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Chapter 8 Further Topics in Moral Hazard

8.1 Efficiency Wages

♦ The aim of an incentive contract is to create a difference between the agent's expected payoff from right and wrong actions.

○ Either with the stick of punishment or the carrot of reward

♦ The Lucky Executive Game

○ Players

√ a corporation (the principal) and an executive (the agent)

- The order of play
  1. The corporation offers the executive a contract which pays $w(q) \geq 0$ depending on profit, q.
  2. The executive decides whether to accept or reject the contract.
  3. If the executive accepts, he exerts effort e of either 0 or 10.
  4. Nature chooses profit according to the table below.

- Payoffs
  - √ Both players are risk neutral.
  - √ If the executive rejects the contract, then $\pi_{agent} = \bar{U} = 5$ and $\pi_{principal} = 0$.
  - √ If the executive accepts the contract, then $\pi_{agent} = U(e, w(q)) = w(q) - e$ and $\pi_{principal} = q - w(q)$.

  - √ Probabilities of Profits in the Lucky Executive Game

	Low profit ($q = 0$)	*High profit* ($q = 400$)
Low effort ($e = 0$)	0.5	0.5
High effort ($e = 10$)	0.1	0.9

- Optimal contracts when the principal and the agent have the <u>same information set</u> and all variables are contractible

    - √ The principal <u>can observe</u> effort.

  - ○ The optimal effort level

    - √ $e^* = 10$

  - ○ Wage w^*

    - √ $0.1U(e^*, w^*) + 0.9U(e^*, w^*) = \bar{U}$

      $0.1(w^* - 10) + 0.9(w^* - 10) = 5$

      $w^* = 15$

- Payoffs π^*_{agent} and $\pi^*_{principal}$
  - √ $\pi^*_{agent} = 5$
  - √ $\pi^*_{principal} = 0.1(0 - 15) + 0.9(400 - 15) = 345$

- Contracts

- Is a first-best contract feasible?

  - The participation constraint

    - √ $\pi_{agent}\ (High\ effort) = 0.1\{w(0) - 10\} + 0.9\{w(400) - 10\} \geq \bar{U}$

    - √ The agent's expected wage must equal 15.

      $0.1w(0) + 0.9w(400) = 15$

  - The incentive compatibility constraint

    - √ $\pi_{agent}\ (High\ effort) \geq \pi_{agent}\ (Low\ effort)$

      $0.1\{w(0) - 10\} + 0.9\{w(400) - 10\} \geq 0.5w(0) + 0.5w(400)$

      $w(400) - w(0) \geq 25$

- √ The gap between the agent's wage for high profit and low profit must equal at least 25.

- ○ A contract that satisfies both constraints is $\{w(0) = -345, w(400) = 55\}$.

  - √ The agent exerts high effort: $e = 10$.
  - √ The agent's expected wage is 15.
  - √ The agent's expected payoff (or utility) is 5.
  - √ The principal's expected payoff is 345.
  - √ The first-best can be achieved by <u>selling the store</u>, putting the entire risk on the agent.

- ○ But this contract is not feasible, because the game requires $w(q) \geq 0$.

  - √ This is an example of the common and realistic bankruptcy constraint.

  - √ The principal cannot punish the agent by taking away more than the agent owns in the first place – zero in the Lucky Executive Game.

- What can be done is to use <u>the carrot</u> instead of the stick and <u>abandon</u> satisfying the participation constraint <u>as an equality</u>.

  - The incentive compatibility constraint

    - √ $\pi_{agent}\,(\textit{High effort}) \;\geq\; \pi_{agent}\,(\textit{Low effort})$

    $$w(400) \;-\; w(0) \;\geq\; 25$$

  - The principal can use the contract $\{w(0) = 0,\, w(400) = 25\}$ and induce high effort.

  - The agent's expected utility is 12.5, <u>more than double</u> his reservation utility of 5.

- The principal's expected payoff is 337.5.
  - √ If the principal paid a lower expected wage, then the agent would exert low effort, and the principal would get 195.

- Since high enough punishments are infeasible, the principal has to use higher rewards.
  - √ The principal is willing to abandon a tight participation constraint.

- The two parts of the idea of the efficiency wage

  - The employer pays a wage higher than that needed to attract workers.

  - Workers are willing to be unemployed in order to get a chance at the efficiency-wage job.

8.2 Tournaments

- Games in which relative performance is important are called tournaments.

    - Like auctions, tournaments are especially useful
      when the principal wants to elicit information from the agents.

    - A principal-designed tournament is sometimes called
      a yardstick competition
      because the agents provide the measure for their wages.

- Farrell (2001) makes a subtler point:

  Although the shareholders of a monopoly maximize profit,
  the managers maximize their own utility, and moral hazard is severe
  without the benchmark of other firms' performances.

- The Firm Apex Game

  - Players

    - √ the shareholders (the principal) and the manager (the agent)

  - The order of play

    1 The shareholders offer the manager a contract which pays $w(c)$ depending on production cost, c.

    2 The manager decides whether to accept or reject the contract.

    3 The firm has two possible production techniques, *Fast* and *Careful*.

    Nature chooses production cost according to the table below.

4 If the manager accepts the contract, he <u>chooses a technique without investigating</u> the costs of both techniques
or does so <u>after investigating</u> them at a utility cost to himself of α.

5 The shareholders <u>can observe</u> the production technique chosen by the manager and the resulting production cost,
but <u>not</u> whether the manager investigates.

- Payoffs

√ If the manager rejects the contract,
then $\pi_{agent} = \bar{U} = \log \bar{w}$ and $\pi_{principal} = 0$.

√ If the manager accepts the contract,

$$\pi_{agent} = \log w(c) \quad \text{if he does not investigate}$$
$$\log w(c) - \alpha \quad \text{if he investigates}$$

$$\pi_{principal} = ? - w(c)$$

√ Probabilities of Production Costs in the Firm Apex Game

	Low cost $(c = 1)$	*High cost* $(c = 2)$
Fast technique	θ	$1 - \theta$
Careful technique	θ	$1 - \theta$

- The contract must satisfy the incentive compatibility constraint and the participation constraint.

  - $w_1 \equiv w(1)$ and $w_2 \equiv w(2)$

  - The incentive compatibility constraint

    - √ π_{agent} (*Investigate*) $\geq$ π_{agent} (*Not investigate*)

      $$\{1-(1-\theta)^2\}\{\log w_1 - \alpha\} + (1-\theta)^2\{\log w_2 - \alpha\}$$

      $$\geq \theta \log w_1 + (1-\theta)\log w_2$$

    - √ It is binding since the shareholders want to keep the manager's compensation to a minimum.

      $$\theta(1-\theta) \log (w_1/w_2) = \alpha$$

- The participation constraint

  - √ $\pi_{agent}\ (Investigate) \; = \; \bar{U}$

    $$\{1-(1-\theta)^2\}\log w_1 + (1-\theta)^2 \log w_2 \; = \; \log \bar{w}$$

  - √ It is binding.

- The contract that satisfies both constraints is

  $$w_1^{o} \; = \; \bar{w}\exp(\alpha/\theta)$$

  and

  $$w_2^{o} \; = \; \bar{w}\exp\{-\alpha/(1-\theta)\}.$$

- The expected <u>cost</u> to the firm is

$$\{1 - (1-\theta)^2\}\, w_1^{\mathrm{o}} + (1-\theta)^2\, w_2^{\mathrm{o}}.$$

  - √ Assume that $\theta = 0.1$, $\alpha = 1$, and $\overline{w} = 1$.

    Then the rounded values are $w_1^{\mathrm{o}} = 22.026$ and $w_2^{\mathrm{o}} = 0.33$.

  - √ The expected cost to the firm is 4.185.

  - √ Quite possibly, the shareholders decide it is not worth making the manager investigate.

- The Apex and Brydox Game

  - The shareholders of each firm can threaten to boil their manager in oil if the other firm adopts a low-cost technology and their firm does not.

  - Apex's <u>forcing contract</u> specifies

    $w_1 = w_2$ to fully insure the manager,

    and

    boiling-in-oil if Brydox has lower costs than Apex.

    √ The contract need satisfy only the <u>participation constraint</u> that

    $$\log w - \alpha = \bar{U} = \log \bar{w}.$$

- √ Assume that $\theta = 0.1$, $\alpha = 1$, and $\overline{w} = 1$.

  Then $w = 2.72$, and
  Apex's <u>cost</u> of extracting the manager's information is only 2.72, not 4.185.

- ○ Competition raises efficiency, not through the threat of firms going bankrupt but through the threat of managers being fired.

♦ Tournaments

- ○ Situations where competition between two agents can be used to <u>simplify</u> the optimal contract

8.3 Institutions and Agency Problems

- ♦ Ways to Alleviate Agency Problems
  - √ When agents are <u>risk averse</u>, the first-best cannot be achieved.

- ○ Reputation
- ○ Risk-sharing contracts
- ○ Boiling in oil
- ○ Selling the store
- ○ Efficiency wages
- ○ Tournaments

- ○ Monitoring
- ○ Repetition
- ○ Changing the type of the agent

- ♦ Government Institutions and Agency Problems
  - ○ Who should bear the cost of an accident, the pedestrian or the driver?
    - √ Who has the most severe moral hazard?
    - √ the least-cost avoider principle
  - ○ Criminal law is also concerned with tradeoffs between incentives and insurance.

- Private Institutions and Agency Problems
  - Agency theory also helps explain the development of many curious private institutions.
  - Having a zero marginal cost of computer time is a way around the moral hazard of slacking on research.
  - Longterm contracts are an important occasion for moral hazard, since so many variables are unforeseen, and hence noncontractible.
    - √ The term opportunism has been used to describe the behavior of agents who take advantage of noncontractibility to increase their payoff at the expense of the principal.
    - √ hold-up potential
- It should be clear from the variety of these examples that moral hazard is a common problem.

8.4 Renegotiation: The Repossession Game

- The players have signed a binding contract,
  but in a subsequent subgame,
  both might agree to scrap the old contract and write a new one,
  using the old contract as a starting point in their negotiations.

- Here we use a model of hidden actions to illustrate renegotiation,
  a model in which a bank that wants to lend money
  to a consumer to buy a car
  must worry about whether he will work hard enough to repay the loan.

  - As we will see, the outcome is Pareto superior
    if renegotiation is not possible.

- ♦ Repossession Game I

  - ○ Players

    - √ a bank and a consumer

  - ○ The order of play

    1 The bank can do nothing or it can at cost 11 offer the consumer an auto loan which allows him to buy a car that costs 11, but requires him to pay back L or lose possession of the car to the bank.

    2 The consumer accepts the loan and buys the car, or rejects it.

    3 The consumer chooses to *Work*, for an income of 15, or *Play*, for an income of 8. The disutility of work is 5.

4 The consumer repays the loan or defaults.

5 If the bank has not been paid L, it repossesses the car.

- Payoffs

  √ If the consumer chooses *Work*, his income is $W = 15$ and his disutility of effort is $D = 5$.

  √ If the consumer chooses *Play*, then $W = 8$ and $D = 0$.

  √ If the bank does not make any loan or the consumer rejects it, the bank's payoff is zero and the consumer's payoff is $W - D$.

  √ The value of the car is 12 to the consumer and 7 to the bank, so the bank's payoff if the loan is made is

$$\pi_{bank} = \begin{cases} L - 11 & \text{if the loan is repaid} \\ 7 - 11 & \text{if the car is repossessed.} \end{cases}$$

√ The consumer's payoff is

$$\pi_{consumer} = \begin{cases} W + 12 - L - D & \text{if the loan is repaid} \\ W - D & \text{if the car is repossessed.} \end{cases}$$

- The model allows commitment in the sense of
  legally binding agreements over transfers of money and wealth
  but it does not allow the consumer to commit directly to *Work*.

- It does not allow renegotiation.

- In equilibrium

    - The bank's strategy is to offer $L = 12$.

    - The consumer's strategy

        - *Accept* if $L \leq 12$

        - *Work* if $L \leq 12$ and he has accepted the loan or
          if he has rejected the loan
          (or if the bank does not make any loan)

        - *Repay* if $W + 12 - L - D \geq W - D$

    - The equilibrium outcome is that the bank offers $L = 12$,
      the concumer accepts, he works, and he repays the loan.

- The bank's equilibrium payoff is 1.

- This outcome is efficient because the consumer does buy the car,
  which he values at more than its cost to the car dealer.

- The bank ends up with the surplus,
  because of our assumption that the bank has
  all the bargaining power over the terms of the loan.

- Repossession Game II

  - Players

    - √ a bank and a consumer

  - The order of play

    1. The bank can do nothing or it can at cost 11 offer the consumer an auto loan which allows him to buy a car that costs 11, but requires him to pay back *L* or lose possession of the car to the bank.

    2. The consumer accepts the loan and buys the car, or rejects it.

    3. The consumer chooses to *Work*, for an income of 15, or *Play*, for an income of 8. The disutility of work is 5.

4 The consumer repays the loan or defaults.

4a The bank offers to settle for an amount *S* and leave possession of the car to the consumer.

4b The consumer accepts or rejects the settlement *S*.

5 If the bank has not been paid *L* or *S*, it repossesses the car.

- Payoffs

  √ If the consumer chooses *Work*, his income is $W = 15$ and his disutility of effort is $D = 5$.

  √ If the consumer chooses *Play*, then $W = 8$ and $D = 0$.

  √ If the bank does not make any loan or the consumer rejects it, the bank's payoff is zero and the consumer's payoff is $W - D$.

- √ The value of the car is 12 to the consumer and 7 to the bank, so the bank's payoff if the loan is made is

$$\pi_{bank} = \begin{cases} L - 11 & \text{if the original loan is repaid} \\ S - 11 & \text{if a settlement is made} \\ 7 - 11 & \text{if the car is repossessed.} \end{cases}$$

- √ The consumer's payoff is

$$\pi_{consumer} = \begin{cases} W + 12 - L - D & \text{if the original loan is repaid} \\ W + 12 - S - D & \text{if a settlement is made} \\ W - D & \text{if the car is repossessed.} \end{cases}$$

- ○ The model does allow renegotiation.

- In equilibrium
  - The equilibrium in Repossession Game I <u>breaks down</u> in Repossession Game II.
    - √ The consumer would deviate by choosing *Play*.
    - √ The bank chooses to <u>renegotiate</u> and offer $S = 8$.
    - √ The offer is accepted by the consumer.
    - √ Looking ahead to this, the bank refuses to make the loan.

- The bank's strategy in equilibrium

  - √ It does not offer a loan at all.

  - √ If it did offer a loan and the consumer accepted and defaulted, then it offers

    $S = 12$ if the consumer chose *Work*

    and

    $S = 8$ if the consumer chose *Play*.

- The consumer's strategy in equilibrium

  - √ *Accept* any loan made, whatever the value of L

  - √ *Work* if he rejected the loan
    (or if the bank does not make any loan)

    Play and Default otherwise

  - √ *Accept* a settlement offer of

    $S = 12$ if he chose *Work*

    and

    $S = 8$ if he chose *Play*

- The <u>equilibrium outcome</u> is that the bank does not offer a loan and the consumer chooses *Work*.

- Renegotiation turns out to be <u>harmful</u>,
  because it results in an equilibrium in which the bank refuses to make the loan, reducing the payoffs of the bank and the consumer to (0,10) instead of (1,10).

  - √ The gains from trade vanish.

♦ Renegotiation is <u>paradoxical</u>.

- In the subgame starting with consumer default, <u>it increases efficiency</u>,
  by allowing the players to make a Pareto improvement over an <u>inefficient punishment</u>.

- In the game as a whole, however, <u>it reduces efficiency</u>
  by preventing players from using punishments to deter inefficient actions.

- The Repossession Game illustrates other ideas too.

  - It is a game of perfect information,
    but it has the feel of a game of moral hazard with hidden actions.

  - This is because it has an implicit bankruptcy constraint,
    so that the contract cannot sufficiently punish the consumer
    for an inefficient choice of effort.

  - Restricting the strategy space has the same effect
    as restricting the information available to a player.

  - It is another example of the distinction between
    observability and contractibility.

8.5 State-Space Diagrams: Insurance Games I and II

♦ Suppose Smith (the agent) is considering buying theft insurance for a car with a value of 12.

♦ A state-space diagram

- ○ A diagram whose axes measure the values of one variable in two different states of the world

- ○ His endowment is $\omega = (12, 0)$.

♦ Insurance Game I: Observable Care

- ○ Players

  - √ Smith and two insurance companies

- The order of play

1. Smith chooses to be either *Careful* or *Careless*, observed by the insurance company.
2. Insurance company 1 offers a contract (x, y), in which Smith pays premium x and receives compensation y if there is a theft.
3. Insurance company 2 also offers a contract of the form (x, y).
4. Smith picks a contract.
5. Nature chooses whether there is a theft, with probability 0.5 if Smith is *Careful* or 0.75 if Smith is *Careless*.

- Payoffs
  - √ Smith is risk-averse and the insurance companies are risk-neutral.
  - √ The insurance company not picked by Smith has a payoff of zero.
  - √ Smith's utility function U is such that $U' > 0$ and $U'' < 0$.
  - √ If Smith chooses *Careful*, the payoffs are

$$\pi_{Smith} = 0.5U(12 - x) + 0.5U(0 + y - x)$$

and

$$\pi_{company} = 0.5x + 0.5(x - y) \quad \text{for his insurer.}$$

  - √ If Smith chooses *Careless*, the payoffs are

$$\pi_{Smith} = 0.25U(12 - x) + 0.75U(0 + y - x) + \epsilon$$

and

$$\pi_{company} = 0.25x + 0.75(x - y) \quad \text{for his insurer.}$$

- The optimal contract with only the *Careful* type
  - If the insurance company <u>can require</u> Smith to park <u>carefully</u>,
    it offers him insurance at a premium of 6,
    with a payout of 12 if theft occurs,
    leaving him with an allocation of $C_1 = (6, 6)$.
    - √ $(x, y) = (6, 12)$
  - This satisfies the <u>competition constraint</u> because it is the most attractive contract any company can offer without making losses.
    - √ An insurance policy (x, y) is <u>actuarially fair</u> if the cost of the policy is precisely its expected value.
    - √ $x = 0.5y$
  - Smith is <u>fully insured</u>.
    - √ His allocation is 6 no matter what happens.

- In equilibrium

  - Smith <u>chooses to be *Careful*</u> because he foresees that otherwise his insurance will be <u>more expensive</u>.

  - Edgeworth box

  - The company is <u>risk-neutral</u>, so its indifference curves are straight lines with a slope of -1.

  - Smith is <u>risk-averse</u>, so (if he is *Careful*) his indifference curves are <u>closest to the origin</u> on the 45^{o} line, where his wealth in the two states is equal.

- The equilibrium contract is C_1.
    - √ It satisfies the competition constraint
      by generating the highest expected utility for Smith.
    - √ It allows nonnegative profits to the company.

- ♦ Insurance Game I is a game of <u>symmetric information</u>.

- ♦ Suppose that Smith's action is a <u>noncontractible variable</u>.
    - We model the situation by putting Smith's move <u>second</u>.

- Insurance Game II: Unobservable Care

  - Players

    - √ Smith and two insurance companies

  - The order of play

    1. Insurance company 1 offers a contract of form (x, y), under which Smith pays premium x and receives compensation y if there is a theft.

    2. Insurance company 2 offers a contract of form (x, y).

    3. Smith picks a contract.

    4. Smith chooses either *Careful* or *Careless*.

5 Nature chooses whether there is a theft, with probability 0.5 if Smith is *Careful* or 0.75 if Smith is *Careless*.

- Payoffs

  - √ Smith is risk-averse and the insurance companies are risk-neutral.

  - √ The insurance company not picked by Smith has a payoff of zero.

  - √ Smith's utility function U is such that $U' > 0$ and $U'' < 0$.

- √ If Smith chooses *Careful*, the payoffs are

$$\pi_{Smith} \;=\; 0.5U(12 - x) \;+\; 0.5U(0 + y - x)$$

and

$$\pi_{company} \;=\; 0.5x \;+\; 0.5(x - y) \quad \text{for his insurer.}$$

- √ If Smith chooses *Careless*, the payoffs are

$$\pi_{Smith} \;=\; 0.25U(12 - x) \;+\; 0.75U(0 + y - x) \;+\; \epsilon$$

and

$$\pi_{company} \;=\; 0.25x \;+\; 0.75(x - y) \quad \text{for his insurer.}$$

- No full-insurance contract will be offered.
  - If Smith is fully insured, his dominant strategy is *Careless*.
  - The company knows the probability of a theft is 0.75.
  - The insurance company must offer a contract with a premium of 9 and a payout of 12 to prevent losses, which leaves Smith with an allocation $C_2 = (3, 3)$.
  - The insurance company's isoprofit curve swivels around ω because that is the point at which the company's profit is independent of how probable it is that Smith's car will be stolen.
    - √ At point ω, the company is not insuring him at all.

- ○ Smith's indifference curve swivels around the intersection of the $\pi_s = 66$ curve with the 45° line, because on that line the probability of theft does not affect his payoff.

- ○ Smith would like to commit himself to being careful, but he cannot make his commitment credible.

- ♦ The outlook is bright because Smith chooses *Careful* if he only has partial insurance, as with contract C_3.

- ○ The moral hazard is "small" in the sense that Smith barely prefers *Careless*.

- ○ Deductibles and coinsurance

- ○ The solution of full insurance is "almost" reached.

- Even when the ideal of full insurance and efficient effort <u>cannot</u> be reached, there exists some best choice like C_5 in the set of feasible contracts, a <u>second-best insurance contract</u> that recognizes the constraints of informational asymmetry.

8.6 Joint Production by Many Agents: The Holmstrom Teams Model

- The existence of a <u>group of agents</u> results in destroying the effectiveness of the individual risk-sharing contracts, because observed output is a joint <u>function</u> of the <u>unobserved effort</u> of many agents.

- The actions of a group of players produce a <u>joint output</u>, and each player wishes that the others would carry out the costly actions.

- A <u>team</u> is a group of agents who <u>independently</u> choose effort levels that result in a <u>single output</u> for the entire group.

- Teams

  - Players
    - √ a principal and n agents

  - The order of play
    1. The principal offers a contract <u>to each agent</u> i of the form $w_i(q)$, where q is total output.
    2. The agents decide whether or not to accept the contract.
    3. The agents simultaneously pick effort levels e_i, $(i = 1, \ldots, n)$.
    4. Output is $q(e_1, \ldots, e_n)$.

  - Payoffs
    - √ If any agent rejects the contract, all payoffs equal zero.
    - √ Otherwise,

$$\pi_{principal} = q - \sum_{i=1}^{n} w_i$$

and

$$\pi_i = w_i - v_i(e_i), \text{ where } v_i' > 0 \text{ and } v_i'' > 0.$$

- The principal can observe output.
- The team's problem is cooperation between agents.

- Efficient contracts
  - Denote the efficient vector of actions by e^*.
  - An efficient contract is

$$w_i(q) = \begin{cases} b_i & \text{if } q \geq q(e^*) \\ 0 & \text{if } q < q(e^*), \end{cases} \tag{8.9}$$

where $\sum_{i=1}^{n} b_i = q(e^*)$ and $b_i > v_i(e_i^*)$.

  - The teams model gives one reason to have a principal:
    he is the residual claimant who keeps the forfeited output.

- Budget balancing and Proposition 8.1

  - The budget-balancing constraint

    - The sum of the wages exactly equal the output.

  - If there is a budget-balancing constraint,
    no differentiable wage contract $w_i(q)$ generates an efficient Nash equilibrium.

    - Agent i's problem is

$$\underset{e_i}{Maximize} \quad w_i(q(e)) - v_i(e_i).$$

His first-order condition is

$$(dw_i/dq)\,(\partial q/\partial e_i) - dv_i/de_i = 0.$$

- $\surd$ The <u>Pareto optimum</u> solves

$$\underset{e_1, \ldots, e_n}{\mathit{Maximize}} \quad q(e) - \sum_{i=1}^{n} v_i(e_i).$$

The first-order condition is that the marginal dollar contribution equal the marginal disutility of effort:

$$\partial q / \partial e_i - dv_i / de_i = 0.$$

- $\surd$ $dw_i / dq \neq 1$

Under budget balancing, <u>not every agent</u> can receive the <u>entire</u> marginal increase in output.

- $\surd$ Because each agent bears the <u>entire burden</u> of his marginal effort and only <u>part of the benefit</u>, the contract <u>does not</u> achieve the first-best.

- Without budget balancing,
  if the agent shirked a little he would gain the entire leisure benefit from shirking, but he would lose his entire wage under the optimal contract in equation (8.9).

- With budget balancing and a linear utility function,
  the Pareto optimum maximizes the sum of utilities.

  - A Pareto efficient allocation is one where consumer 1 is as well-off as possible given consumer 2's level of utility.

    - √ Fix the utility of consumer 2 at $\bar{u}_2$.

- $$\underset{e_1,\, e_2}{\textit{Maximize}} \quad w_1(q(e)) - v_1(e_1)$$

  subject to
  $$w_2(q(e)) - v_2(e_2) \geq \bar{u}_2$$
  and
  $$w_1(q(e)) + w_2(q(e)) = q(e)$$

- $$\underset{e_1,\, e_2}{\textit{Maximize}} \quad w_1(q(e)) - v_1(e_1)$$

  subject to
  $$q(e) - v_2(e_2) - \bar{u}_2 = w_1(q(e))$$

- $$\underset{e_1,\, e_2}{\textit{Maximize}} \quad q(e) - (v_1(e_1) + v_2(e_2)) - \bar{u}_2$$

- ♦ Discontinuities in Public Good Payoffs

  - ○ There is a free rider problem
    if several players each pick a level of effort which increases the level of some public good whose benefits they share.

    - √ Noncooperatively, they choose effort levels lower than if they could make binding promises.

  - ○ Consider a situation in which n identical risk-neutral players produce a public good by expending their effort.

    - √ Let e_i represent player i's effort level, and
      let $q(e_1, \ldots, e_n)$ the amount of the public good produced,
      where q is a continuous function.

√ Player i's problem is

$$\underset{e_i}{Maximize} \quad q(e_1, \ldots, e_n) - e_i.$$

His first-order condition is

$$\partial q/\partial e_i - 1 = 0.$$

√ The greater, first-best n-tuple vector of effort levels e^* is characterized by

$$\sum_{i=1}^{n}(\partial q/\partial e_i) - 1 = 0.$$

○ If the function q were discontinuous at e^* (for example, $q = 0$ if $e_i < e_i^*$ and $q = e_i$ if $e_i \geq e_i^*$ for any i), the strategy profile e^* could be a Nash equilibrium.

- The first-best can be achieved because the discontinuity at e^* makes every player the marginal, decisive player.

  - √ If he shirks a little, output falls drastically and with certainty.

- Either of the following two modifications restores the free rider problem and induces shirking:

  - √ Let q be a function not only of effort but of random noise.
    Nature moves after the players.
    Uncertainty makes the expected output a continuous function of effort.

  - √ Let players have incomplete information about the critical value.
    Nature moves before the players and chooses e^*.
    Incomplete information makes the estimated output a continuous function of effort.

♦ The <u>discontinuity</u> phenomenon is common.

Examples include:

- Effort in teams
  (Holmstrom [1982], Rasmusen [1987])

- Entry deterrence by an oligopoly
  (Bernheim [1984b], Waldman [1987])

- Output in oligopolies with trigger strategies
  (Porter [1983a])

- Patent races

- Tendering shares in a takeover
  (Grossman & Hart [1980])

- Preferences for levels of a public good.

- Pareto optimum

  - $$\underset{e_1,\, e_2}{\mathit{Maximize}} \quad q(e_1, e_2) - e_1$$

    subject to

    $$q(e_1, e_2) - e_2 = \bar{u}_2$$

  - To solve the maximization problem,
    we set up the Lagrangian function:

$$L = q(e_1, e_2) - e_1 - \lambda\{q(e_1, e_2) - e_2 - \bar{u}_2\}.$$

We have the following set of simultaneous equations:

$$\partial L/\partial\lambda \;=\; -\{q(e_1, e_2) - e_2 - \overline{u}_2\} \;=\; 0$$

$$\partial L/\partial e_1 \;=\; \partial q/\partial e_1 \;-\; 1 \;-\; \lambda\partial q/\partial e_1 \;=\; 0 \qquad \text{(A1)}$$

$$\partial L/\partial e_2 \;=\; \partial q/\partial e_2 \;-\; \lambda(\partial q/\partial e_2 - 1) \;=\; 0. \qquad \text{(A2)}$$

Using expressions (A1) and (A2), we obtain

$$(1 \;-\; \lambda)\sum_{i=1}^{2}(\partial q/\partial e_i) \;=\; 1 \;-\; \lambda,$$

which leads to $\displaystyle\sum_{i=1}^{2}(\partial q/\partial e_i) - 1 \;=\; 0.$

8.7 The Multitask Agency Problem

- Holmstrom and Milgrom (1991)
  - Often the principal wants the agent to split his time among several tasks, each with a separate output, rather than just working on one of them.
  - If the principal uses one of the incentive contracts to incentivize just one of the tasks, this "high-powered incentive" can result in the agent completely neglecting his other tasks and leave the principal worse off than under a flat wage.

- ♦ Multitasking I: Two Tasks, No Leisure
  - ○ Players
    - √ a principal and an agent
  - ○ The order of play

    1 The principal offers the agent either an incentive contract of the form $w(q_1)$ or a monitoring contract that pays m under which he pays the agent m_1 if he observes the agent working on Task 1 and m_2 if he observes the agent working on Task 2.

    2 The agent decides whether or not to accept the contract.

    3 The agent picks effort levels e_1 and e_2 for the two tasks such that $\boldsymbol{e_1 + e_2 = 1}$, where 1 denotes the total time available.

    4 Outputs are $q_1(e_1)$ and $q_2(e_2)$, where $dq_1/de_1 > 0$ and $dq_2/de_2 > 0$ but we do not require decreasing returns to effort.

- Payoffs

  - √ If the agent rejects the contract, all payoffs equal zero.

  - √ Otherwise,

$$\pi_{principal} = q_1 + \beta q_2 - m - w - C$$

and

$$\pi_{agent} = m + w - e_1^2 - e_2^2,$$

where C, the cost of monitoring, is $\bar{C}$ if a monitoring contract is used and zero otherwise.

  - √ β is a measure of the relative value of Task 2.

- The principal <u>can observe</u> the output from one of the agent's tasks (q_1) but <u>not</u> from the other (q_2).

- The first best can be found by choosing e_1 and e_2 (subject to $e_1 + e_2 = 1$) and C to maximize the sum of the payoffs.

  - $$\underset{e_1, e_2, C}{Maximize} \quad \pi_{principal} = q_1(e_1) + \beta q_2(e_2) - m - w - C$$

    subject to

    $$\pi_{agent} = m + w - e_1^2 - e_2^2 \geq \bar{U} = 0$$

    and

    $$e_1 + e_2 = 1$$

  - $$\underset{e_1, e_2, C}{Maximize} \quad \pi_{principal} + \pi_{agent} - \bar{U}$$

    subject to

    $$e_1 + e_2 = 1$$

- The first-best levels of the variables

  - √ $C^* = 0$
  - √ $e_1^* = 0.5 + 0.25\{dq_1/de_1 - \beta(dq_2/de_2)\}$ (8.19)
  - √ $e_2^* = 0.5 - 0.25\{dq_1/de_1 - \beta(dq_2/de_2)\}$
  - √ $q_i^* \equiv q_i(e_i^*)$

  - √ Define the minimum wage payment that would induce the agent to accept a contract requiring the first-best effort levels as

    $$w^* \equiv (e_1^*)^2 + (e_2^*)^2.$$

- Can an incentive contract achieve the first best?
  - A profit-maximizing flat-wage contract
    - $w(q_1) = w^o$ or the monitoring contract $\{w^o, w^o\}$
    - The agent chooses $e_1^o = e_2^o = 0.5$.
    - $w^o = 0.5$ satisfies the participation constraint.
  - A sharing-rule incentive contract
    - $dw/dq_1 > 0$
    - The greater the agent's effort on Task 1,
      the less will be his effort on Task 2.
    - Even if extra effort on Task 1 could be achieved for free,
      the principal might not want it – and, in fact, he might be willing
      to pay to stop it.

- The simplest sharing-rule (incentive) contract

  - √ the linear contract

    $$w(q_1) = a + bq_1$$

  - √ The agent will pick e_1 and e_2 to maximize

    $$\pi_{agent} = a + bq_1(e_1) - e_1^2 - e_2^2$$

    subject to $e_1 + e_2 = 1$.

  - √ $e_1^o = 0.5 + 0.25b(dq_1/de_1)$ (8.23)

  - √ If $e_1^* \geq 0.5$, the linear contract will work just fine.

    The contract parameters a and b can be chosen so that
    the linear-contract effort level in equation (8.23) is the same as
    the first-best effort level in equation (8.19),
    with a taking a value to extract all the surplus
    so the participation constraint is barely satisfied.

√ If $e_1^* < 0.5$, the linear contract cannot achieve the first best with a positive value for b.

The contract must actually punish the agent for high output!

○ In equilibrium, the principal chooses some contract that elicits the first-best effort e^*, such as the forcing contract,

$$w(q_1 = q_1^*) \; = \; w^*$$

and

$$w(q_1 = q_1^*) \; = \; 0.$$

- A monitoring contract
  - The cost $\bar{C}$ of monitoring is incurred.
  - The agent will pick e_1 and e_2 to maximize

$$\pi_{agent} = e_1 m_1 + e_2 m_2 - e_1^2 - e_2^2$$

subject to $e_1 + e_2 = 1$.

    - √ The principal finds the agent working on Task i with probability e_i.
    - √ $\pi_{agent} = e_1 m_1 + (1 - e_1) m_2 - e_1^2 - (1 - e_1)^2$
    - √ $d\pi_{agent}/de_1 = m_1 - m_2 - 2e_1 + 2(1 - e_1) = 0$

- If the principal wants the agent to pick e_1^*,
  he should choose m_1^* and m_2^* so that

$$m_1^* = 4e_1^* + m_2^* - 2.$$

  - √ the binding participation constraint

$$e_1^* m_1^* + (1 - e_1^*)m_2^* - (e_1^*)^2 - (1 - e_1^*)^2 = 0$$

- $m_1^* = 4e_1^* - 2(e_1^*)^2 - 1$

  $m_2^* = 1 - 2(e_1^*)^2$

  - √ $e_1^* > e_2^* \quad \Rightarrow \quad m_1^* > m_2^*$
  - √ $dm_1^*/de_1^* > 0$
  - √ $dm_2^*/de_1^* < 0$

♦ Multitasking II: Two Tasks Plus Leisure

- Players
  - √ a principal and an agent

- The order of play

1 The principal offers the agent either an <u>incentive contract</u> of the form $w(q_1)$ or a <u>monitoring contract</u> that pays m under which he pays the agent a base wage of $\overline{m}$ plus
m_1 if he observes the agent working on Task 1 and
m_2 if he observes the agent working on Task 2.

2 The agent decides whether or not to accept the contract.

3 The agent picks effort levels e_1 and e_2 for the two tasks.

4 Outputs are $q_1(e_1)$ and $q_2(e_2)$,
where $dq_1/de_1 > 0$ and $dq_2/de_2 > 0$
but we do not require decreasing returns to effort.

- Payoffs

  - √ If the agent rejects the contract, all payoffs equal zero.

  - √ Otherwise,

    $$\pi_{principal} = q_1 + \beta q_2 - m - w - C$$

    and

    $$\pi_{agent} = m + w - e_1^2 - e_2^2,$$

    where C, the cost of monitoring, is $\overline{C}$ if a monitoring contract is used and zero otherwise.

  - √ β is a measure of the relative value of Task 2.

- The principal <u>can observe</u> the output from one of the agent's tasks (q_1) but <u>not</u> from the other (q_2).

- $\boldsymbol{e_1 + e_2 \leq 1}$

  - √ The amount $(1 - e_1 - e_2)$ represents leisure,
    whose value we set equal to zero in the agent's utility function.

  - √ Here leisure represents not time off the job,
    but time on the job spent shirking rather than working.

- The first-best can be found by choosing e_1, e_2, and C to maximize the sum of the payoffs.

  - $$\underset{e_1,\, e_2,\, C}{\textit{Maximize}} \qquad \pi_{principal} = q_1(e_1) + \beta q_2(e_2) - m - w - C$$

    subject to

    $$\pi_{agent} = m + w - e_1^2 - e_2^2 \geq 0$$

    and

    $$e_1 + e_2 \leq 1$$

  - $$\underset{e_1,\, e_2,\, C}{\textit{Maximize}} \qquad q_1(e_1) + \beta q_2(e_2) - C - e_1^2 - e_2^2$$

    subject to

    $$e_1 + e_2 \leq 1$$

- The first-best levels of the variables

  - √ $C^{**} = 0$

  - √ $e_1^{**} = ?$

  - √ $e_2^{**} = ?$

  - √ $q_i^{**} \equiv q_i(e_i^{**})$

  - √ Define the minimum wage payment that would induce the agent to accept a contract requiring the first-best effort levels as

    $$w^{**} \equiv (e_1^{**})^2 + (e_2^{**})^2.$$

  - √ <u>Positive leisure</u> for the agent in the first-best is a <u>realistic</u> case.

- Can an incentive contract achieve the first best?
  - A <u>flat-wage</u> contract
    - √ $w(q_1) = w^{oo}$ or the monitoring contract $\{w^{oo}, w^{oo}\}$
    - √ The agent chooses $e_1^{oo} = e_2^{oo} = 0$.
    - √ A <u>low-powered</u> incentive contract is disastrous, because pulling the agent away from high effort on Task I does not leave him working harder on Task 2.

- A <u>high-powered</u> sharing-rule incentive contract

  - √ $dw/dq_1 > 0$

  - √ The first-best is <u>unreachable</u> since $e_2^{oo} = 0$.

  - √ The combination ($e_1^{oo} = e_1^{**}$, $e_2^{oo} = 0$) is the <u>second-best</u> incentive-contract solution, since at e_1^{**} the marginal disutility of effort equals the marginal utility of the marginal product of effort.

  - √ In that case, in the second-best the principal would push e_1^{oo} <u>above the first-best level</u>.

- The agent does not substitute between the task with easy-to-measure output and the task with hard-to-measure output,
  but between each task and leisure.

  - The best the principal can do may be
    to ignore the multitasking feature of the problem and
    just get the incentives right for the task whose output
    he can measure.

- A monitoring contract
  - The first-best effort levels can be attained.
  - The monitoring contract might not even be superior to the second-best incentive contract if the monitoring cost $\bar{C}$ were too big.
    - √ But monitoring can induce any level of e_2 the principal desires.
  - The base wage may even be negative, which can be interpreted
    - √ as a bond for good effort posted by the agent or
    - √ as a fee he pays for the privilege of filling the job and possibly earning m_1 or m_2.

- The agent will choose e_1 and e_2 to maximize

$$\pi_{agent} = \overline{m} + e_1 m_1 + e_2 m_2 - e_1^2 - e_2^2$$

subject to $e_1 + e_2 \leq 1$.

  - √ The principal finds the agent working on Task i with probability e_i.

  - √ $\partial\pi_{agent}/\partial e_1 = m_1 - 2e_1 = 0$

    $\partial\pi_{agent}/\partial e_2 = m_2 - 2e_2 = 0$

- The principal will pick m_1^{**} and m_2^{**} to induce the agent to choose e_1^{**} and e_2^{**}.

  - √ $m_1^{**} = 2e_1^{**}$

    $m_2^{**} = 2e_2^{**}$

- The base wage $\overline{m}$

  - √ the binding participation constraint

$$\pi_{agent} = \overline{m} + e_1^{**}m_1^{**} + e_2^{**}m_2^{**} - (e_1^{**})^2 - (e_2^{**})^2$$

$$= \overline{m} + 2w^{**} - w^{**} = 0$$

- $\bar{m} = -w^{**}$

- If the principal finds the agent shirking when he monitors, he will pay the agent an amount of $-w^{**}$.

- In the case where $e_1^{**} + e_2^{**} < 1$, the result is surprising because the principal wants the agent to take some leisure in equilibrium.

- In the case where $e_1^{**} + e_2^{**} = 1$, the result is intuitive.

- The key is that the base wage is important only for inducing the agent to take the job and has no influence whatsoever on the agent's choice of effort.