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Notes on Cournot

1. Teaching Cournot Equilibrium

Typically Cournot equilibrium is taught with identical zero or constant-MC cost functions for the two firms, because that is simpler. I think that's a bad approach. The result of the symmetry is that when you get to the reaction functions and the diagrams, it is very confusing because the two firms look alike. We have $Q_1 = 12 - Q_2/2$ and $Q_2 = 12 - Q_q/2$, and it is hard to tell them apart. It's even worse with the diagrams. Plus, students are tempted to equate $Q_1 = Q_2$, which is true only in equilibrium, before they take the first order conditions. So symmetric cost functions are a bad idea, until you get to $N = 3, 4, ...$, when we really need the simplicity.

How about constant marginal cost? We can use quadratic total cost— linear rising MC— and still get linear reaction functions. That way, too, it helps teach what a multiplant monopoly should do. If $MC_1 = AC_1 = 3$ and $MC_2 = AC_2 = 4$, a monopoly would operate only one plant. If $MC_1 = 4q_1$ and $MC_2 = 5_2$, it would operate both plants. And the algebra doesn't get any harder–it just adds an extra term in . Perhaps most important of all, we usually assume rising MC and U-shaped cost curves when we're teaching perfect competition, so it's good to keep that assumption when we come to imperfect competition.

A drawback is that it's harder to get the numbers to come out even. I haven't tried that in the example below, but I hope someone else does and let's us know of his improved version.

I wonder whether it is even worthwhile teaching the diagrams. They are difficult to understand, and may not even convey the intuition better. This is particularly true for Stackelberg equilibrium, where it is much too difficult to understand the isoprofit lines relative to the understanding gained from them. The intuition from the equations is both easier and better.

I my teaching I do teach the Cournot diagram, but I have not included it in these notes. I also go on, in Cournot, to a symmetric-costs case where I compare what happens to outputs, prices, and profits as the number of firms increases. See http://www.rasmusen.org/g406/chapters/08-monopoly.pdf.

2. Multiplant Monopoly

Suppose a monopolist produces output q_1 and q_2 from his two plants, which have the cost functions

$$\text{Total Cost } 1 = 4 + 2q_1^2 \tag{1}$$

and

$$\text{Total Cost } 2 = 3 + q_2^2. \tag{2}$$

The demand curve is

$$p^d = 24 - (q_1 + q_2) \tag{3}$$

The firm's profit function is

$$\begin{aligned}
\pi &= p(q_1 + q_2) - TC_1 - TC_2 \\
&= [24 - (q_1 + q_2)](q_1 + q_2) - [4 + 2q_1^2] - [3 + q_2^2] \\
&= 24q_1 + 24q_2 - q_1^2 - q_1q_2 - q_2^2 - q_2q_1 - 4 - 2q_1^2 - 3 - q_2^2 \\
&= 24q_1 + 24q_2 - q_1^2 - q_1q_2 - q_2^2 - q_2q_1 - 4 - 2q_1^2 - 3 - q_2^2
\end{aligned} \tag{4}$$

We take the derivative with respect to the firm's two control variables, q_1 and q_2, to get

$$\begin{aligned}
\frac{\partial\pi}{\partial q_1} &= 24 + 0 - 2q_1 - q_2 - 0 - q_2 - 0 - 4q_1 - 0 - 0 = 0 \\
\rightarrow &\qquad 24 - 2q_2 = 6q_1 \\
\rightarrow &\qquad q_1^* = 4 - \tfrac{q_2}{3}
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\frac{\partial\pi}{\partial q_2} &= 0 + 24 - 0 - q_1 - 2q_2 - q_1 - 0 - 0 - 0 - 2q_2 = 0 \\
\rightarrow &\qquad 24 - 2q_1 = 4q_2 \\
\rightarrow &\qquad q_2^* = 6 - \tfrac{q_1}{2}
\end{aligned} \tag{6}$$

Substituting, we get

$$
\begin{aligned}
q_2^* &= 6 - \frac{4 - \frac{q_2^*}{3}}{2} \\
q_2^* &= 6 - 2 + \frac{q_2^*}{6} \\
q_2^* &= -\frac{q_2^*}{6} = 4 \qquad (7) \\
(5/6)q_2^* &= 4 \\
q_2^* &= \frac{6}{5}(4) = 4.8,
\end{aligned}
$$

in which case

$$
\begin{aligned}
q_1^* &= 4 - \frac{q_2^*}{3} \\
&= 4 - \frac{4.8}{3} \qquad (8) \\
&= 2.4
\end{aligned}
$$

With $q_1 = 2.4$ and $q_2 = 4.8$, the price is $p = 24 - (2.4 + 4.8) = 16.8$. Revenue is $p(q_1 + q_2) = 16.8(7.2) \approx 121$. Total cost is $TC_1 + TC_2 = 4 + 2(2.4^2) + 3 + 4.8^2 \approx 42$. Profit is thus $\pi \approx 121 - 42 = 79$.

3. Cournot Duopoly

Now imagine that the two plants operate independently, or that they are split into two separate corporations, each choosing output for the year simultaneously. Again, we maximize profit by choice of output, but now this two separate problems, for firm 1 and firm 2. First there is firm 1's profit:

$$\begin{aligned} \pi_1 &= pq_1 - TC_1 \\ &= [24 - (q_1 + q_2)]q_1 - [4 + 2q_1^2] \\ &= 24q_1 - q_1^2 - q_1 q_2 - 4 - 2q_1^2 \end{aligned} \tag{9}$$

We take the derivative with respect to firm 1's control variable , q_1, to get firm 1's reaction function:

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &= 24 - 2q_1 - q_2 - 0 - 4q_1 = 0 \\ \rightarrow & \quad 24 - q_2 = 6q_1 \\ \rightarrow & \quad q_1^* = 4 - \frac{q_2}{6}. \\ \rightarrow & \end{aligned} \tag{10}$$

Now let's solve firm 2's profit maximization problem. Its profit is:

$$\begin{aligned} \pi_2 &= pq_2 - TC_2 \\ &= [24 - (q_1 + q_2)]q_2 - [3 + q_2^2] \\ &= 24q_2 - q_1 q_2 - q_2^2 - 3 - q_2^2 \end{aligned} \tag{11}$$

We take the derivative with respect to firm 2's control variable , q_2, to get firm 2's reaction function:

$$\begin{aligned}
\frac{\partial \pi_2}{\partial q_2} &= 24 - q_1 - 2q_2 - 0 - 2q_2 = 0 \\
\rightarrow \quad & 24 - q_1 = 4q_2 \\
\rightarrow \quad & q_2^* = 6 - \frac{q_1}{4}.
\end{aligned} \tag{12}$$

Substituting, we get

$$\begin{aligned}
q_2^* &= 6 - \frac{q_1}{4} \\
q_2^* &= 6 - \frac{4 - \frac{q_2}{6}}{4} \\
q_2^* &= 6 - \frac{4 - \frac{q_2}{6}}{4} \\
q_2^* &= 6 - 1 + \frac{q_2}{24} \\
\frac{23}{24} q_2^* &= 5 \\
q_2^* &= \frac{24}{23}(5) \approx 5.2,
\end{aligned} \tag{13}$$

so

$$\begin{aligned}
q_1^* &= 4 - \frac{q_2^*}{6} \\
&\approx 4 - \frac{5.2}{6} \\
&\approx 3.13
\end{aligned} \tag{14}$$

With $q_1 = 3.13$ and $q_2 = 5.2$, the price is $p = 24 - (3.13 + 5.2) \approx 15.7$. Industry revenue, the sum of the two firms' revenues, is $p(q_1 + q_2) \approx 15.7(8.3) \approx 130$. Total industry cost is $TC_1 + TC_2 \approx 4 + 2(3.13^2) + 3 + 5.2^2 \approx 53$. Industry profit is thus $\pi \approx 130 - 53 = 77$. Revenue is higher than under monopoly, but price and profit are lower.

4. Stackelberg Duopoly

Now imagine that the two plants operate independently, or that they are split into two separate corporations, but firm 1 chooses its output first. Firm 2 observes q_1 and then decides its own output. This means that firm 1 can commit to a high output knowing that firm 2, appalled to see firm 1 producing so much, will contract its own output. There will be a first-mover advantage for firm 1 (though in this example, it will still end up that $q_1^* < q_2^*$ because of firm 1's inferior cost function).

Firm 1's profit maximization problem starts out the same as in Cournot:

$$\begin{aligned}\pi_1 &= pq_1 - TC_1 \\ &= [24 - (q_1 + q_2)]q_1 - [4 + 2q_1^2] \qquad (15) \\ &= 24q_1 - q_1^2 - q_1 q_2 - 4 - 2q_1^2\end{aligned}$$

Now, though, firm 1 can predict exactly how much firm 2 will produce. It knows, from the analysis in the Cournot model, that firm 2's reaction function is $q_2^* = 6 - \frac{q_1}{4}$. Thus,

$$\begin{aligned}\pi_1 &= 24q_1 - q_1^2 - q_1 q_2 - 4 - 2q_1^2 \\ &= 24q_1 - q_1^2 - q_1(6 - \tfrac{q_1}{4}) - 4 - 2q_1^2 \qquad (16) \\ &= 24q_1 - q_1^2 - 6q_1 + \tfrac{q_1^2}{4}) - 4 - 2q_1^2\end{aligned}$$

We take the derivative with respect to firm 1's control variable , q_1, to get firm 1's reaction function:

$$\begin{aligned} \tfrac{\partial \pi_1}{\partial q_1} &= 24 - 2q_1 - 6 + 2\tfrac{q_1}{4} - 0 - 4q_1 = 0 \\ \rightarrow & \quad 18 = 6q_1 - \tfrac{q_1}{2} \\ \rightarrow & \quad 18 = 5.5q_1 \\ \rightarrow & \quad q_1^* \approx= 3.3 \end{aligned} \tag{17}$$

We can use firm 2's reaction function to find its output:

$$q_2^* = 6 - \frac{q_1}{4} \approx 6 - \frac{3.3}{4} \approx 5.17. \tag{18}$$

With $q_1 = 3.3$ and $q_2 = 5.17$, the price is $p = 24 - (3.3 + 5.17) \approx 15.5$. Industry revenue, the sum of the two firms' revenues, is $p(q_1 + q_2) \approx 8.47(15.5) \approx 131$. Total industry cost is $TC_1 + TC_2 \approx 4 + 2(3.3^2) + 3 + 5.17^2 \approx 55$. Industry profit is thus $\pi \approx 131 - 55 = 76$. Revenue is higher than under monopoly or Cournot duopoly, but price and profit are lower. Firm 1's profit, however, is higher than under Cournot duopoly— but this has depressed firm 2's profit more than it raised firm 1's.