## G751, Spring 2011: Test 1

There are 100 points in total. Please start each of the 3 questions on a new sheet of paper. This is a closed-book exam.

Question 1 (30 points).

**Finding Equilibria**

| | | **Operating System Company** | | |
|---|---|---|---|---|
| | | *Cheap* | *Moderate* | *Careful* |
| | *Lots* | 0,0 | $-1, 0$ | $3, 4$ |
| **Appwriter:** | *Medium* | $3, 1$ | 0,-2 | $2, -1$ |
| | *Few* | $0, 1$ | 1,-1 | $1, -2$ |

(a) How far can you get using iterated deletion of weakly dominated strategies?

*Answer.*Moderate is dominated by Cheap for the OS Company, so it can be dropped. Few is then dominated by Lots for the Appwriter and can be dropped.

(b) What are the pure-strategy Nash equilibria?

*Answer.*(Medium, Cheap) and (Lots, Careful).

(c) What is the mixed-strategy Nash equilibrium?

*Answer.* Appwriter choose Medium with probability $\gamma$ and Lots with probability $(1-\gamma)$. OS Company chooses Cheap with probability $\theta$ and Careful with probability $1-\theta$.

$$\pi_{App}(Medium) = \theta(3) + (1-\theta)(2) = \pi_{App}(Lots) = \theta(0) + (1-\theta)(3) \qquad (1)$$

so $3\theta + 2 - 2\theta = 3 - 3\theta$ and $4\theta = 1$ so $\theta = .25$.

$$\pi_{OS}(Cheap) = \gamma(1) + (1-\gamma)(0) = \pi_{OS}(Careful) = \gamma(-1) + (1-\gamma)(4) \qquad (2)$$

so $\gamma = -\gamma + 4 - 4\gamma$ and $6\gamma = 4$ so $\gamma = 2/3$.

Question 2 (30 points). The ability of the VP of a company takes an integer value from 1 to 10. Initially, the CEO and the VP both think all values are equally probable. Then, they each see how the VP performed in designing a new business plan, and their information partitions become different. We assume, following Harsanyi, that each player knows the other player's partition, though not the particular state of the world that has been reached.

The CEO's information partition is (1,2) (3) (4,6) (5,7) (8) (9) (10). The VP's is (1,2,3) (4,5,6) (7) (8) (9,10).

(a) What is the finest common coarsening of the two information partitions?

*Answer.* (1,2,3) (4,5,6,7) (8) (9,10)

(b) Suppose we define the VP's ability to be high if it is 5,6,7, 8, 9 or 10. For what abilities is his high ability common knowledge?

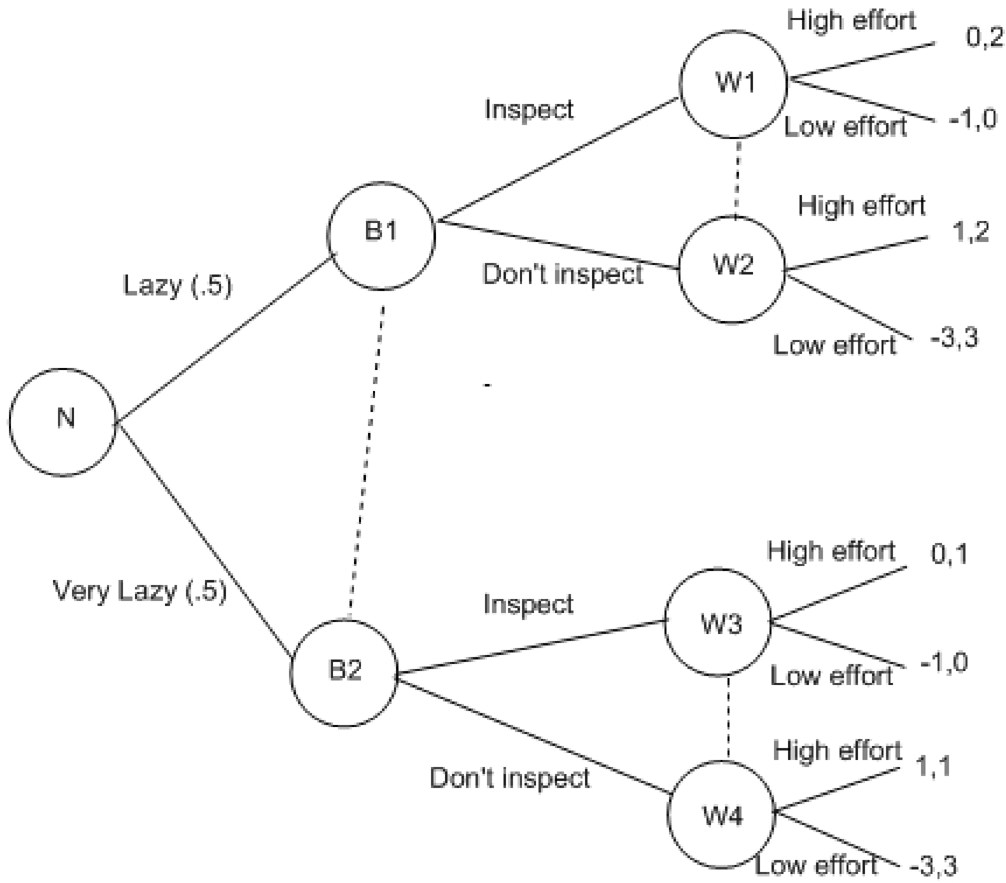
*Answer.* For 8, 9 or 10.

(c)For what abilities do both players know that the VP's ability is high?

*Answer.* For abilities 7,8, 9, and 10.

Question 3 ( 40 points). There are two types of workers. Type 1 workers are lazy, with a cost of 1 for high effort, and type 2 workers are even lazier, with a cost of 2 for high effort. Each type is half of the population. The boss must decide each day with what probability to inspect a worker's production, which costs the boss 1. The boss and worker simultaneously choose effort and whether to inspect. The boss earns value 4 from high effort production and 0 from low effort production. If he find the worker has not exerted high effort, the boss fires the worker with no pay,. If the boss does not inspect or finds that effort was high, the boss pays the worker 3. The boss's payoff is the value of production minus his effort cost and the amount the worker is paid. The worker's payoff is his pay minus his effort cost. The equilibrium will involve mixed strategies. Denote the type 1 worker's probability of high effort as $\theta_1$ and the type 2's as $\theta_2$, and the boss's probability of inspecting by $\gamma$.

(a) Draw the extensive form, putting the boss's decision before the worker's.




(b) Why are there no equilibria in which mixed strategies are not used by at least one player?

<u>Answer.</u> If the boss always inspects, then both types of worker would always choose high effort, yielding payoffs of 1 for the boss. But then the boss could deviate to Don't Inspect and raise his payoff to 2 by avoiding the cost of inspection. If the boss always chooses not to inspect, then the workers will always choose low effort, in which case the boss could increase his payoff by deviating to Inspect. Thus it is clear that the boss, at least, will use mixed strategies in any Nash equilibrium.

The way to answer a question like this is to show that some player would deviate from the hypothesized equilibrium. An equilibrium in which at least one player does not use mixed strategies is one in which the boss does not use pure strategies and the worker does not use pure strategies. Thus, start with the possible strategy profiles in which all the players use pure strategies, and show that in each of them at least one player has incentive to deviate. If even one player deviates, then the profile is not an equilibrium, so it is enough to show that the boss would deviate from any profile in which he plays a pure strategy and the worker plays the worker's best response.

(c) What would be each player's pure-strategy payoff as a function of the other player's mixing probabilities?

*Answer.* For the worker the payoff functions would be

$$\pi_1(high) = -1 + 3 = \pi_1(low) = \gamma(0) + (1-\gamma)(3)$$

$$\pi_2(high) = -2 + 3 = \pi_2(low) = \gamma(0) + (1-\gamma)(3)$$

Note that no one value of $\gamma$ can make both of these payoff equations true. Thus, one type of worker has to play a pure strategy. Either the Type 1 workers will always choose high effort, or the Type 2 workers will always choose low effort.

$$\pi_{boss}(inspect) = -1 + .5(\theta_1(4-3) + (1-\theta_1)(0)) + .5(\theta_2(4-3) + .5 + .5(1-\theta_2)(0)) = -1 + .5\theta_1 + .5\theta_2$$

$$\pi_{boss}(not\ inspect) = .5(\theta_1(4-3) + .5(1-\theta_1)(-3)) + .5(\theta_2(4-3) + .5(1-\theta_2)(-3)) = 2\theta_1 + 2\theta_2 - 3$$

I gave full credit if you instead answered with the players' overall payoff as a function of his own mixing probability too.

(d) What are the equilibrium strategies?

*Answer.* The key is to realize that since no one boss's mixing probability $\gamma$ can equate both the Type 1 and the Type 2 worker's payoffs, it must be that one type of worker plays a pure strategy. It could be that the Lazy type 1 always chooses high effort, or that the Very Lazy Type 2 always chooses low effort. The boss has the very low payoff of -3 when the worker is not inspected and chooses low effort. Let's guess that that means he will inspect enough that both types of workers have positive probability of high effort— which means the Type 1 workers will choose high effort with probability 1.

If we set $\theta_1 = 1$, then we can equate the boss's payoffs thus:

$$\pi_{boss}(inspect) = -1 + .5 + .5\theta_2 = \pi_{boss}(not\ inspect) = 2 + 2\theta_2 - 3$$

This results in $-.5 + .5\theta_2 = -1 + 2\theta_2$ and $\theta_2 = 1/3$.

Using Type 2's equating of payoffs,

$$\pi_2(high) = -2 + 3 = \pi_2(low) = \gamma(0) + (1 - \gamma)(3)$$

This yields $1 = 3 - 3\gamma$ so $\gamma = 2/3$.

The equilibrium is thus for all of the Type 1 workers to choose high effort, for .35 of the Type 2 workers to, and for the boss to inspect with probability 2/3.

What about the possibility of an equilibrium with $\theta_2 = 1$ and with type 1 workers mixing? If we set $\theta_2 = 1$ and try to equate the boss's pure-strategy payoffs we get:

$$\pi_{boss}(inspect) = -1 + .5\theta_1 + 0 = \pi_{boss}(not\ inspect) == 2\theta_1 + 2(0) - 3$$

This yields $2 = 1.5\theta_1$ and $\theta_1 = 4/3$, which is impossible. The problem is that if the type 2 workers always choose low effort, the boss will respond with $\gamma = 1$, always inspecting, even if all of the type 1 workers high choose effort.