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Follow the Leader I has three pure strategy Nash equilibria of which only one is reasonable.

Equilibrium	Strategies	Outcome
E_1	{*Large, (Large, Large)*}	**Both pick** $Large$**.**
E_2	{*Large, (Large, Small)*}	**Both pick** $Large$**.**
E_3	{*Small, (Small, Small)*}	**Both pick** $Small$**.**

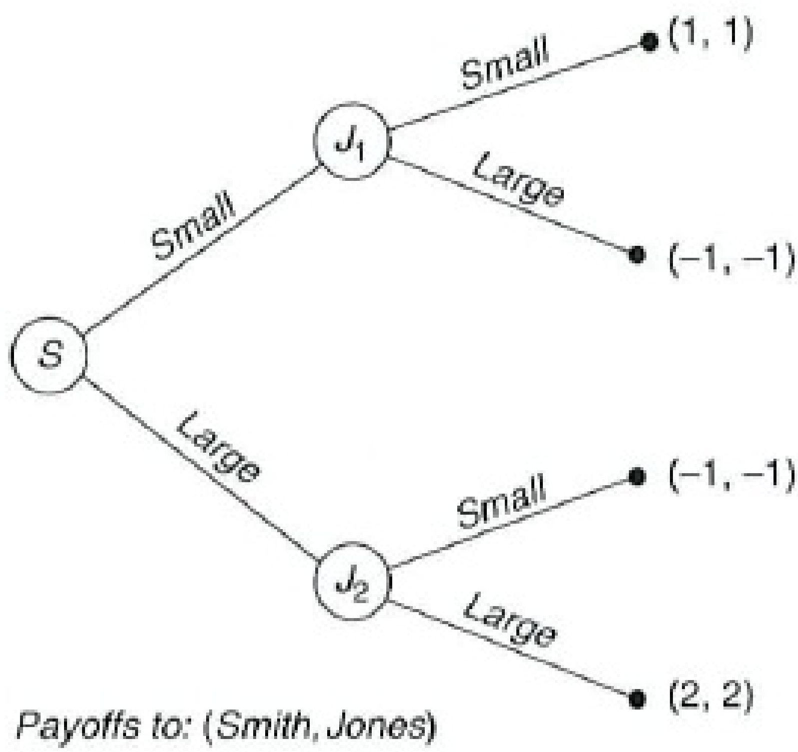


Figure 1: *Follow the Leader I*

A strategy profile is a perfect equilibrium if it remains an equilibrium on all possible paths, including not only the equilibrium path but all the other paths, which branch off into different "subgames."

A **subgame** *is a game consisting of a node which is a singleton in every player's information partition, that node's successors, and the payoffs at the associated end nodes.*

(Note: pedantic people will call this a "proper subgame".)

A strategy profile is a **subgame perfect Nash equilibrium** *if (a) it is a Nash equilibrium for the entire game; and (b) its relevant action rules are a Nash equilibrium for every subgame.*

This is an application of BACKWARDS INDUCTION or SEQUENTIAL RATIONALITY.

Reasons why we use perfect equilibrium

(1) sequential rationality

(2) robustness

On (2): Suppose there is small probability ϵ of a "tremble": a player might pick the wrong move by mistake.

Non-perfect Nash equilibria are all weakly dominated (why?) and so would then disappear.

The tremble approach is NOT equivalent to sequential rationality.

Nash equilibria, all weak:
(Out, Down), *(Out, Up)*, **and** *(In, Up)*.

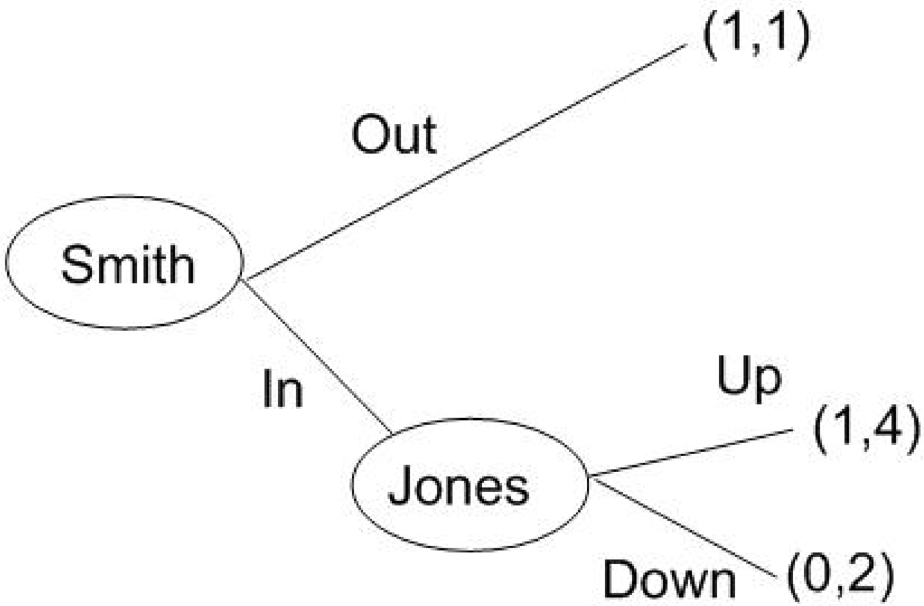


Figure 2: The Tremble Game: Trembling Hand Versus Subgame Perfectness

Think of the basic Bertrand Game too. The only Nash equilibrium is in weakly dominated strategies—picking price to equal marginal cost. The Tremble idea rules this out.

Predatory Pricing

McGee (1958): a price war would hurt the incumbent more than collusion with the entrant.

Entry Deterrence I

Players
Two firms, the entrant and the incumbent.

The Order of Play

1 The entrant decides whether to *Enter* or *Stay Out.*

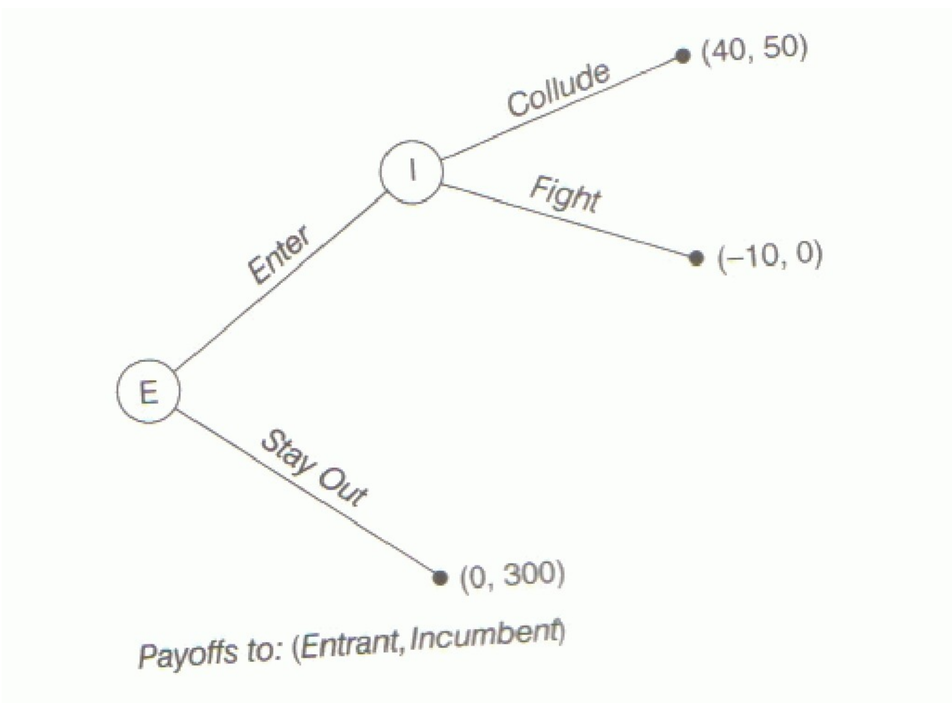
2 If the entrant enters, the incumbent can $Collude$ with him, or $Fight$ by cutting the price drastically.

Payoffs
Market profits are 300 at the monopoly price and 0 at the fighting price. Entry costs are 10. Duopoly competition reduces market revenue to 100, which is split evenly.

Table 1: Entry Deterrence I

		Incumbent		
		Collude		*Fight*
	Enter	**40,50**	$\leftarrow$	$-10, 0$
Entrant:		$\uparrow$		$\downarrow$
	Stay Out	$0, 300$	$\leftrightarrow$	**0,300**

Two Nash equilibria :
(*Enter***,** *Collude***) and (***Stay Out***,** *Fight* **). Perfectness rules out threats that are not credible. (Schelling idea)**



Nuisance Suits I: Simple Extortion

Players

A plaintiff and a defendant.

The Order of Play

1. **The plaintiff decides whether to bring suit against the defendant at cost c.**
2. **The plaintiff makes a take-it- or-leave-it settlement offer of $s > 0$.**
3. **The defendant accepts or rejects the settlement offer.**
4. **If the defendant rejects the offer, the plaintiff decides whether to give up or go to trial at a cost p to himself and d to the defendant.**
5. **If the case goes to trial, the plaintiff wins amount x with probability γ and otherwise wins nothing.**

Payoffs

Figure 4 shows the payoffs. Let $\gamma x < p$, so the plaintiff's expected winnings are less than his marginal cost of going to trial.

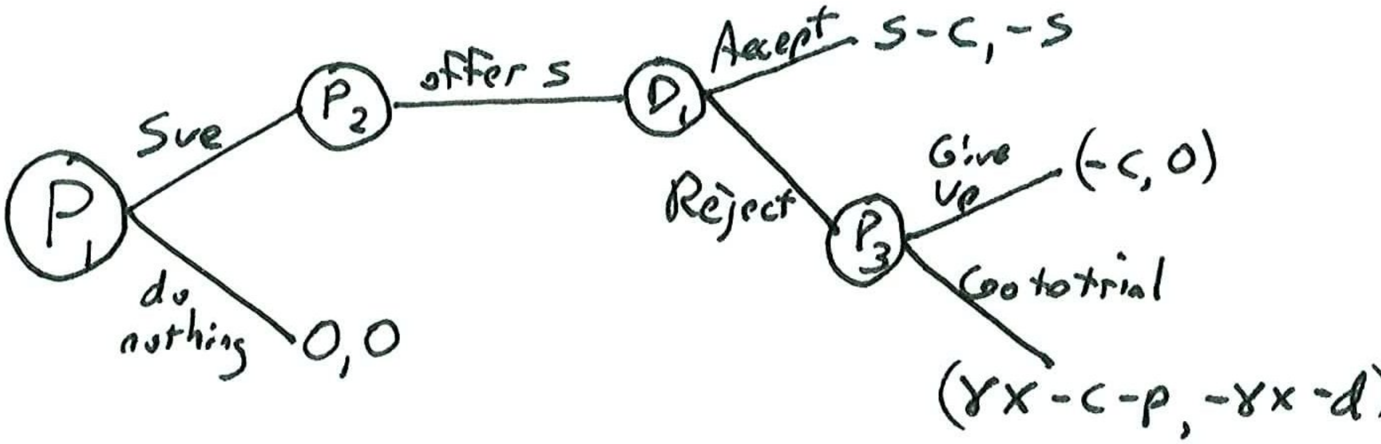


Figure 4 The Extensive Form for Nuisance Suits

The perfect equilibrium is

Plaintiff: *Do nothing*, *Offer* s, *Give up*
Defendant: *Reject*
Outcome: The plaintiff does not bring a suit.

The equilibrium settlement offer s can be any positive amount.

Introducing Risk Aversion

Add a final move by Nature to decide who wins.

γx represented the expected value of the award. If both the defendant and the plaintiff are equally risk averse, γx can still represent the expected payoff from the award— one simply interprets x and 0 as the utility of the cash award and the utility of an award of 0, rather than as the actual cash amounts.

If the defendant is more risk averse, the payoffs from *Go to trial* would change to $(-c - p + \gamma x, -\gamma x - y - d)$, where y represents the extra disutility of risk to the defendant.

This, however, makes no difference to the equilibrium.

Nuisance Suits II Using Sunk Costs Strategically

Now change the order of moves. The plaintiff pays his lawyer the amount p in advance.

This inability to obtain a refund actually helps the plaintiff, by changing the payoffs from the game so his payoff from *Give up* is $-c-p$, compared to $-c-p+\gamma x$ from *Go to trial*. Having sunk the legal costs, he will go to trial if $\gamma x > 0$.

This, in turn, means that the plaintiff would only prefer settlement to trial if $s > \gamma x$. The defendant would prefer settlement to trial if $s < \gamma x + d$, so there is a positive settlement range of $[\gamma x, \gamma x + d]$ within which both players are willing to settle.

Here, allowing the plaintiff to make a take-it-or-leave-it offer means $s = \gamma x + d$ in equilibrium, and if $\gamma x + d > p + c$, the nuisance suit will be brought even though $\gamma x < p + c$.Thus, the plaintiff is bringing the suit only because he can extort d, the amount of the defendant's legal costs.

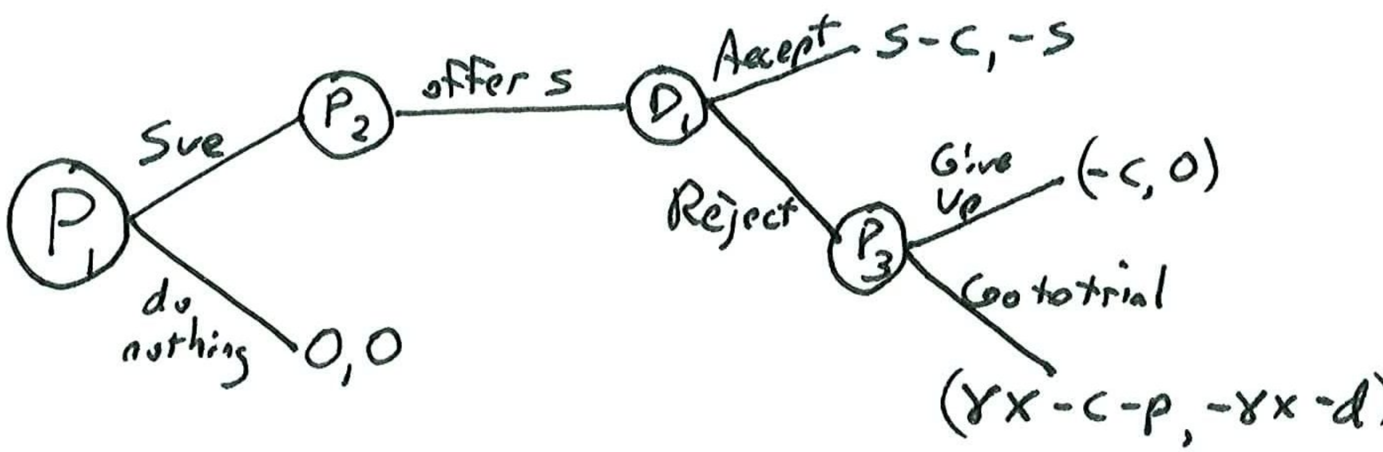
If

$$-c - p + \gamma x + d \geq 0 \quad (1)$$

then the perfect equilibrium is :

Plaintiff: *Sue*, *Offer* $s = \gamma x + d$, *Go to trial*
Defendant: *Accept* $s \leq \gamma x + d$
Outcome: Plaintiff sues and offers to settle, to which the defendant agrees.

Change the payoffs below appropriately to see how this works:



The Open-Set Problem

The equilibrium in Nuisance Suits II is only a weak Nash equilibrium. The plaintiff proposes $s = \gamma x + d$, and the defendant has the same payoff from accepting or rejecting, but in equilibrium the defendant accepts the offer with probability one, despite his indifference.

Shouldn't the plaintiff propose a slightly lower settlement to give the defendant a strong incentive to accept it and avoid the risk of having to go to trial?

Why would the plaintiff risk holdi out for 60 when he might be rejected and receive 0 at trial, when he could offer 59 and give the defendant a strong incentive to accept?

(1) No other equilibrium exists besides $s = 60$.

(2) The objection's premise is false because the plaintiff bears no risk whatsoever in offering $s = 60$

(3) The problem is an artifact of using a model with a continuous strategy space

MORE ON: (3) The problem is an artifact of using a model with a continuous strategy space

Assume that s can only take values in multiples of 0.01, so it could be 59.0, 59.01, 59.02, and so forth, but not 59.001 or 59.002.

The settlement part of the game will now have two perfect equilibria. In the strong equilibrium E1, $s = 59.99$ and the defendant accepts any offer $s < 60$.

In the weak equilibrium E2, $s = 60$ and the defendant accepts any offer $s \leq 60$.

An alternative version of the lawsuits game in which going to trial is credible

(Nuisance Suits II arrives at a different bargaining solution because in it, p is a sunk cost by the time of the settlement negotiation.)

If plaintiff and defendant go to trial,

$$\pi_{plaintiff} = -c + \gamma x - p, \pi_{defendant} = -\gamma x - d$$

Otherwise,

$$\pi_{plaintiff} = -c + s$$

Not going to trial, plaintiff has already saved p. So he gets a further $\frac{d-p}{2}$ if $s = \gamma x + \frac{d-p}{2}$.

$$\pi_{pl}(settle) = -c + \gamma x + \frac{d-p}{2}, \pi_{def}(settle) = -\gamma x - \frac{d-p}{2},$$

By settling, the plaintiff's payoff has risen by $p + \frac{d-p}{2} = \frac{d+p}{2}$.

By settling, the defendant's payoff has risen by $d - \frac{d-p}{2} = \frac{d+p}{2}$.

More systematically, if we want to split the surplus equally we want

$$\pi_{pl}(settle) - \pi_{pl}(trial) = \pi_d(settle) - \pi_d(trial)$$

$$(-c + s) - (-c + \gamma x - p) = -s - (-\gamma x - d)$$

so

$$s - \gamma x + p = -s + \gamma x + d$$

so

$$2s = -s + 2\gamma x + d - p$$

$$s = \gamma x + \frac{d-p}{2}.$$

The Ultimatum Game

1. Smith proposes how to share \$10. He offers Jones share x.

2. Jones accepts or rejects.

If Jones accepts, the payoffs are $(10 - x)$ for Smith and x for Jones. If Jones rejects, the payoffs are 0 for both players.

There are many non-perfect Nash equilibria. The unique perfect equilibrium is ($x = 0$, Jones accepts any offer of $x \geq 0$).

But experiments show that Jones would not follow this strategy. Why?

Nuisance Suits III: Malice

How would we model the idea that the plaintiff dislikes the defendant?

Nuisance Suits III: let $\gamma = 0.1$, $c = 3$, $p = 14$, $d = 50$, **and** $x = 100$, **and the plaintiff receives additional utility of 0.1 times the defendant's disutility.**

Let the settlement s **be in the middle of the settlement range.**

The payoffs conditional on suit being brought are

$$\pi_{plaintiff}(Defendant\ accepts) = s - c + 0.1s = 1.1s - 3$$

$$\pi_{plaintiff}(Go\ to\ trial) = \gamma x - c - p + 0.1(d + \gamma x)$$

$$= 10 - 3 - 14 + 6 = -1.$$

$$\pi_{plaintiff}(give\ up) = -3.$$

The overall payoff from bringing a suit that eventually goes to trial is still -1, **which is worse than the payoff of 0 from not bringing suit in the first place, but if** s **is high enough, the payoff from bringing suit and settling is higher still.**

If s is greater than 1.82 ($= \frac{-1+3}{1.1}$, rounded), the plaintiff prefers settlement to trial, and if s is greater than about 2.73 ($= \frac{0+3}{1.1}$, rounded), he prefers settlement to not bringing the suit at all.

In determining the settlement range, the relevant payoff is the expected incremental payoff since the suit was brought. The plaintiff will settle for any $s \geq 1.82$, and the defendant will settle for any $s \leq \gamma x + d = 60$, as before. The settlement range is $[1.82, 60]$, and $s = 30.91$.

Plaintiff: *Sue*, *Go to Trial*
Defendant: *Accept any* $s \leq 60$
Outcome: The plaintiff sues and offers $s = 30.91$, and the defendant accepts the settlement.

4.4 Recoordination to Pareto-Dominant Equilibria in Subgames: Pareto Perfection

Suppose we think Pareto-dominant equilibria are what will be played out.

That idea has further implications in dynamic games.

Coalition-proof Nash equilibrium: no coalition of players could form a self-enforcing agreement to deviate from it.

Combining this with sequential rationality: no coalition would deviate in future subgame— renegotiation proofness (most common name), recoordination pareto perfection

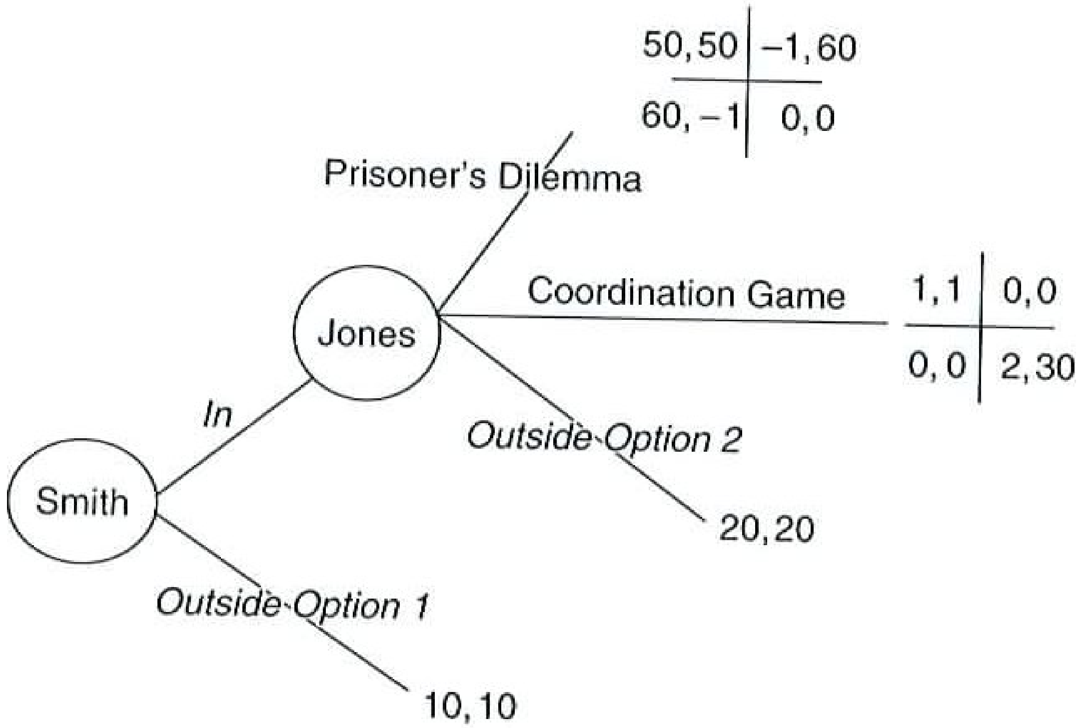


Figure 4.5 The Pareto Perfection Puzzle.

Figure 5: The Pareto Perfection Puzzle

The perfect equilibria of the Pareto Perfection Puzzle are:

E1: (In, outside option 2$|In$, the actions yielding (1,1) in the coordination subgame, the actions yielding (0,0) in the Prisoner's Dilemma subgame). The payoffs are (20,20).

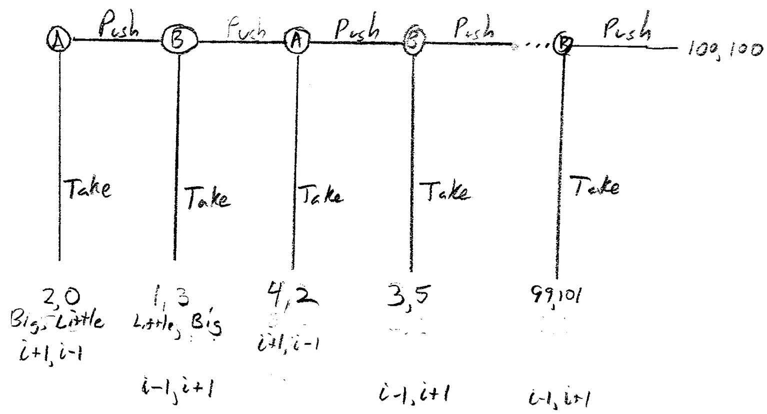
E2: (outside option 1, coordination game$|In$, the actions yielding (2,30) in the coordination subgame, the actions yielding (0,0) in the Prisoner's Dilemma subgame). The payoffs are (10,10).

The Centipede Game

Consider the perfect-information game in the figure below, adapted from Rosenthal (1981, JET). Players A and B are seated at a table, and Player A has two plates in front of him, a big plate with one gold coins and a littler plate with no gold coins. He can take the big plate and get the coin on it, leaving the little plate for Player B, or he can push both plates across the table, in which case the referee will add one coin to each plate. In round 2, Player B can take the big plate and its 2 coins, or push both plates across the table, the referee again adding one coin to each pile. This continues until the 100th round, when if player B does not take the big plate and get 101 coins, leaving 99 for Player A, the referee splits the coins equally, each player getting 100 of them.

In the unique subgame perfect equilibrium, each player follows the strategy of always $Take$. The equilibrium outcome is for Player A to $Take$ in round 1, for payoffs of (2,0). Yet in experiments people do $Push$ for a while, to their benefit. Palacios-Huerta & Volij (2009, AER) survey the theoretical and empirical literature well. They found in their 6-round version of the Centipede Game that chess players do tend to $Take$ early playing against each other, and Grandmasters *always* choose $Take$ in the first round. Playing against non-chess-players, though, even chess players choose $Push$ more.

The lesson is that when games are iterated like this, common knowledge of the ability to do complicated inductive reasoning becomes very important to the result.



Palacios-Huerta, I. & Volij, O. (2009) "Field Centipedes," American Economic Review 99(4): 1619–1635.

Rosenthal, R. (1981) "Games of Perfect Information, Predatory Pricing, and the Chain Store," Journal of Economic Theory 25:92–100.

A Paradox of Sequential Rationality

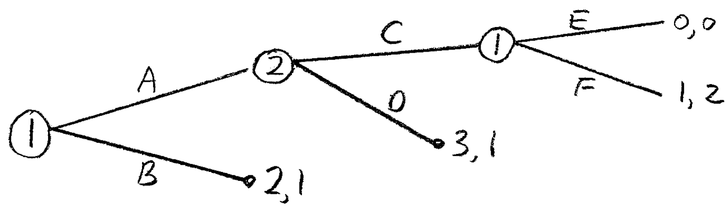
Consider the perfect-information game in the figure below. Player 1's strategy set is: $(A, E|AC), (A, F|AC), (B, E|A$ **Player 2's strategy set is:** $C|A, D|A$.

Using backwards induction, Player 1 uses $F|AC$**, Player 2 uses** $C|A$**, and Player 1 uses** B **in the subgame perfect Nash equilibrium.**

The other Nash equilibrium is for Player 1 to use $(A, E|AC)$ **and for Player 2 to use** $D|A$.

Player 1 prefers the non-perfect equilibrium with its (3,1) instead of (2,1) payoffs. The story justifying subgame perfectness that we tell ourselves is that before the game starts, if Player 1 threatens Player 2, saying: "I am going to choose A, so you'd better choose D in response, or I'll go ahead and choose E and we'll both get 0," then Player 2 will respond, "I don't believe you. I'm calling your bluff. You are rational, and so I know you would never choose E instead of F." Then Player 1 would give up and choose B instead.

But what if Player 1 makes his little speech, and then actually does choose A? What should Player 2 think? In equilibrium, choosing A isn't supposed to happen. It seems to refute the assumption of Player 1 being rational. So maybe Player 2 should respond by choosing D after all. If he does that, however, then Player 1's action has turned out to be rational after all.



(This is inspired by Section 6.4 of Osborne and Rubinstein's 1994 *A Course in Game Theory*, which has a similar but not identical game.)