**How Incomplete Information Can Solve the Coordination Game Problem in Repeated Games** (April 10, 2007)

Consider a ranked coordination game with $n \geq 2$ players indexed by $i$ who simultaneously choose actions $x_1, ..., x_n$ from the interval [0,100]. If $m$ players choose the same action $x_i$, the per-period payoff to player $i$ is $\pi_i(x_i, m)$, with:

$$
\begin{aligned}
&(a) \frac{\partial \pi_i(x_i, x_{-i}, m)}{\partial x_i} > 0 \quad (b) \frac{\Delta \pi_i(x_i, x_{-i}, m)}{\Delta m} > 0, \quad (c) \frac{\partial^2 \pi_i(x_i, x_{-i}, m)}{\partial x_i \partial x_j} = 0 \\
&(d) \pi_i(0, x_{-i}, n) > \pi_i(1, x_{-i}, n-1), \quad (e) \pi_i(w, x_{-i}, l) > \pi_i(w', x_{-i}, l-1) \forall l, w, w' \neq x
\end{aligned}
\tag{1}
$$

(e) is a more general form of (d). I think I need either (c) or (e), but I've forgotten why.

We will normalize $\pi_i(0, x_{-i}, n) = 0$, which is to say that the payoff when all the players choose $x = 0$ is 0. The assumptions then imply that full coordination on $x > 0$ yields positive payoffs and incomplete coordination yields negative payoffs.

The game is repeated a finite number $T$ times, with the players observing each other's choices after each round and with no discounting (an easily relaxed assumption).

If $T = 1$, the game has a continuum of pure strategy equilibria, with $x$ on the continuum from 0 to 1, as well as mixed strategy equilibria. All players prefer the equilibrium in which $x = 1$, the pareto-optimal outcome.

In a **time-dependent equilibrium**, some player's strategy in a round depends on which round it is. If the strategies are the same each round, the equilibrium is **time-independent**.

In a **history-dependent equilibrium**, some player's strategy in a round depends on the history of play up to that point. If the strategies do not depend on past play, the equilibrium is **history-independent**.

*Players are of two types. With some small probability $p > 0$, a player $i$ is "constrained" to always play $x_i = z$, in every round of the game, where $z$ is chosen from [0, 100] using the atomless density $f(z)$.*

*If T is large enough, the game now has a much smaller interval of equilibria, and the average payoff becomes arbitrarily close to 100. Formally:*

*Proposition 1: For any $\epsilon$, there exists T large enough that in all equilibria $\frac{\sum_{t=1}^{T} \pi_{it}}{T} > 100 - \epsilon$.*

*AN ALTERNATIVE ASSUMPTION: With small probability p, player 1 is "constrained" to play tit-for-tat: in equilibrium he begins with whatever $x_1 = z$ maximizes his equilibrium payoff, but thereafter he plays the action y that is played by the most other players in $(t-1)$, randomizing among the possibilities if there is a tie for action y.*