Getting Carried Away in Auctions as Imperfect Value Discovery

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(Comments welcomed)

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Abstract

If estimating his private value is costly for a bidder, he may wait to do so until the middle of an auction and rationally revise his reservation price upwards, to his ex post regret– apparently "getting carried away".

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Sometimes bidders seem to get carried away in auctions, paying more than the reservation price they entered the auction willing to pay. This might be due to irrational emotion, as examined in Ku, Malhotra & Murnighan. Or, if the auction is common-value, the bidder might revise his own value estimate upwards on seeing other bidders' confidence. I here suggest an alternative using the idea in Compte & Jehiel (2000, 2004) and Rasmusen (2003) that even if an auction is private-value, other players' bids may stimulate a player to learn more about his own value during the auction, and sometimes his estimate rises.

Let there be two possible bidders, both risk-neutral and with private values, in open-exit auction. The price starts at zero and rises continuously until one player drops out at his "bid ceiling" and the other player wins.

Bidder 1's value is v_1, which has three components: $v_1 = \mu + u + \epsilon$. He knows μ but not u or ϵ, which are independently distributed according to symmetric densities $f(u)$ and $g(\epsilon)$ with mean zero and $Min(\mu + u + \epsilon) \geq 0$. His initial expectation of v_1 thus equals μ. If he wishes, at any time he can pay c and learn u's value, but he cannot discover ϵ until after the auction.

Bidder 2 knows his value, v_2, which is distributed according to the atomless and differentiable density $h(v_2)$ where $h(v_2) > 0$ on the support $[\alpha, \beta]$ and $0 < \alpha < \mu$ and $\beta > \mu$.

The optimal bid ceiling for a player equals his expected value for the object being auctioned. If he bids less, he might lose when the winning bid was below his expected value. If he bids more, he might win when the price exceeded his expected value. If Bidder 1 does not acquire any information about his value, his best strategy is to bid up to μ, the expected value of the object to him. If he does discover u, it is to bid up to $(\mu + u)$, his updated estimate of v_1. Bidder 2's optimal strategy is a bid ceiling of v_2. Bidder 2 will not change his bid ceiling to affect the timing of Bidder 1's value discovery, because value discovery is instantaneous, unlike in Rasmusen (2003a).

Bidder 1 has three possible value discovery strategies:

Early discovery. Discover u when the bid level reaches the optimal "discovery level" $p^* \in [0, \alpha)$.

Late discovery. Discover u if the bid level reaches $p^* \geq \alpha$.

No discovery. Never discover u.

Suppose (contrary to the assumptions) that Bidder 1 knows v_2, and that $p \leq \mu$, so there is positive probability that Bidder 1 pays c and discovers u.

If $v_2 < p$ Bidder 1 wins at price v_2, for an expected payoff of $(\mu - v_2)$. If $v_2 > p$ he pays c to discover u. He loses the auction if $\mu + u < p$; otherwise, he wins. Overall, if $v_2 > p$ his expected payoff is

$$\pi_1(v_2|v_2 > p) = -c + \int_{u=-\infty}^{v_2-\mu} (0) f(u)du + \int_{u=v_2-\mu}^{\infty} (\mu + u - v_2) f(u)du. \tag{1}$$

Integrating over the possible values of v_2 yields

$$pi_1 = \int_{v_2=\alpha}^{p} (\mu - v_2)h(v_2)dv_2 + \int_{v_2=p}^{\beta} \left(-c + \int_{u=v_2-\mu}^{\infty} (\mu + u - v_2) f(u)du \right) h(v_2)dv_2 \tag{2}$$

If, on the other hand, $p > \mu$, then Bidder 1 is following the policy of no discovery, and his expected payoff is the first part of equation (2):

$$\pi_1(p > \mu) = \int_{v_2=\alpha}^{\mu} (\mu - v_2) h(v_2) dv_2. \tag{3}$$

Proposition: *The optimal discovery level, p^*, rises with c, rising strictly if $p^* \in (\alpha, \mu)$. Bidder 1 will follow a policy of early discovery ($p^* \in [0, \alpha)$) if c is low enough, late discovery ($p^* \in [\alpha, \mu]$) for higher levels of c, and no discovery ($p^* \in (\mu, \infty]$) if c is sufficiently high.*

Proof: Differentiating equation (2) with respect to p yields

$$\begin{aligned}
\frac{d\pi_1}{dp} &= (\mu - p)h(p) - \left(-c + \int_{u=p-\mu}^{\infty} (\mu + u - p) f(u) du\right) h(p) \\
&= \left[c + (\mu - p) - \int_{u=p-\mu}^{\infty} (\mu - p + u) f(u) du\right] h(p) \\
&= \left[c + - \int_{u=-\infty}^{\infty} (\mu - p + u) f(u) du - \int_{u=p-\mu}^{\infty} (\mu - p + u) f(u) du\right] h(p) \\
&= \left[c + \int_{u=-\infty}^{p-\mu} (\mu - p + u) f(u) du\right] h(p)
\end{aligned} \tag{4}$$

If $h(p) > 0$ (true between α and μ) and c is small enough, derivative (4) is negative. If c is small enough, $\frac{d\pi_1}{dp} < 0$ for $p \in [\alpha, \mu]$, and the payoff rises if p is reduced to below α – that is, to early discovery. If $p < \alpha$, then $h(p) = 0$, so further reductions are unimportant.

If c is greater, $\frac{d\pi_1}{dp} > 0$ at $p = \alpha$, and the optimal p exceeds α. At the optimal p,

$$\begin{aligned}
\frac{d^2\pi_1}{dp^2} &= \left((\mu - p + [p - \mu]) f(u) + \int_{u=-\infty}^{p-\mu} (-1) f(u) du\right) h(p) + \left[c + \int_{u=-\infty}^{p-\mu} (\mu - p + u) f(u) du\right] h'(p) \\
&= 0 - \left(\int_{u=-\infty}^{p-\mu} f(u) du\right) h(p) + (0) h'(p) < 0
\end{aligned} \tag{5}$$

where we use the fact that $\frac{d\pi_1}{dp} = 0$ at the optimum to obtain the term $(0)h'(p)$. Since

$$\frac{d^2\pi_1}{dpdc} = (1)h(p) > 0, \tag{6}$$

the implicit function theorem tells us that $\frac{dp}{dc} > 0$ when $h(p) > 0$, i.e., the optimal discovery level rises with the discovery cost. Thus, there exist levels of c such that the optimal discovery level lies within (α, μ) and late discovery is optimal. As c increases, the optimal discovery level exceeds μ, so “no discovery” becomes optimal. ■

If the discovery cost is low enough, early discovery is best, because the bidder averts the possibility that he might pay more than his value by winning even at the other bidder’s lowest possible value. If the discovery cost is somewhat higher, it is not worth avoiding that possibility–thus, late discovery. How late depends on the size of the discovery cost, and the optimal discovery level rises smoothly with the discovery cost. If the discovery cost is too high, then no discovery becomes optimal.

This model provides an interpretation for "getting carried away". Suppose we see a bidder winning an auction at a price higher than his initial reservation price, and he later regrets having won— an "unhappy victory." Here is the model's interpretation. At the start of the auction, μ was the most he intended to bid. The auction began, and the bidding rose to μ. he reconsidered, at cost c, and raised his bid ceiling to $(\mu + u)$. This new ceiling exceeded β, the most other bidders would pay, so he won, at price β. After the auction is over, however, he learned ϵ and found that $\mu + u + \epsilon < \beta$. His reaction is, "I got carried away and bid too much. I wish I'd stuck with my original ceiling of μ." This will not happen in every auction for every bidder, but the winning bidder is likely to be one who revised his value upwards, and if the final estimate is unbiased with a symmetric error it is as likely to be too high as too low. Thus, roughly half the time the revising bidder will be willing to pay too much, and often competition will lead him to do so. Observationally, this will look like getting carried away by emotion. An empirical test would be whether bidders who revise their reservation prices upwards regret doing so *on average* (emotion) or not (rational value discovery).

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