

5 Reputation and Repeated Games with Symmetric Information

January 27, 2014

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The Chainstore Paradox

Suppose that we repeat Entry Deterrence I 20 times in the context of a chainstore that is trying to deter entry into 20 markets where it has outlets.

First, though, let's look at the Prisoner's Dilemma.

Prisoner's Dilemma

		Column	
		<i>Silence</i>	<i>Blame</i>
Row:	<i>Silence</i>	5,5 →	-5,10
	<i>Blame</i>	10,-5 →	0,0

What if we repeat it twice? N times? An infinite number of times?

Because the one-shot Prisoner's Dilemma has a dominant-strategy equilibrium, blaming is the only Nash outcome for the repeated Prisoner's Dilemma, not just the only perfect outcome.

The backwards induction argument does not prove that blaming is the unique Nash outcome. Why not? See the next page of slides.

Here is why blaming is the only Nash outcome:

1. No strategy in the class that calls for *Silence* in the last period can be a Nash strategy, because the same strategy with *Blame* replacing *Silence* would dominate it.

2. If both players have strategies calling for blaming in the last period, then no strategy that does not call for blaming in the next-to-last period is Nash, because a player should deviate by replacing *Silence* with *Blame* in the next-to-last period. And then keep going to 2nd-to-last period, etc.

Uniqueness is only on the equilibrium path. Nonperfect Nash strategies could call for cooperation at nodes away from the equilibrium path. The strategy of always blaming is not a dominant strategy, not even weakly.

If the one-shot game has multiple Nash equilibria, the perfect equilibrium of the finitely repeated game has not only the one-shot outcomes, but others. Benoit & Krishna (1985).

What if we repeat the Prisoner's Dilemma an infinite number of times?

Defining payoffs in games that last an infinite number of periods presents the problem that the total payoff is infinite for any positive payment per period.

1 Use an overtaking criterion. Payoff stream π is preferred to $\tilde{\pi}$ if there is some time T^* such that for every $T \geq T^*$,

$$\sum_{t=1}^T \delta^t \pi_t > \sum_{t=1}^T \delta^t \tilde{\pi}_t.$$

2 Specify that the discount rate is strictly positive, and use the present value. Since payments in distant periods count for less, the discounted value is finite unless the payments are growing faster than the discount rate.

3 Use the average payment per period, a tricky method since some sort of limit needs to be taken as the number of periods averaged goes to infinity.

Here is a strategy that yields an equilibrium with SILENCE.

The Grim Strategy

1 Start by choosing Silence.

2 Continue to choose Silence unless some player has chosen Blame, in which case choose Blame forever.

The GRIM STRATEGY is an example of a trigger strategy.

Robert Porter (1983) Bell J. Economics, “A study of cartel stability: The Joint Executive Committee, 1880-1886,” examines price wars between railroads in the 19th century. The classic reference.

Slade (1987) concluded that price wars among gas stations in Vancouver used small punishments for small deviations rather than big punishments for big deviations.

Now think back to the 20-repeated Entry Deterrence game.

Not every strategy that punishes blaming is perfect. A notable example is the strategy of Tit-for-Tat.

Tit-for-Tat

1 Start by choosing *Silence*.

2 Thereafter, in period n choose the action that the other player chose in period $(n - 1)$.

Tit-for-Tat is almost never perfect in the infinitely repeated Prisoner's Dilemma because it is not rational for Column to punish Row's initial *Blame*.

The deviation that kills the potential equilibrium is not from *Silence*, but from the off-equilibrium action rule of *Blame* in response to a *Blame*.

Adhering to Tit-for-Tat's punishments results in a miserable alternation of *Blame* and *Silence*, so Column would rather ignore Row's first *Blame*.

Problem 5.5 asks you to show this formally.

Theorem 1 (the Folk Theorem)

In an infinitely repeated n-person game with finite action sets at each repetition, any profile of actions observed in any finite number of repetitions is the unique outcome of some subgame perfect equilibrium given

Condition 1: The rate of time preference is zero, or positive and sufficiently small;

Condition 2: The probability that the game ends at any repetition is zero, or positive and sufficiently small; and

Condition 3: The set of payoff profiles that strictly Pareto dominate the minimax payoff profiles in the mixed extension of the one-shot game is n- dimensional.

Condition 1: Discounting

The Grim Strategy imposes the heaviest possible punishment for deviant behavior.

The Prisoner's Dilemma

		Column	
		<i>Silence</i>	<i>Blame</i>
Row:	<i>Silence</i>	5,5 →	-5,10
	<i>Blame</i>	10,-5 →	0,0

$$\pi(\text{equilibrium}) = 5 + \frac{5}{r}$$

$$\pi(\text{BLAME}) = 10 + 0$$

These are equal at $r = 1$, so $\delta = \frac{1}{1+r} = .5$

Condition 2: A probability of the game ending

If $\theta > 0$, the game ends in finite time with probability one. The expected number of repetitions is finite.

The probability that the game lasts till infinity is zero.

Compare with the Cauchy distribution (Student's t with one degree of freedom) which has no mean.

It still behaves like a discounted infinite game, because the expected number of future repetitions is always large, no matter how many have already occurred. It is “stationary”.

The game still has no Last Period, and it is still true that imposing one, no matter how far beyond the expected number of repetitions, would radically change the results.

“1 The game will end at some uncertain date before T .”

“2 There is a constant probability of the game ending.”

Amazing Grace on Stationarity

*When we've been there ten thousand years,
Bright shining as the sun,
We've no less days to sing God's praise
Than when we'd first begun.*

Condition 3: Dimensionality

The “minimax payoff” is the payoff that results if all the other players pick strategies solely to punish player i , and he protects himself as best he can.

The set of strategies s_{-i}^{i*} is a set of $(n - 1)$ minimax strategies chosen by all the players except i to keep i 's payoff as low as possible, no matter how he responds. s_{-i}^{i*} solves

$$\underset{S_{-i}}{\text{Minimize}} \underset{S_i}{\text{Maximum}} \pi_i(s_i, s_{-i}). \quad (1)$$

Player i 's minimax payoff, minimax value, or security value: his payoff from this.

We'll come back and talk about this more after finishing up the dimensionality condition.

The dimensionality condition is needed only for games with three or more players.

It is satisfied if there is some payoff profile for each player in which his payoff is greater than his minimax payoff but still different from the payoff of every other player.

Thus, a 3-person Ranked Coordination game would fail it.

The condition is necessary because establishing the desired behavior requires some way for the other players to punish a deviator without punishing themselves.

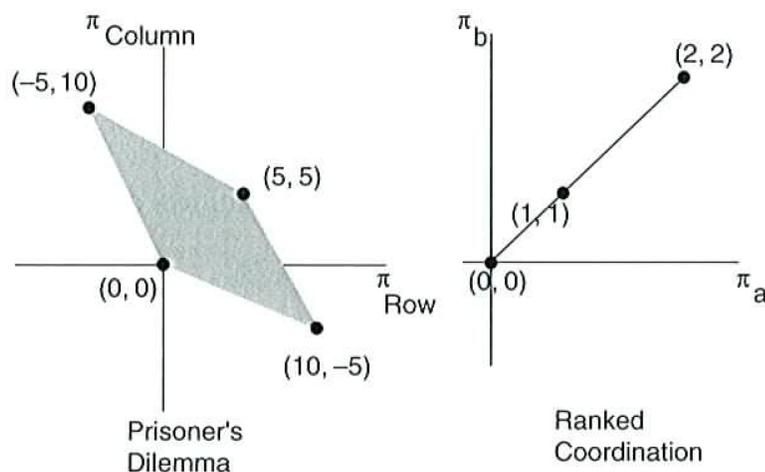


Figure 5.1 The dimensionality condition.

The Dimensionality Condition

Minimax and Maximin

The strategy s_i^* is a **maximin strategy** for player i if, given that the other players pick strategies to make i 's payoff as low as possible, s_i^* gives i the highest possible payoff. In our notation, s_i^* solves

$$\underset{s_i}{\text{Maximize}} \quad \underset{s_{-i}}{\text{Minimum}} \quad \pi_i(s_i, s_{-i}). \quad (2)$$

The minimax and maximin strategies for a two-player game with Player 1 as i :

$$\text{Maximin: } \underset{s_1}{\text{Maximum}} \quad \underset{s_2}{\text{Minimum}} \quad \pi_1$$

$$\text{Minimax: } \underset{s_2}{\text{Minimum}} \quad \underset{s_1}{\text{Maximum}} \quad \pi_1$$

In the Prisoner's Dilemma, the minimax and maximin strategies are both *Blame*.

Another Minimizing Game

		Tom	
		<i>Left</i>	<i>Right</i>
Joe:	<i>Up</i>	0,0	1,-1
	<i>Down</i>	1,2	3,3

If Tom picks Left, the most Joe can get is 1, from DOWN. Tom minimaxes Joe using LEFT.

If Joe picks Up, the most Tom can get is 0 from LEFT. Joe minimaxes Tom using UP.

If Joe picks Down, the worst he can do is 1, from Tom picking LEFT. That is Joe's maximin strategy.

If Tom picks Left, the worst he can get is 0, if Joe picks UP. That is Tom's maximin strategy.

Joe's Maximin value: The highest payoff Joe can assure himself if the other players are out to get him.

Joe's Maximin strategy: A strategy that assures Joe of his maximin payoff.

Joe's Minimax value: The lowest payoff Joe's opponent can limit him to.

Tom's Minimax strategy against Joe: Tom's strategy that limits Joe to Joe's minimax payoff.

The minimax and maximin strategies for a two-player game :

1's maximin strategy	<i>Maximum</i>	<i>Minimum</i>	π_1
	s_1	s_2	
2's strategy to minimax 1:	<i>Minimum</i>	<i>Maximum</i>	π_1
	s_2	s_1	

Under minimax, Player 2 is purely malicious but must choose his mixing probability first, in his attempt to cause player 1 the maximum pain.

Under maximin, Player 1 chooses his mixing probability first, in the belief that Player 2 is out to get him.

In variable-sum games, minimax is for sadists and maximin for paranoids.

The maximin strategy need not be unique.

Since maximin behavior can also be viewed as minimizing the maximum loss that might be suffered, decision theorists refer to such a policy as a minimax criterion.

The Minimax Illustration Game

		Column	
		<i>Left</i>	<i>Right</i>
	<i>Up</i>	-2, 2	1 , -2
Row:	<i>Middle</i>	1 , -2	-2, 2
	<i>Down</i>	0, 1	0, 1

In the Minimax Illustration Game Row can guarantee himself a payoff of 0 by choosing *Down*, so that is his maximin strategy.

Column cannot hold Row's payoff down to 0 by using a pure strategy, so his minimax strategy must be mixed.

Column's minimax strategy is (*Probability 0.5 of Left, Probability 0.5 of Right*).

Row would respond with *Down*, for a minimax payoff of 0, since either *Up*, *Middle*, or a mixture of the two would give him a payoff of -0.5 ($= 0.5(-2) + 0.5(1)$).

It happens that *Down*, (*Probability 0.5 of Left, Probability 0.5 of Right*) is a Nash equilibrium too.

The Minimax Illustration Game

		Column	
		<i>Left</i>	<i>Right</i>
<i>Up</i>		-2, 2	1 , -2
Row: <i>Middle</i>		1 , -2	-2, 2
<i>Down</i>		0, 1	0, 1

Row's strategy for minimaxing Column is (*Probability 0.5 of Up, Probability 0.5 of Middle*). **Row then gets 0** with left, right, or a mixture.

Column's maximin strategy is (*Probability 0.5 of Left, Probability 0.5 of Right*), **and his minimax payoff is 0.**

The Minimax Theorem (von Neumann [1928]), says that a minimax equilibrium exists in pure or mixed strategies for every two-person zero-sum game and is identical to the maximin equilibrium.

Precommitment

What if we allow players to commit at the start to a strategy for the rest of the game?

If precommitted strategies are chosen simultaneously, the equilibrium outcome of the finitely repeated Prisoner's Dilemma calls for always blaming.

What about in sequence?

The outcome depends on the particular values of the parameters, but one possible equilibrium is the following:

Row moves first and chooses the strategy (*Silence* until Column *Blames*; thereafter always *Blame*), and Column chooses (*Silence* until the last period; then *Blame*).

The observed outcome? Why is it Nash? The game has a second-mover advantage.

The One-Sided Prisoner's Dilemma (Reputation)

		Consumer (Column)	
		<i>Buy</i>	<i>Boycott</i>
Seller (Row):	<i>High Quality</i>	5,5	0,0
	<i>Low Quality</i>	10, -5	0,0

←

↓

→

↑

The Nash and iterated dominance equilibria are (*Low Quality, Boycott*), but it is not a dominant-strategy equilibrium.

Buyer does not have a dominant strategy, because if Seller were to choose *High Quality*, Buyer would choose *Buy*, to obtain the payoff of 5; but if Row chooses *Low Quality*, Column would choose *Boycott*, for a payoff of zero.

Low Quality is however, weakly dominant for Seller, which makes (*Low Quality, High Quality*) the iterated dominant strategy equilibrium.

Product Quality, Klein & Leffler (1981)

The Order of Play

1 An endogenous number n of firms decide to enter the market at cost F .

2 A firm that has entered chooses its quality to be *High* or *Low*, incurring the constant marginal cost c if it picks *High* and zero if it picks *Low*. The choice is unobserved by consumers. The firm also picks a price p .

3 Consumers decide which firms to buy from. The amount bought from firm i is denoted q_i .

4 All consumers observe the quality of all goods purchased in that period.

5 The game returns to (2) and repeats.

Payoffs

Consumers buy $q(p) = \sum_{i=1}^n q_i$ of high quality, 0 of low quality. where $\frac{dq}{dp} < 0$.

If a firm stays out, its payoff is zero.

If firm i enters, it receives $-F$ immediately. Its current end-of-period payoff is $q_i p$ if it produces *Low* quality and $q_i(p - c)$ if it produces *High* quality. The discount rate is $r \geq 0$.

An equilibrium:

Firms. \tilde{n} firms enter. Each produces high quality and sells at price \tilde{p} . If a firm ever deviates from this, it thereafter produces low quality (and sells at the same price \tilde{p}).

Buyers. Buyers start by choosing randomly among the firms charging \tilde{p} . Thereafter, they remain with their initial firm unless it changes its price or quality, in which case they switch randomly to a firm that has not changed its price or quality.

The equilibrium must satisfy three constraints: incentive compatibility, competition, and market clearing.

The incentive compatibility constraint says that the individual firm must be willing to produce high quality.

$$\frac{q_i p}{1+r} \leq \frac{q_i(p-c)}{r} \quad (\text{incentive compatibility}). \quad (3)$$

That means the price must satisfy:

$$\tilde{p} \geq (1+r)c. \quad (4)$$

The second constraint is that competition drives profits to zero, so firms are indifferent between entering and staying out of the market.

$$\frac{q_i(p-c)}{r} = F \quad (\text{competition}) \quad (5)$$

Replacing p gives

$$q_i = \frac{F}{c}. \quad (6)$$

Third, the output must equal the quantity demanded by the market.

$$nq_i = q(p). \quad (\text{market clearing}) \quad (7)$$

Combining equations (3), (6), and (7) yields

$$\tilde{n} = \frac{cq([1+r]c)}{F}. \quad (8)$$

What if there were no entry cost?

Would profits be dissipated?

Reputation: Umbrella Branding

What if there are two goods? Could a firm do better by using umbrella branding, selling both under the threat of losing its entire reputation if one of them turns out to be defective?

What is your intuition?

Would it matter if the seller was a monopoly or not?

Customer Switching Costs, Farrell & Shapiro (1988)

Players

Firms Apex and Brydox, and a series of customers, each of whom is first called a youngster and then an oldster.

The Order of Play

1a Brydox, the initial incumbent, picks the incumbent price p_1^i .

1b Apex, the initial entrant, picks the entrant price p_1^e .

1c The oldster picks a firm.

1d The youngster picks a firm.

1e Whichever firm attracted the youngster becomes the incumbent.

1f The oldster dies and the youngster becomes an oldster.

2a Return to (1a), possibly with new identities for entrant and incumbent.

Payoffs

The discount factor is δ . The customer reservation price is R and the switching cost is c . The per period payoffs in period t are, for $j = (i, e)$,

Payoff for firm j :

$$\left\{ \begin{array}{ll} 0 & \text{if no customers are attracted.} \\ p_t^j & \text{if just oldsters or just youngsters} \\ 2p_t^j & \text{if both oldsters and youngsters} \end{array} \right.$$

The payoff for an oldster:

$$\left\{ \begin{array}{ll} R - p_t^i & \text{if he buys from the incumbent.} \\ R - p_t^e - c & \text{if he switches to the entrant.} \end{array} \right.$$

The payoff for a youngster:

$$\left\{ \begin{array}{ll} R - p_t^i & \text{if he buys from the incumbent.} \\ R - p_t^e & \text{if he buys from the entrant.} \end{array} \right.$$

*A **Markov strategy** is a strategy that, at each node, chooses the action independently of the history of the game except for the immediately preceding action (or actions, if they were simultaneous).*

Here, a firm's Markov strategy is its price as a function of whether the particular is the incumbent or the entrant, and not a function of the entire past history of the game.

There are two ways to use Markov strategies:

(1) The right way. Look for equilibria that use Markov strategies (perfect Markov equilibrium)

(2) The wrong way. Disallow non-Markov strategies and then look for equilibria.

Brydox, the initial incumbent, moves first. It does not want Bertrand competition and zero profits. So it chooses p^i low enough that Apex is not tempted to choose $p^e < p^i - c$ and steal away the oldsters.

Entrant Apex's profit is p^i if it chooses $p^e = p^i$ and serves just youngsters (we need for it to get ALL the youngsters in equilibrium—open-set problem) and $2(p^i - c)$ if it chooses $p^e = p^i - c$ and serves both oldsters and youngsters. Brydox chooses p^i to make Apex indifferent between these alternatives, so

$$p^i = 2(p^i - c), \quad (9)$$

and

$$p^i = 2c. \quad (10)$$

Apex will get all the entrants, and therefore in equilibrium, Apex and Brydox take turns being the incumbent. Also, Apex charges the same price as Brydox, which is the most it can get away with charging the youngsters:

$$p^e = p^i = 2c.$$

Let's compute the payoffs. First, note that the Oldsters are getting a better price than the Youngsters, even though they are the captive customers.

The equilibrium payoff of the current entrant is the immediate payment of p^e plus the discounted value of being the incumbent in the next period:

$$\pi_e^* = p^e + \delta\pi_i^*. \quad (11)$$

The incumbent's payoff is the immediate payment of p^i plus the discounted value of being the entrant next period:

$$\pi_i^* = p^i + \delta\pi_e^*. \quad (12)$$

In equilibrium the incumbent and the entrant sell the same amount at the same price, so $\pi_i^* = \pi_e^*$ and

$$\pi_i^* = 2c + \delta\pi_i^*. \quad (13)$$

It follows that

$$\pi_i^* = \pi_e^* = \frac{2c}{1 - \delta}. \quad (14)$$

5.6 Evolutionary Equilibrium: Hawk-Dove

A strategy s^* is an **evolutionarily stable strategy**, or **ESS**, if, using the notation $\pi(s_i, s_{-i})$ for player i 's pay-off when his opponent uses strategy s_{-i} , for every other strategy s' either

$$\pi(s^*, s^*) > \pi(s', s^*) \quad (15)$$

or

$$\begin{aligned} (a) \quad & \pi(s^*, s^*) = \pi(s', s^*) \\ \text{and} & \\ (b) \quad & \pi(s^*, s') > \pi(s', s'). \end{aligned} \quad (16)$$

If condition (17) holds, then a population of players using s^* cannot be invaded by a deviant using s' . If condition (18) holds, then s' does well against s^* , but badly against itself, so that if more than one player tried to use s' to invade a population using s^* , the invaders would fail.

A strategy s^* is an **evolutionarily stable strategy**, or **ESS**, if, using the notation $\pi(s_i, s_{-i})$ for player i 's payoff when his opponent uses strategy s_{-i} , for every other strategy s' either

$$\pi(s^*, s^*) > \pi(s', s^*) \quad (17)$$

or

$$\begin{aligned} (a) \quad & \pi(s^*, s^*) = \pi(s', s^*) \\ \text{and} \quad & \\ (b) \quad & \pi(s^*, s') > \pi(s', s'). \end{aligned} \quad (18)$$

Condition (17) is satisfied when s^* is a strong Nash equilibrium (although not every strong Nash strategy is an ESS).

Condition (18) is satisfied if s^* is only a weak Nash strategy, but the weak alternative s' is not a best response to itself.

ESS is a refinement of Nash: Nash plus:

(a) it has the highest payoff of any strategy used in equilibrium (which rules out equilibria with asymmetric payoffs),

(b) any other best response s' is not as good a response as s^* to itself.

ESS is a refinement of Nash: Nash plus:

(a) it has the highest payoff of any strategy used in equilibrium (which rules out equilibria with asymmetric payoffs),

(b) Any other best response s' does better against s^* than it does against s' .

Example: The Battle of the Sexes. The mixed strategy equilibrium is an ESS, because a player using it has as high a payoff as any other player. The two pure strategy equilibria are not made up of ESS's, though, because in each of them one player's payoff is higher than the other's.

Ranked Coordination has two pure strategy equilibria. They both use ESS's. The "bad" equilibrium strategy is an ESS, because given that the other players are using it, no player could do as well by deviating.

The mixed-strategy equilibrium is a best response to itself.

Example: The Utopian Exchange Economy. In Utopia, each citizen can produce either one or two units of individualized output. He will then go into the marketplace and meet another citizen.

If either of them produced only one unit, trade cannot increase their payoffs.

If both of them produced two, they can trade one unit for one unit, and both end up happier with more variety.

The Utopian Exchange Economy Game

		Jones		
		<i>Low Output</i>	<i>High Output</i>	
Smith:	<i>Low Output</i>	1, 1	\leftrightarrow	1, 1
	<i>High Output</i>	1, 1	\rightarrow	2, 2

This game has three Nash equilibria, one of which is in mixed strategies. *High Output* is an ESS by condition (a): it is a strict Nash equilibrium.

Low Output fails to meet condition (b). *High output* is weakly best response to it, and *High output* does even better against itself.

If the economy began with all citizens choosing *Low Output*, then if Smith deviated to *High Output* he would not do any better, but if *two* people deviated to *High Output*, they would do better in expectation because they might meet each other and receive (2,2).

An Example of ESS: Hawk-Dove

A resource worth $V = 2$ “fitness units” is at stake when the two birds meet. If they both fight, the loser incurs a cost of $C = 4$, which means that the expected payoff when two Hawks meet is -1 ($= 0.5[2] + 0.5[-4]$) for each of them.

Table 5 Hawk-Dove: Economics Notation

		Bird Two	
		<i>Hawk</i>	<i>Dove</i>
Bird One:	<i>Hawk</i>	-1,-1 →	2,0
	<i>Dove</i>	0, 2 ←	1,1

Payoffs to: (Bird One, Bird Two). Arrows show how a player can increase his payoff.

Table 6 Hawk-Dove: Biology Notation

		Bird Two	
		<i>Hawk</i>	<i>Dove</i>
Bird One:	<i>Hawk</i>	-1	2
	<i>Dove</i>	0	1

Payoffs to: (Bird One)

Hawk-Dove has no symmetric pure-strategy Nash equilibrium, and hence no pure-strategy ESS, since in the two asymmetric Nash equilibria, *Hawk* gives a bigger payoff than *Dove*, and the doves would disappear from the population.

In the mixed-strategy ESS, the equilibrium strategy is to be a hawk with probability 0.5 and a dove with probability 0.5, which can be interpreted as a population 50 percent hawks and 50 percent doves.

The equilibrium is stable in a sense similar to the Cournot equilibrium. If 60 percent of the population were hawks, a bird would have a higher fitness level as a dove. If “higher fitness” means being able to reproduce faster, the number of doves increases and the proportion returns to 50 percent over time.

The bourgeois strategy (a correlated strategy) is an ESS. Under this strategy, the bird behaves as a hawk if it arrives first, and a dove if it arrives second.

The bourgeois strategy has an expected payoff of 1 from meeting itself, and behaves exactly like a 50:50 randomizer when it meets a strategy that ignores the order of arrival, so it can successfully invade a population of 50:50 randomizers.

The ESS is suited to games in which all the players are identical and interacting in pairs.

The approach follows three steps:

- (1) the initial population proportions and the probabilities of interactions,**
- (2) the pairwise interactions,**
- (3) the dynamics by which players with higher payoffs increase in number in the population.**

Slow dynamics also makes the starting point of the game important, unlike the case when adjustment is instantaneous. Figure 2, taken from David Friedman (1991), shows a way to graphically depict evolution in a game in which all three strategies of *Hawk*, *Dove*, and *Bourgeois* are used. A point in the triangle represents a proportion of the three strategies in the population. At point E_3 , for example, half the birds play *Hawk*, half play *Dove*, and none play *Bourgeois*, while at E_4 all the birds play *Bourgeois*.

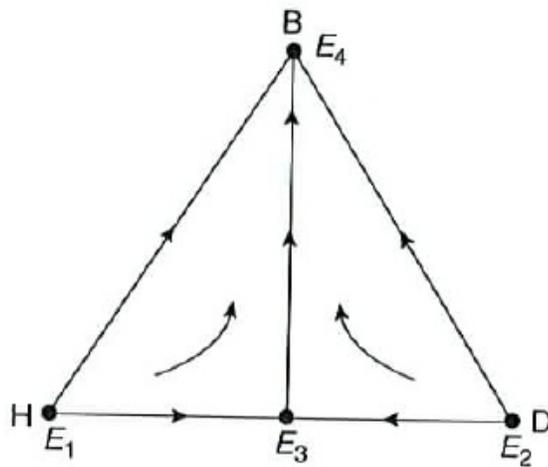


Figure 5.2 Evolutionary Dynamics in the Hawk–Dove – Bourgeois game.

Evolutionary Dynamics in the Hawk-Dove-Bourgeois Game

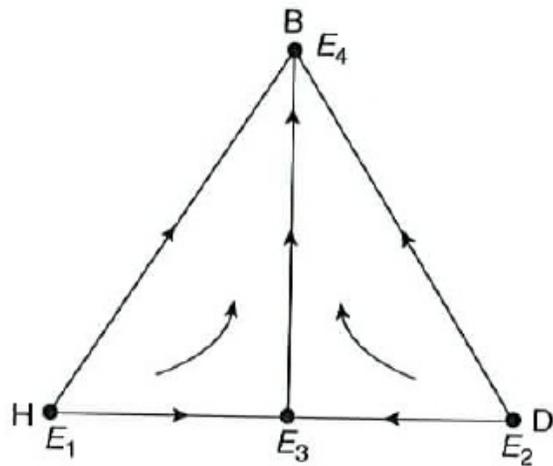


Figure 5.2 Evolutionary Dynamics in the Hawk–Dove – Bourgeois game.

The figure also shows the importance of mutation in biological games. If the population of birds is 100 percent dove, as at E_2 , it stays that way in the absence of mutation, since if there are no hawks to begin with, the fact that they would reproduce at a faster rate than doves becomes irrelevant.