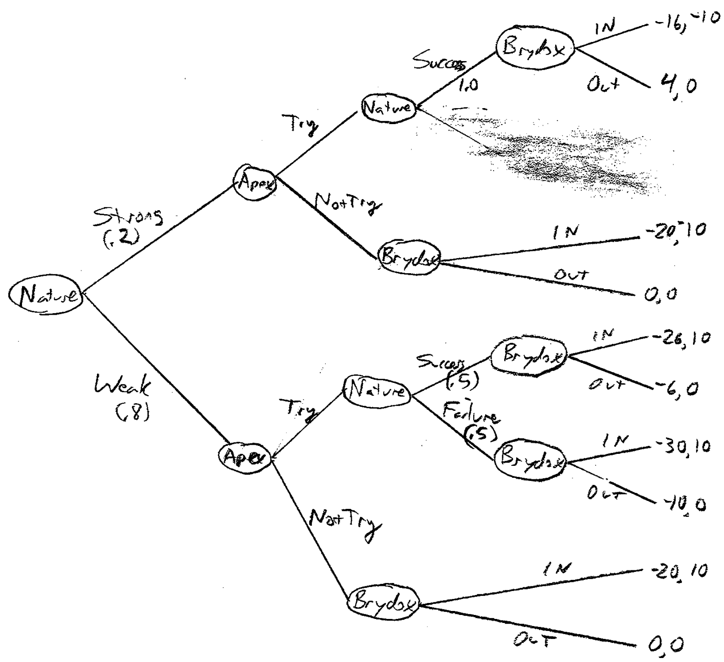
G751: Old Test Questions Relevant to Test 2

January 24, 2011

1. (long version– see the last part) Apex is currently the only company making widgets, but Brydox is thinking about entering the industry. Initially, Brydox thinks that Apex is a Weak company with probability .8 and a Strong company with probability .2. With no new product or entry, Apex's payoff is 0. Apex must decide whether to Try or $NotTry$ to introduce a new product, the superwidget, which would add 4 to its payoff. If Apex is strong, trying costs 0 and it always succeeds. If Apex is weak, trying costs 10, and it succeeds with probability .5. Brydox must decide whether to be In or Out of the industry after observing whether Apex starts selling superwidgets ($Success$) or not ($Failure$). If Brydox chooses In, that reduces Apex's payoff by 20. Brydox receives a payoff of 0 if it chooses Out. If Brydox chooses In and Apex is strong, Brydox's payoff is -10, but if Apex is weak, Brydox's payoff is +10.

(a) (3 points) Draw the extensive form of this game if both companies observe every move.

ANSWER. A couple of special notes:
1. Even if Apex chooses $NotTry$, Brydox chooses In or Out.
2. The weak Apex's expected gain from the superwidget product is -8 = .5(4) -10, but in the extensive form it shows up as -10 or -6, depending on Success or Failure.



(b) (3 points) If Brydox observes Apex's type, what is the Nash equilibrium?

ANSWER. Apex: $Try|Strong, NotTry|Weak$.
Brydox:$Enter|(Weak, NotTry), Enter|(Weak, Try, Success), Enter|(Weak, Try, Failure)$, $Out|(Strong, NotTry), Out|(Strong, Try, Success)$.

(c) (3 points) For the rest of the question, assume Brydox does not observe Apex's type, but does see if Apex sells the superwidget (Success) or not (Failure).

What is the strong Apex's strategy in any Nash equilibrium? Explain why there is no equilibrium in which the strong Apex chooses $NotTry$ and Brydox chooses $In|Success$? (or $In|Try$).

ANSWER. Apex's Nash strategy is $Try|Strong$. If we disregard Brydox's response to $Success$, Try yields 4 in extra payoff to Apex. Thus, the only reason for Apex not to choose Try would be if Try or $Success$ made Brydox enter but $NotTry$ or $Failure$ did not.

That is a conceivable reason. Consider the strategy profile,

$(NotTry|Strong, Try|Weak, In|Try, Out|NotTry)$.

The strong Apex would not deviate, because his equilibrium payoff is 0 and his deviation payoff is $-20+4$. This shows that $Try|Strong$ is not a dominant or weakly dominant strategy. The profile is not a Nash equilibrium, however, because the weak Apex would deviate to $NotTry$ in order to prevent Brydox from choosing In. In any equilibrium in which the strong Apex would choose $NotTry$, so would the weak Apex, since the weak Apex gets an expected 8 less in payoff from trying to invent the superwidget.

(d) (3 points) Why is it not an equilibrium for Apex to use a pooling strategy of always choosing Try?

ANSWER. We must look at Brydox's response. Bayes's Rule gives us Brydox's posterior probability.

$$Prob(Weak|Success) = \frac{Prob(Success|Weak)Prob(Weak)}{Prob(Success)}$$

$$= \frac{.5(.8)}{.5(.8)+(1)(.2)} = \frac{.40}{.60} = 2/3. \quad (1)$$

Thus, in the conjectured pooling equilibrium, Brydox would think Apex was weak with probability greater than .5 and Brydox would choose $In|Success$ as well as $In|Failure$ ($Failure$ would be a sure sign of a weak Apex). So the weak Apex gains no advantage from choosing Try and just has a payoff reduced by 8 (=.5(4)-10). Apex would deviate to $NotTry|Weak$.

(e) (3 points)Show why in equilibrium Apex will not use a separating strategy of $(Try|Strong, NotTry|Weak)$.

ANSWER. If Apex does, then Brydox will respond with $In|Failure, Out|Success$, because he knows that $Success$ is a perfect indicator of $Strong$. The weak Apex wants to deter entry if possible, because that hurts Apex by 20. Apex's equilibrium payoff is -20. If Apex deviates to $Try|Weak$, given Brydox's strategy, Apex's payoff is $-10+.5(4)+.5(-20) = -18$. Thus, Apex gains by deviating.

(f) (3 points) Suppose that in equilibrium Apex uses a strategy in which he chooses Try with probability .9 if he is strong, but only with probability

.2 if he is weak. If Brydox observes Apex selling the superwidget, what probability does Brydox assign to Apex being weak?

ANSWER. Note that $Prob(Success|Weak)$ is not .5, but .5(.2). That's because although $Prob(Success|Weak, Try) = .5$, if Apex is Weak, he tries with probability only .2.

$$Prob(Weak|Success) = \frac{Prob(Success|Weak)Prob(Weak)}{Prob(Success)}$$

$$= \frac{.5(.2)(.8)}{.5(.2)(.8)+(.9)(.2)} = \frac{.8}{.26} = 4/13. \tag{2}$$

(g) (3 points) What is an equilibrium of this game?

ANSWER. Earlier, we saw that there is no pure-strategy pooling or separating equilibrium. We have seen that $Try|Strong$ must be part of any equilibrium. It then follows that $In|Failure$ must be part of Brydox's equilibrium strategy, since $Failure$ is a sure sign of Apex being weak.

What must happen is that the weak Apex must succeed often enough so that $Success$ leads to a high enough probability that Apex is strong that Brydox is indifferent between In and Out.

Suppose Apex chooses $Try|Weak$ and $NotTry|Weak$ with probabilityies α and $1 - \alpha$, and Brydox chooses $In|Success$ and $Out|Success$ with probabilities β and $1 - \beta$.

First, equate the payoffs from Apex's pure strategies.

$$\pi(Try|Weak) \quad = -10 + .5(-20) + .5(\beta(4 - 20) + (1 - \beta)(4)) \tag{3}$$

and

$$\pi(NotTry|Weak) \quad = -20. \tag{4}$$

Equating these yields $-10 - 10 - 8\beta + 2 - 2\beta = -20$ so $2 = 10\beta$ and $\beta = .2$.

Now equate the payoffs from Brydox's pure strategies. These need a couple of posterior probabilities. Note that $Prob(Success|Weak)$ is not .5, but $.5(\alpha)$. That's because although $Prob(Success|Weak, Try) = .5$, if Apex is Weak, he tries with probability only α.

$$Prob(Weak|Success) = \frac{Prob(Success|Weak)Prob(Weak)}{Prob(Success)}$$

$$= \frac{.5\alpha(.8)}{.5\alpha(.8)+(1)(.2)} = \frac{40\alpha}{40\alpha+20} = \frac{2\alpha}{2\alpha+1}. \quad (5)$$

and

$$Prob(Strong|Success) = \frac{Prob(Success|Strong)Prob(Strong)}{Prob(Success)}$$

$$= \frac{(1)(.2)}{.5\alpha(.8)+(1)(.2)} = \frac{20}{40\alpha+20} = \frac{1}{2\alpha+1}. \quad (6)$$

Then

$$\pi(In|Success) = \frac{2\alpha}{2\alpha+1}(10) + \frac{1}{2\alpha+1}(-10) \quad (7)$$

and

$$\pi(Out|Success) = 0 \quad (8)$$

Equating these yields $20\alpha - 10 = 0$ so $\alpha = .5$.

The equilibrium is:

Apex: $Try|Strong$ with probability 1. $Try|Weak$ with probability 5.

Brydox:$In|Failure$ with probability 1. $In|Success$ with probability .2.

2. A monopoly with zero marginal costs rents a movie-playing device to two customers at rental rate P, chosen each year. In the first year, each customer gets utility of $20-P_1$ from the device if he rents it. At the end of the first year, they simultaneously decide whether to pay 5 to buy a large set of Western movies which raises a customer's utility from rental to $28 - P_2$ in the second year. The monopoly must charge the same P to both customers, though the rental rate can change for the second year. It observes who bought the set of Westerns before it sets P_2.

Clarification: The two customers are named Smith and Jones. I did not specify that the discount rate was positive, but if it was, that would not affect the equilibrium prices but it would affect the mixing probability (it has the same effect as increasing the price of the Westerns).

Comment: This is a question about what is called "the hold-up problem." It proved surprisingly difficult— most people missed the point that once a customer has bought the Westerns, the payment of 5 is a sunk cost and he is willing to rent for any price less than 28, even though his total payoff will turn out to be negative. Thus, most students got even part (a) wrong. Nobody noticed that there was a mixed strategy equilibrium.

(a) What is the monopoly's equilibrium price strategy?

Answer: Charge $P_1 = 20$ the first year. Charge $P_2 = 20$ the second year if zero or one customer buys the Western series. Charge $P_2 = 28$ if both did.

(b) What is a symmetric equilibrium strategy for Smith and Jones?

Answer: First, note that the seller chooses P_2 after the customers decide whether to buy the Westerns, so a customer can't condition his buying decision on P_2.

For the rental decision, in period 1 the customer should rent if $P_1 \leq 20$. If he hasn't bought the Westerns, then he rents in the second period if $P_2 < 20$; otherwise, he rents if $P_2 \leq 28$.

If a customer doesn't buy, his payoff is zero. If he does, it is +3 or -5 in the second year, depending on whether P_2 equals 20 or 28, which in turn depends on how whether both customers buy. The symmetric equilibrium is in mixed strategies, because if neither were to buy, $P_2 = 20$ and the payoff from deviating to buying would be +3, but if both were to buy, $P_2 = 28$ and their payoffs would both be -5 and one could deviate to not buying, then not rent in the second year either, and his payoff would be 0 instead of -5.

Suppose each customer buys the Western series with probability θ. The payoffs are:

$$\pi(NotBuy) = 0 = \pi(Buy) = \theta(-5) + (1-\theta)(3).$$

Solving yields $\theta(5) = (1-\theta)(3)$, $5\theta = 3 - 3\theta$, $\theta = 3/8$.

Another, trivial, equilibrium that I hadn't thought of is for neither customer to buy or to rent in either period. Their payoffs are zero in equilibrium, and if one of them deviates and rents, his payoff will remain equal to zero. Or, they could both rent in period 1 and then both not not buy and not rent in period 2.

There are also asymmetric equilibria, in which one customers buys and the other does not.