Rethinking Bargaining Under Complete Information with Outside Options

7 August 2008 . Pdf'd August 7, 2008.

David P. Myatt and Eric B. Rasmusen

Abstract

Myatt: Nuffield College, Room L. University of Oxford, New Road, Oxford, England, OX1 1NF. 011-44-1865 278-578 or (01865) 278-578. David.myatt@economics.ox.ac.uk.

Rasmusen: Dan R. and Catherine M. Dalton Professor, Department of Business Economics and Public Policy, Kelley School of Business, Indiana University, BU 438, 1309 East Tenth Street, Bloomington, Indiana, USA 47405, (812) 855-9219 (secretary), (812) 855-3356 (direct), Erasmuse@indiana.edu, http://www.rasmusen.org, http://www.rasmusen.org/papers/bargaining.pdf.

We would like to thank xxx for helpful comments and Nuffield College for its hospitality.

1. Introduction

The most basic problem in bargaining is what happens when two players with perfect information about each other must both agree on how to split a surplus or else the surplus vanishes. The ideal model of this situation would:

1. Give plausible results (e.g., a 50-50 split between symmetric players)

2. Be simple.

3. Allow us to parameterize bargaining strength.

For many years, the standard model was from Nash (1950). The Nash Bargaining Solution was an idea from cooperative game theory that used plausible axioms to construct a reduced-form solution. It gives us the 50-50 bargaining split result and is simple, but it assumes equal bargaining strengths. It can be seen, however, as a justification of the even simpler Split the Difference reduced-form model: assign Player 1 a bargaining strength of λ and Player 2 of $(1-\lambda)$ and assume that those are their equilibrium shares of the surplus.

Both of these are reduced-form models, which can make it unclear how to fit them into larger models. Also, they both evade the question of what "the surplus" means, something which we will discuss more below. One simple way to build a structural model is to let each player have probability λ of making a take-it-or-leave-it offer. Let us call this the Probabilistic Ultimatum Game. If the players have equal bargaining strength, this results in an expected payoff for each of them of half the surplus. Otherwise, we can use λ to parameterize bargaining strength. The Probabilistic Ultimatum Game is a structural model, but its assumptions about the bargaining process are unrealistic, so it has not been popular.

The model most used today is that of Rubinstein (1982). In this model, the two players alternate making offers in discrete periods, and the game ends when one player accepts the other's offer. Delay is costly, and in equilibrium the very first offer is accepted. If the two players have equal discount rates, the equilibrium split is close to 50-50, but otherwise the more patient player has an advantage, providing a way to parameterize bargaining strength.

The Rubinstein Model does have drawbacks. First, its feature of alternating offers leads to the complication that it matters who gets to offer first and that the equilibrium strategies depend on who is making the offer in a given period. Second, the equilibrium split is not quite 50-50 with symmetric players. It approaches 50-50 as discount rates go to zero, but one must always add that qualification– and if discount rates do equal zero, the model breaks down. Third, it does not resolve the question of what "surplus" means any better than the Nash solultion. Fourth, it is troubling that bargaining strength is parameterized by discounting when in most bargaining situations the delay between offer and counter-offer is trivial.

As Sutton (1986) explains, the availability of the outside option in case exogenous breakdown occurs is crucial. There are two ways to run Rubinstein (1982): time discounting, and breakdown. It is crucial whether breakdown removes the possibility of the outside option.

The Rubinstein model has some good properties, but it doesn't have the right feel. It is really a model of alternating monopolies. The new model below will have that same problem.

The Rubinstein Model

Ann and Bob are splitting a pie of size 1. First Ann makes an offer of a split of x_a for herself and $(1 - x_a)$ for Bob. If Bob accepts, the game is over and the payoffs are

$$\pi_a = x_a \qquad\qquad \pi_b = 1 - x_a. \tag{1}$$

If Bob rejects, a period of time passes, at the end of which he makes an offer of a split of $(1 - x_b)$ for himself and x_b for Ann. If Ann accepts, the game is over and the payoffs are (viewed from the start of the game)

$$\pi_a = \delta_a x_b \qquad\qquad \pi_b = \delta_b(1 - x_b) \tag{2}$$

because second-period payoffs are discounted by discount factors of $\delta_a = \frac{1}{1+\rho_a} < 1$ and $\delta_b = \frac{1}{1+\rho_b} < 1$. Our main interest is in what happens when the periods are short, so the discount rates approach $\rho_a = \rho_b = 0$ and the discount factors approach $\delta_a = \delta_b = 1$.

There are two possible origins for discounting here: time preference, and the possibility of the game ending exogenously. We will discuss both. Use time discounting for now.

If Ann rejects Bob's offer, another period of time elapses and she makes the next offer. The two players make alternating offers until agreement is reached, or forever if agreement is not reached.[1]

In any subgame perfect equilibrium, Ann's first offer will be accepted immediately. The only reason to wait is that Ann can get a bigger share x'_a later. She would prefer to offer $x'_a + \epsilon$ now.

It follows that the equilibrium is stationary. The players' payoffs cannot get bigger or smaller, or they would make more generous offers now that would equal any bigger payoffs without having to pay the bargaining costs.

Denote Ann's equilibrium offer by x_a^* and Bob's by x_b^*. In making her offer, Ann realizes that Bob could reject it and, if both players follow their equilibrium strategies, offer x_b^* next period and have it accepted. Thus, rejection gives Bob a payoff of $\delta_b(1 - x_b^*)$, as in equation (2). Ann can make Bob willing to accept her offer by making it generous enough to give him the same payoff, i.e.

$$1 - x_a^* = \delta_b(1 - x_b^*) \tag{3}$$

In making his offer, Bob realizes that Ann could reject it and, if both players follow their equilibrium strategies, offer x_a^* next period and have it accepted. Thus, rejection gives Ann a

[1]Unlike in some games, it makes little difference whether the game is infinitely repeated or just repeated a large finite number of times.

payoff of $\delta_a x_a^*$. Bob can make Ann willing to accept his offer by making it generous enough to give her the same payoff, i.e.

$$x_b^* = \delta_a x_a^*. \tag{4}$$

These two equations solve to

$$x_a^* = \frac{1-\delta_b}{1-\delta_a\delta_b} \tag{5}$$

and

$$x_b^* = \frac{\delta_a(1-\delta_b)}{1-\delta_a\delta_b} \tag{6}$$

This solution is asymmetric because it gives the first offeror, Ann, a bigger share, even if the number of possible offers is very large. If discounting is small, though, it is not very asymmetric.

Also, it allows us to use the discount factors to parametrize bargaining strength. We do think impatient people have less bargaining strength, though that is not the only source.

As Rubinstein found, we cannot let $\delta_a = \delta_b = 1$ or the solution technique fails to work.

No Time Preference, but an Exogenous Risk of Breakdown

Another version of the model, with properties almost identical to those of the standard Rubinstein model, has an exogenous risk of breakdown replacing time discounting. Let there be no time preference, but suppose that with probability $1-\delta$ the game ends suddenly and both players receive payoffs of 0.

Ann and Bob are splitting a pie of size 1. First Ann makes an offer of a split of x_a for herself and $(1-x_a)$ for Bob. If Bob accepts, the game is over and the payoffs are

$$\pi_a = x_a \qquad\qquad \pi_b = 1 - x_a. \tag{7}$$

If Bob rejects, then a period of time passes, at the end of which either Bob makes an offer of a split of x_b for Ann and $(1-x_b)$ for Ann, which has probability δ; or the game ends, which has probability $(1-\delta)$. If Bob offers and Ann accepts, the game is over. The payoffs are (viewed from the start of the game)

$$\pi_a = \delta x_b \qquad\qquad \pi_b = \delta(1-x_b). \tag{8}$$

If Ann rejects Bob's offer, another period of time elapses and she makes the next offer. The two players make alternating offers until agreement is reached, or forever if agreement is not reached.

In any subgame perfect equilibrium, Ann's first offer will be accepted immediately. The only reason to wait is that Ann can get a bigger share x_b' later. She would prefer to offer $x_b' + \epsilon$ now.

It follows that the equilibrium is stationary. The players' payoffs cannot get bigger or smaller, or they would make more generous offers now that would equal any bigger payoffs without having to pay the bargaining costs.

Denote Ann's equilibrium offer by x_a^* and Bob's by x_b^*. In making her offer, Ann realizes that Bob could reject it and, if both players follow their equilibrium strategies, offer x_b^* next period and have it accepted. Thus, rejection gives Bob a payoff of $\delta(1 - x_b^*)$, as in equation (8). Ann can make Bob willing to accept her offer by making it generous enough to give him the same payoff, i.e.

$$1 - x_a^* = \delta(1 - x_b^*) \tag{9}$$

In making his offer, Bob realizes that Ann could reject it and, if both players follow their equilibrium strategies, offer x_a^* next period and have it accepted. Thus, rejection gives Ann a payoff of δx_a^*. Bob can make Ann willing to accept his offer by making it generous enough to give her the same payoff, i.e.

$$x_b^* = \delta x_a^*. \tag{10}$$

These two equations solve to

$$x_a^* = \frac{1-\delta}{1-\delta^2} \tag{11}$$

and

$$x_b^* = \frac{\delta(1-\delta)}{1-\delta^2} \tag{12}$$

This solution is asymmetric because it gives the first offeror, Ann, a bigger share, even if the number of possible offers is very large. If discounting is small, though, it is not very asymmetric.

Definition: The **Rubinstein price** *is the price generated by equation* (11) *in a bargaining game.*

If the surplus size is one, and the buyer offers first, the Rubinstein price is

$$p = 1 - \frac{1-\delta}{1-\delta^2} = \frac{1-\delta^2-1+\delta}{1-\delta^2} = \frac{\delta-\delta^2}{1-\delta^2}. \tag{13}$$

We cannot let $\delta = 1$ or the solution technique fails to work.

This is even simpler than the time preference model, but now we have lost any way to parameterize bargaining strength.

Outside Options: Binmore-Rubinstein-Wolinsky (1986)

Now let us add outside options of w_a and w_b. These are alternatives that the players have if they fail to reach agreement.

See the Sutton (1986) paper for the source of the outside option idea.

What others have shown is that the outside options matter only if the constraints they provide are binding.

This is puzzling. Suppose Bob has no outside option. The outside option principle says that the bargaining split will be about .5 even if Ann has an outside option of about .5. We would think that if Ann could get about .5 by ceasing to bargain, she could get more by bargaining. The Split the Difference principle would give her .75 instead. Why doesn't that happen here?

The reason is that the outside option is a threat, and the threat is not credible. Both players expect agreement on about .5 will be reached with certainty. Hence, Ann saying that she could withdraw and not be hurt as much as Bob would be is irrelevant. It is not that Ann can withdraw and do better than the Rubinstein solution; what is true is merely that she could withdraw and not do so much worse as Bob would do. Instead of getting about 50, she would end up with 20; but Bob would end up with 0. That, however, is irrelevant to the bargaining.

If this outcome is implausible, it is because something is missing from the model. And it is indeed implausible, as the following example will show.

Competition Does Not Reduce Prices

Let's consider an example: bargaining between a buyer and a seller. We will start with bilateral monopoly, and then apply the model to one buyer facing two sellers.

The buyer values the object being sold at $v > c$ and each seller has a marginal cost of $c \geq 0$. The discount rate is $r > 0$. Bargaining with a seller costs the buyer $k \geq 0$, so if he deals with one seller his cost is b but if he deals with two his cost is $2k$. There is an infinite number of periods. The buyer might have an outside option with payoff w to him.

We will use a running example in which $v = 100, c = 0, r = .01$ (so $\delta \approx .99$), and $k = 0$.

The order of play is that the buyer chooses a seller and makes him a price offer, which the celler accepts or rejects, all happening at the start of the period. If it is rejected, then immediately the buyer may switch sellers and make an offer to the second seller, or if the buyer does not switch, after one unit of time the first seller can make an offer to the buyer that the buyer accepts or rejects. This continues, with either alternating offers within the same bargaining pair or switching and a new start to bargaining with a new bargaining pair.

One Seller

If there were just one seller, this would be the Rubinstein (1982) model. The bargaining range is prices in the interval $[0, 100]$. The buyer payoff range is in the range of $[0, 100]$. The equilibrium buyer surplus from (5) would be about $\pi_{buyer} = \left(\frac{1-\delta}{1-\delta^2}\right) 100 = \left(\frac{1-.99}{1-.9801}\right) 100 \approx 50.03$, and the equilibrium price would be the Rubinstein price, $p \approx 49.97$.

Already, we can see two defects of the Rubinstein model. The equilibrium price is not 50, and for a simple discount rate such as $r = .01$ the equilibrium price cannot be represented with just two decimals. We could instead say that the equilibrium price approaches 50 as the discount rates approaches zero, but it would be nice to be able to avoid the verbiage of such qualifications and just say $p^* = 50$. This would be especially true if our bargaining model were embedded in a more complex model where every step after the bargaining process would have to include these qualifications, and the reader would have to check whether the qualifications arise from the bargaining submodel used or from some other assumption of the complex model.

Monopoly with an Outside Price of 80

With equal bargaining power, discounting near zero, and a fixed outside option of $p = 80$ from some outside seller of a substitute product (so $w = 20$), things do not change. The equilibrium price remains the Rubinstein price of $p \approx 49.97$.

This outcome is disconcerting because the social surplus from bargaining would seem to be not 100, but 80, in which case the seller is getting 3/5 of it, not 1/2, from the Rubinstein price.

Two Sellers, No Switching Cost

Now add a second seller to the model. We have two identical sellers competing to sell to one buyer, so we would expect an equilibrium price of $p = c = 0$ and an equilibrium buyer payoff of $\pi_{buyer} = 100$. But that is not the only equilibrium.

The Rubinstein (1982) bargaining model has now become a Binmore- Rubinstein-Wolinsky (1986) model with an outside option. The impasse payoff, the payoff the buyer gets if bargaining breaks down and he takes no other option, is about 0. The outside option payoff, the payoff the buyer gets if he stops bargaining with his current partner, is his payoff from bargaining with the other player, which we will denote by π^2_{buyer}.

The outside option payoff π^2_{buyer} is the buyer's payoff from bargaining with Seller 2. The value of π^2_{buyer} depends on the buyer's outside option in that bargaining game, which is the buyer's payoff, π^1_{buyer}, from quitting and bargaining with Seller 1 instead. Thus, we have a simultaneous system to solve: we need to find values of π^1_{buyer} and π^2_{buyer} that are consistent with each other.

Consider the values $(p = 0, \pi^1_{buyer} = \pi^2_{buyer} = 100)$. This is an equilibrium outcome. In it, the buyer starts by making an offer of $p = 0$ to Seller 1. Seller 1 knows that if he rejects it, the buyer will make an offer of $p = 0$ to Seller 2 and Seller 2's equilibrium strategy is to accept that offer. Thus, the buyer's outside option has a payoff of 100, which is better than the 50.03 of the Rubinstein solution. Knowing this, Seller 1 is willing to accept $p = 0$, and the buyer's payoff is 100.

But consider instead $p = 37$. This, too, is an equilibrium outcome. The buyer makes an offer of $p = 37$ to Seller 1. Seller 1 knows that if he rejects it, the buyer will make an offer of $p = 37$ to Seller 2 and Seller 2's equilibrium strategy is to accept that offer, so Seller 1 will end up with a payoff of 0. The buyer's outside option has a payoff of 63, which is better than the 50.03 of the Rubinstein solution. Thus, Seller 1 will accept $p = 37$.

What if the buyer deviates by offering a lower price, $p = 35$? Seller 1 believes that if he rejects this offer as too low, the buyer will not have a positive incentive to switch (since Seller 2 would require a price of 37). Instead, he, Seller 1, will get to make an offer of 37 in the second round, and the buyer will accept it. So the buyer cannot gain by deviating from $p = 37$.

On the other hand, consider the price $p = 96$. This is not an equilibrium outcome. Suppose the buyer deviates to $p = 49.97$. The seller need not fear that the buyer will go to the other seller if he rejected $p = 49.97$, because the other seller will hold out for $p = 96$ in our postulated equilibrium. The first seller does, however, know that with the other seller irrelevant, he is in a simple Rubinstein bargaining game with the buyer, and the equilibrium has $p = 49.97$.

In fact, any price in $[0, 49.97]$ and any buyer payoff in $[50.03, 100]$ is an equilibrium. It all depends on expectations. The Rubinstein model does better than the simultaneous offer game in narrowing down the number of equilibria– it has ruled out prices in $[49.97, 100]$– but it still has a continuum of them, almost all of which yield positive profit to a seller.

At least the Bertrand equilibrium is one equilibrium, if not the unique one. But it turns out not to be robust to a small switching cost. Only the Rubinstein equilibrium will survive in that case.

Two Sellers, Positive Switching Cost

Now let us restore the switching cost, so $k = 1$. This has a drastic effect: now the only equilibria are near $p = 49.97$.

Consider the values $p = 37$ that we looked at before. The buyer has made an offer of $p = 37$. Suppose Seller 1 rejects this and counterproposes $p = 37.5$, which if accepted yields him a payoff in period-1 dollars of about 37.1. The buyer's payoff from accepting is about $.99(62.5) \approx 61.9$. The buyer's payoff from rejecting and bargaining with Seller 2 is about $.99(63 - 1) \approx 61.4$, since the buyer would have to incur the switching cost. Thus, the buyer will accept Seller 1's offer and the seller's deviation has been profitable.

Let's make the argument more generally. Let p^* be the equilibrium price and let $\hat{p}$ denote the deviation offer by the seller if he rejects the buyer's offer in the first round. The seller chooses $\hat{p}$ to maximize it subject to two constraints: (1) $\hat{p}$ must not exceed the Rubinstein price of $\left(\frac{\delta - \delta^2}{1-\delta^2}\right) 100$ and (2) the buyer must be willing to accept in the second round rather than wait and switch to the second seller in the third round. If the second constraint is binding,

$$\begin{aligned}
\pi_{buyer}(accept\ \hat{p}) &= \pi_{buyer}(switch,\ and\ get\ p^*) \\
\delta(100-\hat{p}) &= \delta^2(100-p^*-k) \\
\hat{p} &= (1-\delta)100+\delta p^*+\delta k
\end{aligned} \tag{14}$$

For the deviation to be profitable, the seller must strictly benefit from his deviation to $\hat{p}$. This will be true unless

$$\begin{aligned}
\pi_{seller}(equilibrium, p^*) &\geq \pi_{seller}(deviation, \hat{p}) \\
p^* &\geq \delta\hat{p} = \delta[Min\{\left(\frac{\delta-\delta}{1-\delta^2}\right), 100(1-\delta)100+\delta p^*+\delta k\}]
\end{aligned} \tag{15}$$

If it is constraint (2) that is binding,

$$\begin{aligned}
p^*(1-\delta^2) &\geq \delta(1-\delta)100+\delta^2 k] \\
p^* &\geq \frac{\delta(1-\delta)100}{1-\delta^2}+\frac{\delta^2 k}{1-\delta^2}.
\end{aligned} \tag{16}$$

We know from before that p^* cannot exceed the Rubinstein solution, so if it is constraint (2) that is binding,

$$\begin{aligned}
p^* \in [\frac{\delta(1-\delta)100}{1-\delta^2}+\frac{\delta^2 k}{1-\delta^2}, \left(\frac{\delta-\delta^2}{1-\delta^2}\right)100] &\approx [\frac{.99(1-.99)100}{1-.99^2}+\frac{.99^2(1)}{1-\delta^2}, 49.97] \\
&\approx [\frac{.99}{.0199}+\frac{.9801}{.0199}, 49.97] \\
&\approx [49.7+49.3, 49.97]
\end{aligned} \tag{17}$$

With the parameter values from our running example, it is constraint (1) that is binding, so

$$\begin{aligned}
p^* \in [\delta\left(\frac{\delta-\delta^2}{1-\delta^2}\right)100, \left(\frac{\delta-\delta^2}{1-\delta^2}\right)100] &\approx [.99(49.97), 49.97] \\
&\approx [49.47, 49.97]
\end{aligned} \tag{18}$$

Thus, only p^* in that interval can be supported in equilibrium.

As δ gets closer to 1 (i.e. as discounting falls), the interval of equilibrium prices shrinks towards the Rubinstein price, $\tilde{p}$. If constraint (1) is binding that is because the interval is $[\delta\tilde{p}, \tilde{p}]$. If constraint (2) is binding that is because using L'Hospital's Rule the lower bound approaches

$$\begin{aligned}
&\frac{\frac{d}{d\delta}[\delta(1-\delta)100+\delta^2 k]}{\frac{d}{d\delta}[1-\delta^2]} \\
&= \frac{100-200\delta+2\delta k]}{-2\delta} = 100-k
\end{aligned} \tag{19}$$

xxx SOMETHING IS WRONG IN THE CONSTRAINT TWO MAIN FORMULA

Thus, imposing a small transactions cost eliminates the sensible equilibrium in which price is competed down to marginal cost and leaves an equilibrium in which the presence of a competing seller is completely irrelevant.

A New Model

As we said earlier, it is crucial whether or not the players receive their outside options if bargaining breaks down. If they do, then the model does not behave at all like the model with time preference. Rather, it concludes with the split-the-difference rule. It would still have the other disadvantages of (1) An advantage to the first offeror, (2) No way to parametrize bargaining strength. So let us abandon the alternating-offer framework and use the probabilistic-offer framework of Baron et al. That will solve the two problems and also be much less confusing to work with, since we won't have to keep track of who offers in which period.

Let x be Ann's share of the pie, so $(1-x)$ is Ben's. There is an infinite number of periods. In each period, Ann has probability α of making the offer and Ben has probability $(1-\alpha)$. The non-offering player either accepts or rejects. If a player accepts an offer, the game is over. If he rejects, the game continues to the next period with probability δ but ends without agreement with probability $(1-\delta)$. If it ends without agreement, then next period Ann gets her outside option of w_a and Ben gets his outside option of w_b. In any period, a player may take his outside option instead of making or receiving an offer.

We will denote the equilibrium Ann's-share offer that Ann would make in period t by x_t^a and Ben's by x_t^b.

We don't need to let the periods get small to get even results, and the offers are the same in each period rather then differing in even and odd periods, unlike in alternating-offer bargaining.

Equilibrium

We will just work out Ann's equilibrium offer, not Bob's, since in equilibrium her first offer is accepted.

Think about Ann's payoff in period t. With probability α, she makes the offer and will offer x_t^1, which she will choose so it is accepted by Ben. With probability $(1-\alpha)$, Ben makes the offer, which he will make so she will just accept it. Thus,

$$\pi_t^a = \alpha x_t^a + (1-\alpha) x_t^b \tag{20}$$

Ann will choose x_t^a to make Ben indifferent about accepting. The value of waiting for him is that with probability $(1-\delta)$ the game will end and he will get the outside option w_b next period and with probability δ he will get π_{t+1}^b. Ann must subtract that from the size of the pie (=1) to get what she retains for herself in her offer. Thus,

$$x_t^a = 1 - [\delta\pi_{t+1}^b + (1-\delta)w_b] \tag{21}$$

Of course, it may be that w_b is so large that equation (21) cannot be solved because even by giving Bob the entire pie Ann cannot make him willing to accept her offer, or it may be that her offer to him must be so high that she herself would rather take the outside offer immediately.

Similarly,

$$x_t^b = \delta\pi_{t+1}^a + (1-\delta)w_a \tag{22}$$

Substituting these in to (20) gives

$$\pi_t^a \quad = \alpha[1 - [\delta\pi_{t+1}^b + (1-\delta)w_b]] + (1-\alpha)[\delta\pi_{t+1}^a + (1-\delta)w_a] \tag{23}$$

This is a stationary problem, so $\pi_{t+1}^a = \pi_t^a$. There will be immediate agreement in equilibrium, so the payoffs of the two players add up to 1 and $\pi_t^b = 1 - \pi_t^a$. Thus, we can drop the time subscripts, and

$$\pi^a = \alpha[1 - [\delta(1-\pi^a) + (1-\delta)w_b]] + (1-\alpha)[\delta\pi^a + (1-\delta)w_a] \tag{24}$$

We want to solve equation (24) for π^a.

$$\begin{aligned}
\pi^a &= \alpha - \alpha\delta + \alpha\delta\pi^a - \alpha(1-\delta)w_b] + (1-\alpha)\delta\pi^a + (1-\alpha)(1-\delta)w_a \\
(1 - \alpha\delta - (1-\alpha)\delta)\pi^a &= \alpha - \alpha\delta - \alpha(1-\delta)w_b] + (1-\alpha)(1-\delta)w_a \\
\pi^a &= \tfrac{\alpha - \alpha\delta - \alpha(1-\delta)w_b] + (1-\alpha)(1-\delta)w_a}{1 - \alpha\delta - (1-\alpha)\delta} \\
\pi^a &= \tfrac{(1-\delta)[\alpha(1-w_b) + (1-\alpha)w_a]}{1-\delta} \\
\pi^a &= \alpha + (1-\alpha)w_a - \alpha w_b
\end{aligned} \tag{25}$$

We know that $x_a = \pi^a$.

If the bargaining strengths are equal, so $\alpha = .5$, equation (25) boils down to

$$\pi^a \quad = \quad .5 + \tfrac{w_a - w_b}{2}. \tag{26}$$

If the bargaining strengths differ, but there are no outside options, so $w_a = w_b = 0$, equation (25) boils down to

$$\pi^a \quad = \quad \alpha. \tag{27}$$

Thus, we have a model which is simpler, more plausible, and allows parameterization of bargaining power.

The Pricing Game

What if we use the new bargaining model for our pricing game?

In monopoly, the outcome becomes simpler. It is exactly $p = 50$.

Monopoly with an Outside Price of 80

With equal bargaining power, discounting near zero, and a fixed outside option of $p = 80$ from some outside seller of a substitute product (so $w = 20$), things do change. The equilibrium price changes to, from (27),

$$\pi_{buyer} \quad = \quad (.5)(100) + \frac{w}{2} = 50 + 10 = 60, \tag{28}$$

so $p^* = 40$.

Duopoly

With duopoly, things change more. When there is no switching cost, so $k = 0$, the only equilibrium will be (explain more a nd prove xxxx):

$$\begin{aligned} \pi_{buyer} &= .5(100) + \frac{w}{2} \\ &= 50 + \frac{w}{2} \\ &= 100. \end{aligned} \tag{29}$$

The buyer gets the entire surplus and the price equals marginal cost.

Having a positive cost of switching makes only a tiny change in this result. The change is that now the seller would refuse $p = 0$ and could make a counteroffer of $p \leq k$ which the buyer would accept rather than switch. If k is small, the change is small too.

References

Binmore, K. (1985): "Bargaining and Coalitions," *Game-Theoretic Models of Bargaining*, pp. 269–304.

Binmore, K., A. Rubinstein, and A. Wolinsky (1986): "The Nash Bargaining Solution in Economic Modelling," *The RAND Journal of Economics*, 17(2), 176–188.

Muthoo, A. (1999): *Bargaining Theory with Applications*. Cambridge University Press.

Nash, J. F. (1950): " The bargaining problem," *Econometrica*, 18(2), 155–162.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50(1), 97–109.

Shaked, A., and J. Sutton (1984): "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model," *Econometrica*, 52(6), 1351–1364.

Sutton, J. (1986): "Non-Cooperative Bargaining Theory: An Introduction," *Review of Economic Studies*, 53(5), 709–724.