, and can still be reached by iterated dominance, but it has lower payoffs than the other Nash equilibrium.

Focal Points

1 Circle one of the following numbers: 100, 14, 15, 16, 17, 18.

2 Circle one of the following numbers 7, 100, 13, 261, 99, 666.

3 Name Heads or Tails.

4 Name Tails or Heads.

5 You are to split a pie, and get nothing if your proportions add to more than 100 percent.

6 You are to meet somebody in New York City. When? Where?

Online auction bidding. Two bidders. Ties split the prize.

Bidder 1 has value v_1 . He bids b_1 .

$\pi_1 = 0$ if $b_1 < b_2$.

$\pi_1 = (v_1 - b_2)/2$ if $b_1 = b_2$

$\pi_1 = [v_1 - b_2]$ if $b_1 > b_2$.

Bidder 1's best response function is

$b_1 > b_2$ if $b_2 < v_1$

b_1 takes any value if $b_2 = v_1$

$b_1 < b_2$ if $b_2 > v_1$

Suppose $v_1 = 5$ and you knew that $v_2 = 9$. Then one of the Nash equilibria is $b_1 = 2, b_2 = 9$. Player j would get the prize at a price of 2, rather than 5.

Another equilibrium is $b_1 = 100, b_2 = 2$. If bidder 2 thinks he is going to lose the auction anyway because of bidder 1's crazy bidding, he is willing to bid very low.

A weakly dominant strategy is $b_1 = v_1$.

Bertrand best responses.

N = 2 firms with equal costs c choose price. The customers buy one unit from whoever has the lowest price, splitting their purchases evenly otherwise.

The payoff for firm 1 is

$$\pi_1 = 0 \text{ if } p_1 > p_2$$

$$\pi_1 = (p_1 - c)/2 \text{ if } p_1 = p_2$$

$$\pi_1 = p_1 - c \text{ if } p_1 < p_2$$

Firm 1's best response is

$$p_1 > p_2 \text{ if } p_2 < c,$$

$$p_1 \geq p_2 \text{ if } p_2 = c,$$

$$p_1 = p_2 + \epsilon \text{ if } p_2 > c$$

We can draw the best response functions .

They will intersect only at $p_1 = p_2 = c$.

There is another way to see that this is the unique Nash equilibrium.

First, there is no Nash equilibrium with one price below c , because that firm would capture the market and have negative profits.

Second, if both firms are above $p=c$ then

the highest one can deviate to just below the lower firm and increase its profits.

Third, if just one is above $p=c$, then the $p=c$ firm would deviate to price just below the higher firm.

Something weird: In the Nash equilibrium with $p_1=p_2=c$, both firms are playing weakly dominated strategies. They earn zero profits, whereas a price of $p > c$ would earn either zero profits, or positive profits if the other firm charged an even higher price.

But this is the only Nash equilibrium, so we accept it. It is the only stable outcome.

This is like a first-price auction (not second-price, as above), where both bidders know each other's values and the values are equal. Firm 1 is not saying the least he will charge; he is saying exactly what he will charge.