Information

- ♦ Information partition

  - ○ Player *i*'s information partition is a collection of his information sets such that

    - √ each path is represented by one node in a single information set in the partition, and

    - √ the predecessors of all nodes in a single information set are in one information set.

  - ○ The information partition refers to a stage of the game, not chronological time.

  - ○ We say that partition II is coarser, and partition I is finer.

- We categorize the information structure of a game in four different ways.
  - In a game of perfect information, each information set is a singleton.
    - √ Otherwise, the game is one of imperfect information.
  - A game of certainty has no moves by Nature after any player moves.
    - √ Otherwise, the game is one of uncertainty.
  - In a game of symmetric information, a player's information set at
    - √ any node where he chooses an action, or
    - √ an end node

    contains at least the same elements as the information sets of every other player.
    - √ Otherwise, the game is one of asymmetric information.

- ○ In a game of incomplete information, Nature moves first and is unobserved by at least one of the players.

  - √ Otherwise, the game is one of complete information.

♦ Bayes' Rule

- ○ For Nature's move x and the observed data,

$$Prob(x \mid data) = Prob(data \mid x)\, Prob(x) \,/\, Prob(data)$$

Chapter 7 Moral Hazard: Hidden Actions

7.1 Categories of Asymmetric Information Models

- ♦ We will make heavy use of the principal-agent model.
  - ○ The principal hires an agent to perform a task,
    and the agent acquires an informational advantage about his type,
    his actions, or the outside world at some point in the game.
  - ○ It is usually assumed that the players can make a binding contract
    at some point in the game.
  - ○ The principal (or uninformed player) is the player
    who has the coarser information partition.
  - ○ The agent (or informed player) is the player
    who has the finer information partition.

- Categories of asymmetric information models

  - Moral hazard with hidden actions

    - The moral hazard models are games of complete information with uncertainty.

  - Postcontractual hidden knowledge

  - Adverse selection

    - Adverse selection models have incomplete information.

- Signalling
  - √ A "signal" is different from a "message" because it is not a costless statement, but a costly action.
- Screening
  - √ If the worker acquires his credentials in response to a wage offer made by the employer, the problem is screening.
  - √ Many economists do not realize that screening and signalling are different and use the terms interchangeably.

7.2 A Principal-Agent Model: The Production Game

- ♦ The Production Game

  - ○ Players

    - √ the principal and the agent

  - ○ The order of play

    1. The principal offers the agent a wage w.
    2. The agent decides whether to accept or reject the contract.
    3. If the agent accepts, he exerts effort e.
    4. Output equals $q(e)$, where $q' > 0$.

- Payoffs
  - √ If the agent rejects the contract, then $\pi_{agent} = \overline{U}$ and $\pi_{principal} = 0$.
  - √ If the agent accepts the contract, then $\pi_{agent} = U(e, w)$ and $\pi_{principal} = V(q - w)$, where $\partial U / \partial e < 0$, $\partial U / \partial w > 0$, and $V' > 0$.

- An assumption common to most principal-agent models
  - Other principals compete to employ the agent, so the principal's equilibrium profit equals zero.
  - Or many agents compete to work for the principal, so the agent's equilibrium utility equals the minimum for which he will accept the job, called the reservation utility, $\overline{U}$.

- Production Game I: Full Information
  - Every move is common knowledge and the contract is a function $w(e)$.
  - The principal must decide what he wants the agent to do and what incentive to give him to do it.
  - The agent must be paid some amount $\tilde{w}(e)$ to exert effort e,

    where $U(e, \tilde{w}(e)) = \bar{U}$.
  - The principal's problem is

$$\underset{e}{Maximize} \quad V(q(e) - \tilde{w}(e)).$$

- At the optimal effort level, e^*, the marginal utility to the agent which would result if he kept all the marginal output from extra effort equals the marginal disutility to him of that effort.

  - √ $(\partial U / \partial \tilde{w})\,(\partial q / \partial e) \;=\; -\,\partial U / \partial e$

  - √ $q(e)$ denotes the monetary value of output at an effort level e.

- Under perfect competition among the principals, the profits are zero.

  - √ at the profit-maximizing effort e^*

    $\tilde{w}(e^*) \;=\; q(e^*)$

    $U(e^*, q(e^*)) \;=\; \bar{U}$

  - √ The principal selects the point (e^*, w^*) on the indifference curve $\bar{U}$.

- The principal must then design a contract that will induce the agent to choose this effort level.

- The following contracts are equally effective under full information.

  - √ The forcing contract sets $w(e^*) = w^*$ and $w(\mathrm{e} \neq e^*) = 0$.

  - √ The threshold contract sets $w(e \geq e^*) = w^*$ and $w(e < e^*) = 0$.

  - √ The linear contract sets $w(e) = \alpha + \beta e$,
    where α and β are chosen so that $w^* = \alpha + \beta e^*$ and
    the contract line is tangent to the indifference curve $\bar{U}$ at e^*.

- Utility function $U(e, w) = log(w - e^2)$ is also a <u>quasilinear</u> function, because it is just a monotonic function of $U(e, w) = w - e^2$.

- Utility function $U(e, w) = log(w - e^2)$ is <u>concave</u> in w, so it represents a <u>risk-averse</u> agent.

- As with utility function $U(e, w) = w - e^2$, the <u>optimal effort</u> does not depend on the agent's wealth w.

- Production Game II: Full Information

  - Every move is <u>common knowledge</u> and the contract is a <u>function</u> $w(e)$.

  - The agent moves first.

    - √ The agent, not the principal, <u>proposes</u> the contract.

  - The order of play

    1. The agent offers the principal a contract $w(e)$.
    2. The principal decides whether to accept or reject the contract.
    3. If the principal accepts, the agent exerts effort e.
    4. Output equals $q(e)$, where $q' > 0$.

- In this game, <u>the agent</u> has all the bargaining power, not the principal.
  - √ The agent will maximize his own payoff by driving the principal to exactly zero profits.
  - √ $w(e) = q(e)$

- The maximization problem for the agent can be written as

$$\underset{e}{Maximize} \quad U(e, q(e)).$$

- The optimality equation is <u>identical</u> in Production Games I and II.
  - √ At the optimal effort level, e^*, the marginal utility of the money derived from marginal effort <u>equals</u> the marginal disutility of effort.
  - √ $(\partial U/\partial w)\,(\partial q/\partial e) = -\,\partial U/\partial e$

- Although the form of the optimality equation is the same,
  the optimal effort might not be, because except in the special case in which the agent's reservation payoff in Production Game I equals his equilibrium payoff in Production Game II,
  the agent ends up with higher wealth
  if he has all the bargaining power.

  - √ If the utility function is not quasilinear,
    the wealth effect will change the optimal effort.

- If utility is quasilinear, the efficient effort level is independent of
  which side has the bargaining power
  because the gains from efficient production are independent of how those gains are distributed
  so long as each party has no incentive to abandon the relationship.

  - √ This is the same lesson as the Coase Theorem's:
    under general conditions the activities undertaken will be efficient and independent of the distribution of property rights.

- Production Game III: A Flat Wage under Certainty
  - The principal can condition the wage <u>neither</u> on effort <u>nor</u> on output.
    - √ The principal observes neither effort nor output, so information is asymmetric.
  - The outcome of Production Game III is simple and <u>inefficient</u>.
    - √ If the wage is nonnegative, the agent accepts the job and exerts zero effort, so the principal offers a wage of zero.

- Moral hazard

  - √ the problem of the agent choosing the wrong action because the principal cannot use the contract to punish him

  - √ the danger to the principal that the agent, constrained only by his morality, not punishments, cannot be trusted to behave as he ought

  - √ a temptation for the agent, a hazard to his morals

- A clever contract can overcome moral hazard
  by conditioning the wage on something that is observable and correlated with effort, such as output.

- ♦ Production Game IV: An Output-based Wage under Certainty

  - ○ The principal cannot observe effort but can observe output and specify the contract to be $w(q)$.

  - ○ It is possible to achieve the efficient effort level e^* despite the unobservability of effort.

    - √ The principal starts by finding the optimal effort level e^*.

    - √ $q^* = q(e^*)$

    - √ To give the agent the proper incentives, the contract must reward him when output is q^*.

- √ A variety of contracts could be used.

- √ The forcing contract, for example, would be any wage function such that

$$U(e^*, w(q^*)) = \bar{U} \quad \text{and} \quad U(e, w(q)) < \bar{U} \;\; \text{for } e \neq e^*.$$

- ○ The unobservability of effort is <u>not a problem</u> in itself, if the contract can be conditioned on something which is <u>observable</u> and perfectly <u>correlated with effort</u>.

- Production Game V: Output-based Wage under Uncertainty

  - The principal cannot observe effort but can observe output and specify the contract to be $w(q)$.

  - Output, however, is a function $q(e, \theta)$ both of effort and the state of the world $\theta \in \mathbf{R}$, which is chosen by Nature according to the probability density $f(\theta)$.

  - The principal cannot deduce $e \neq e^*$ from $q \neq q^*$.

- Even if the contract does induce the agent to choose e^*,
  if it does so by penalizing him heavily when $q \neq q^*$,
  it will be <u>expensive</u> for the principal.

  √ The agent's expected utility must be kept equal to $\bar{U}$.

  √ If the agent is sometimes paid a low wage
  because output happens not to equal q^* despite his correct effort,
  he must be <u>paid more</u> when output does equal q^* to make up for it.

  √ There is a <u>tradeoff</u> between incentives and insurance against risk.

- Moral hazard is a problem when $q(e)$ is not a one-to-one function and a single value of e might result in any of a number of values of q, depending on the value of θ.

  - √ The output function is not invertible.

- The combination of unobservable effort and lack of invertibility means that no contract can induce the agent to put forth the efficient effort level without incurring extra costs, which usually take the form of extra risk imposed on the agent.

- We will still try to find a contract that is efficient
  in the sense of maximizing welfare
  given the informational constraints.

- The terms "first-best" and "second-best" are used to distinguish
  these two kinds of optimality.

  - √ A first-best contract achieves the same allocation as the contract
    that is optimal when the principal and the agent have
    the same information set and all variables are contractible.

  - √ A second-best contract is Pareto optimal
    given information asymmetry and constraints on writing contracts.

  - √ The difference in welfare between the first-best and
    the second-best is the cost of the agency problem.

- How do we find a second-best contract?

  - √ Because of the tremendous variety of possible contracts,
    finding the optimal contract
    when a forcing contract cannot be used
    is a hard problem without general answers.

  - √ The rest of the chapter will show how the problem may be approached, if not actually solved.

7.3 The Incentive Compatibility and Participation Constraints

- The Participation Constraint and the Incentive Compatibility Contraint

  - The principal's problem is

$$\underset{w(\cdot)}{Maximize} \quad EV(q(\tilde{e}, \theta) - w(q(\tilde{e}, \theta)))$$

subject to

$$\tilde{e} = \underset{e}{argmax} \quad EU(e, w(q(e, \theta)))$$

(incentive compatibility constraint)

$$EU(\tilde{e}, w(q(\tilde{e}, \theta))) \geq \bar{U}$$

(participation constraint).

√ the first-order condition approach

- The Three-Step Procedure
  - The first step is to find for each possible effort level the set of wage contracts that induce the agent to choose that effort level.
  - The second step is to find the contract which supports that effort level at the lowest cost to the principal.
  - The third step is to choose the effort level that maximizes profits, given the necessity to support that effort with the costly wage contract from the second step.

√ Mathematically, the problem of finding the least cost $C(\tilde{e})$ of supporting the effort level $\tilde{e}$ combines steps one and two.

$$C(\tilde{e}) = \underset{w(\cdot)}{Minimum} \quad Ew(q(\tilde{e}, \theta))$$

subject to

$$\tilde{e} = \underset{e}{argmax} \quad EU(e, w(q(e, \theta)))$$

$$EU(\tilde{e}, w(q(\tilde{e}, \theta))) \geq \bar{U}$$

√ Step three takes the principal's problem of maximizing his payoff, and restates it as

$$\underset{\tilde{e}}{Maximize} \quad EV(q(\tilde{e}, \theta) - C(\tilde{e})). \qquad (7.27)$$

- √ After finding which contract most cheaply induces each effort, the principal discovers the optimal effort by solving problem (7.27).

- ○ Breaking the problem into parts makes it easier to solve.

- ○ Perhaps the most important lesson of the three-step procedure is to reinforce the points

  - √ that the goal of the contract is to induce the agent to choose a particular effort level

    and

  - √ that asymmetric information increases the cost of the inducements.

7.4 Optimal Contracts: The Broadway Game

- A peculiarity of optimal contracts
  - Sometimes the agent's reward should not increase with his output.

- Broadway Game I
  - Players
    - √ producer and investors
  - The order of play
    1. The investors offer a wage contract $w(q)$ as a function of revenue q.
    2. The producer accepts or rejects the contract.
    3. The producer chooses: *Embezzle* or *Do not embezzle*.
    4. Nature picks the state of the world to be *Success* or *Failure* with equal probability.

- √ Revenues (or profits)

State of the World

Effort

	Failure (0.5)	*Success* (0.5)
Embezzle	− 100	+ 100
Do not embezzle	− 100	+ 500

- ○ Payoffs

- √ The producer is risk averse and the investors are risk neutral.
- √ The producer's payoff is $U(100)$ if he rejects the contract, where $U' > 0$ and $U'' < 0$, and the investors' payoff is 0.

- √ Otherwise,

$$\pi_{producer} = \begin{cases} U(w(q) + 50) & \text{if he embezzles} \\ U(w(q)) & \text{if he is honest} \end{cases}$$

$$\pi_{investors} = q - w(q)$$

- Boiling-in-oil contract
  - The investors will observe -100, $+100$, or $+500$.
    - √ $w(-100)$, $w(+100)$, and $w(+500)$
  - The producer's expected payoffs
    - √ $\pi(\textit{Do not embezzle}) = 0.5U(w(-100)) + 0.5U(w(+500))$
    - √ $\pi(\textit{Embezzle}) = 0.5U(w(-100)+50) + 0.5U(w(+100)+50)$
  - The incentive compatibility constraint
    - √ $\pi\,(\textit{Do not embezzle}) \geq \pi\,(\textit{Embezzle})$

- The participation constraint
  - √ π (*Do not embezzle*) $\geq$ $U(100)$

- The investors want to impose <u>as little risk</u> on the producer <u>as possible</u>, since he requires a higher expected wage for higher risk.
  - √ $w(-100) = w(+500)$, which provides <u>full insurance</u>.
  - √ The outcome $+100$ cannot occur unless the producer chooses the undesirable action.

- The following <u>boiling-in-oil contract</u> provides both <u>riskless wages</u> and <u>effective incentives</u>.
  - √ $w(+500) = 100$
    $w(-100) = 100$
    $w(+100) = -\infty$

- √ The participation constraint is satisfied, and is <u>binding</u>.
- √ The incentive compatibility constraint is satisfied, and is <u>nonbinding</u>.

- ○ The producer chooses *Do not embezzle* in equilibrium.
- ○ The cost of the contract to the investors is 100 in equilibrium, so that their overall expected payoff is 100.

- The sufficient statistic condition

  - It says that for incentive purposes,
    if the agent's utility function is separable in effort and money,
    wages should be based on whatever evidence best indicates effort,
    and only incidentally on output.

  - In equilibrium, the datum $q = +500$ contains
    exactly the same information as the datum $q = -100$.

- ♦ Milder contracts

  - ○ Two wages will be used in equilibrium,
    a low wage $\underline{w}$ for an output of $q = +100$ and
    a high wage $\overline{w}$ for any other output.

  - ○ To find the mildest possible contract,
    the modeller must specify a function for utility $U(w)$.

    - √ $U(w) = 100w - 0.1w^2$

  - ○ The participation constraint

    - √ Solving for the full-insurance high wage, we obtain
      $\overline{w} = w(-100) = w(+500) = 100$
      and a reservation utility of 9,000.

- The incentive compatibility constraint
  - √ Substituting into the incentive compatibility constraint, we obtain $\underline{w} \leq 5.6$.
  - √ A low wage of $-\infty$ is far more severe than what is needed.

♦ One of the oddities of Broadway Game I is that the wage is higher for an output of -100 than for an output of $+100$.

- This illustrates the idea that the principal's aim is to reward input, not output.
- If the principal pays more simply because output is higher, he is rewarding Nature, not the agent.
- Higher effort usually leads to higher output, but not always. Thus, higher pay is usually a good incentive, but not always, and sometimes low pay for high output actually punishes slacking.

- ♦ The <u>decoupling</u> of reward and result has broad applications.

- ♦ Shifting support scheme

  - ○ The contract depends on the support of the output distribution being <u>different</u> when effort is <u>optimal</u> than when effort is other than optimal.

  - ○ The set of possible outcomes <u>under optimal effort</u> must be <u>different from</u> the set of possible outcomes under any other effort level.

    - √ As a result, particular outputs show without doubt that the producer embezzled.

    - √ Very heavy punishments inflicted only for those outputs achieve the first-best.

- The conditions favoring boiling-in-oil contracts are
  - The agent is not very risk averse.
  - There are outcomes with high probability under shirking that have low probability under optimal effort.
  - The agent can be severely punished.
  - It is credible that the principal will carry out the severe punishment.

- Selling the Store
  - Another first-best contract that can sometimes be used is selling the store.
  - Under this arrangement, the agent buys the entire output for a flat fee paid to the principal, becoming the residual claimant.
  - This is equivalent to fully insuring the principal, since his payoff becomes independent of the moves of the agent and of Nature.
  - The drawbacks are that
    - √ the producer might not be able to afford to pay the investors the flat price of 100, and
    - √ the producer might be risk-averse and incur a heavy utility cost in bearing the entire risk.

- Public Information That Hurts the Principal and the Agent

    - Having more public <u>information</u> available <u>can hurt</u> both players.

    - Revenues (or profits) in Broadway Game II

State of the World

Effort

	Failure (0.5)	*Minor Success* (0.3)	*Major Success* (0.2)
Embezzle	– 100	– 100	+ 400
Do not embezzle	– 100	+ 450	+ 575

- √ Each player's initial information partition is ({*Failure*, *Minor Success*, *Major Success*}).

- Under the optimal contract,
  $w(-100) = w(+450) = w(+575) > w(+400) + 50.$

  - √ This is so because the producer is risk-averse and only the datum $q = +400$ is proof that the producer embezzled.

  - √ The optimal contract must do two things:
    deter embezzlement and
    pay the producer as predictable a wage as possible.

  - √ $w(-100) = w(+450) = w(+575) = 100$
    $w(+400) = -\infty$

  - √ The punishment would not have to be infinitely severe, and the minimum effective punishment could be calculated.

  - √ The producer chooses *Do not embezzle* in equilibrium.

  - √ The investors' expected payoff is 100 in equilibrium.

- Broadway Game III

  - √ Each player's initial information partition is ({*Failure*, *Minor Success*}, {*Major Success*}).

  - √ If the investors could still hire the producer and prevent him from embezzling at a cost of 100, the payoff from investing in a <u>major success</u> would be 475.

    But the payoff from investing in a show given the information set {*Failure*, *Minor Success*} would be about 6.25.

    So the improvement in information is <u>no help</u> with respect to the decision of when to invest.

- √ The refinement does, however, ruin the producer's incentives.

  If he observes {*Failure*, *Minor Success*},
  he is free to embezzle without fear of the oil-boiling output of $+400$.

- √ Better information reduces welfare,
  because it increases the producer's temptation to misbehave.