Isoperfect Price Discrimination in a Hotelling Duopoly

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Abstract

When duopolists compete by haggling with consumers, the form of the bargaining model is very important, whether in a Bertrand model or a Hotelling model.

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1. Introduction

There is a literature on posted pricing versus bargaining: Wang (1995, EER). Gill & Thanassoulis, Anderson & Renault (2003) JET, Camera & Delacroix (2004) *Review of Economic Dynamic.*, Cason, Friedman, . and Milam, (2003) IJIO. Desai & Purohit (2004) *Marketing Science.*

2. The Model

A Hotelling (1929) city has linear transportation costs. The city consists of a single street, the unit interval $[0, 1]$. Consumers are uniformly distributed along this street. Two firms located at 0 and 1 sell a product with a constant marginal cost that we normalize to zero. Each consumer has a maximum willingness-to-pay of the same number $v > 0$ for a single unit of a product, but there are linear transportation costs, so a consumer located at $x \in [0, 1]$ purchasing from a firm located at $y \in [0, 1]$ is willing to pay at most $v - t|x - y|$.

All functions and parameters are common knowledge. The firms know each consumer's reservation price, can identify each consumer, and can prevent resale.

Let us define terms as follows:

Definition: Under **"posted pricing"** : the firm posts a single price, which buyers may only accept or reject.

Definition: Under **"Pigouvian perfect price discrimination"** : a firm bargains with each buyer separately, and captures the entire surplus.

Definition: Under **"isoperfect price discrimination"** : a firm bargains with each buyer separately, and captures fraction λ of the surplus from each buyer.

Definition: Under **"balanced isoperfect price discrimination" or "IPD"** : a firm bargains with each buyer separately, and captures half the surplus from each buyer. i.e., $\lambda = .5$.

We have just described a reduced-form model of the pricing process. This flows easily from natural structural models. The monopolist engages in simultaneous discrete-time bargaining games with all the consumers.

In each period, the consumer picks a firm. He bargains with that firm according to our bargaining game. Then he can switch.

With probability λ the seller makes the offer in a period, and with probability $(1 - \lambda)$ the buyer does. The buyer offers price p_b and the seller offers p_s. The non-offeror accepts or rejects. If agreement is not reached, a period elapses. At the end of the period the buyer can go to the other seller if he wishes, and start bargaining there.

This is a model in the style of Sutton (1986), except with a probabilistic offeror instead of alternating offerors. (so it is more like Baron et al.)

There are two ways that bargaining between two players might break down: in such a way as to end the game entirely, or in such a way that the players are free to seek alternatives to the current potential surplus.

Definition: **(1) Sudden-Death Breakdown**: *This occurs when bargaining breaks down exogenously and the players cannot replace the gains from trade by dealing with someone else instead.*

Definition: **(2) Bargain-Specific Breakdown**: *This occurs when bargaining breaks down exogenously and the players can replace part of the gains from trade by dealing with someone else instead.*

With probability $1-\beta$, bargaining between the particular buyer and seller breaks down. With probability γ this is bargain-specific breakdown and the buyer can visit another seller instead. With probability $1-\gamma$ this is sudden-death breakdown and the game ends with zero payoffs for all players.

Sudden-death breakdown can represent time preference, with the time preference rate r such that $\beta = \frac{1}{1+r}$.

In general, the difference between the two kinds of breakdown is in the way they affect disagreement points and outside options. After sudden-death breakdown, the players receive their disagreement payoffs, usually normalized to zero. Like the bargaining surplus, their outside options suddenly vanish. After bargain-specific breakdown, the players receive their outside options as payoffs.

We describe a bargaining game here in which the monopolist bargains with all consumers simultaneously. We could instead have used a model in which alternating-offer games were played with consumers in an exogenous sequence or in a sequence chosen by the monopolist.

Equilibrium under Monopoly

Consider a monopolist located at $y = 0$. If he charges price p then a consumer located at x is willing to buy one unit if and only if $v - tx \geq p$. Hence a monopolist supplying z units faces the demand curve

$$p(z) = v - tz. \tag{1}$$

Let us write z^* for the efficient quantity, where if $t \geq v$ then $z^* = v/t$, and if $t \leq v$ then $z^* = 1$.

The monopolist faces a competitive fringe which charges the price $\bar{p}$ for a product with the location $y = 1$.

The market demand curve for the monopolist's product is linear up to the reservation price $\bar{p}$ if marginal cost is high enough that not all consumers are served when price equals marginal cost.

The profit-maximizing quantity, monopoly price, and profits under posted pricing are, if the competitive fringe's price is not a binding constraint,

$$z^m = \frac{v}{2t} \quad \Rightarrow \quad p^m = .5v, \text{ and } \pi^m = \frac{v^2}{4t}. \tag{2}$$

If $.5v > \bar{p}$ then the presence of the competitive fringe constrains the monopolist and he sets his price equal to $\bar{p}$ instead of $.5v$.

This yields a solution so long as $z^m \leq 1$, or, equivalently, $t \leq .5v$. If $t < .5v$ then the constraint $z \leq 1$ binds. the monopolist chooses to supply the entire market, setting a price $p^m = v - t$ to ensure that the consumer located at $x = 1$ is willing to buy. Hence:

$$t < .5v \quad \Rightarrow \quad z^m = 1 \quad \Rightarrow \quad p^m = v - t \text{ and } \pi^m = v - t. \tag{3}$$

Under posted pricing the profits are thus

$$\pi^m = \begin{cases} vt - t^2 & t \leq .5v \\ .25v^2 & t \geq .5v \end{cases} \tag{4}$$

Monopoly Price Discrimination

If the monopolist uses price discrimination, he sells the efficient quantity z^*.

Under our assumptions, with probability $1 - \lambda$ the buyer makes the offer in a period, and with probability λ the seller does. The buyer offers price p_b and the seller offers p_s. The non-offeror accepts or rejects. If agreement is not reached, a period elapses. At the end of the period, with probability $1-\beta$ breakdown occurs. With probability γ the buyer can purchase from the competitive fringe at price $\bar{p}$, for a payoff of $v - \bar{p}$. With probability $1 - \gamma$ the buyer cannot purchase from the competitive fringe and his payoff is zero.

In equilibrium, there is immediate agreement. Hence:

$$\begin{aligned} \pi_s^{offer} &= p_s \\ \pi_s^{accept} &= p_b \\ \pi_b^{offer} &= v - p_b \\ \pi_b^{accept} &= v - p_s. \end{aligned} \tag{5}$$

The seller's payoff from rejecting is

$$\pi_s^{reject} = (1 - \beta)(0) + \beta[(1 - \lambda)\pi_s^{accept} + \lambda\pi_s^{offer}], \tag{6}$$

because in the next period, if the bargaining does not fall through, the seller will accept the buyer's offer in equilibrium if the buyer offers.

Substituting $\pi_s^{accept} = p_b$ and $\pi_s^{offer} = p_s$ from (5) yields

$$\pi_s^{reject} = \beta((1-\lambda)p_b + \lambda p_s). \tag{7}$$

If the buyer makes the offer, he does it to make the seller indifferent between accepting and rejecting, so $\pi_s^{reject} = \pi_s^{accept} = p_b$. Thus,

$$p_b = \beta(1-\lambda)p_b + \beta\lambda p_s \tag{8}$$

Then,

$$p_s = p_b\left(\frac{1-\beta(1-\lambda)}{\beta\lambda}\right) \tag{9}$$

The buyer's payoff from rejecting is

$$\pi_b^{reject} = (1-\beta)(1-\gamma)(0) + (1-\beta)\gamma(v-\bar{p}) + \beta((1-\lambda)\pi_b^{offer} + \lambda\pi_b^{accept}). \tag{10}$$

The form of equation (10) is different from the seller's analog, equation (6), because after bargain-specific breakdown (probability $(1-\beta)\gamma$) the buyer can still buy from the competitive fringe at price $\bar{p}$.

We can substitute $\pi_b^{accept} = v - p_s$ and $\pi_b^{offer} = v - p_b$ from (5) to get

$$\pi_b^{reject} = (1-\beta)\gamma(v-\bar{p}) + \beta((1-\lambda)(v-p_b) + \lambda(v-p_s)). \tag{11}$$

The seller chooses p_s to make the buyer indifferent between accepting and rejecting, so $\pi_b^{reject} = \pi_b^{accept} = v - p_s$ and we can write

$$v - p_s = (1-\beta)\gamma v - (1-\beta)\gamma\bar{p} + \beta v - \beta(1-\lambda)p_b - \beta\lambda p_s \tag{12}$$

Then

$$v - (1-\beta)\gamma v - \beta v + (1-\beta)\gamma\bar{p} + \beta(1-\lambda)p_b = p_s - \beta\lambda p_s \tag{13}$$

and

$$p_s = \frac{v - (1-\beta)\gamma v - \beta v + (1-\beta)\gamma\bar{p} + \beta(1-\lambda)p_b}{1-\beta\lambda}. \tag{14}$$

Equating our two expressions for p_s from (9) and (14) yields

$$p_b\left(\frac{1-\beta(1-\lambda)}{\beta\lambda}\right) = \frac{v - (1-\beta)\gamma v - \beta v + (1-\beta)\gamma\bar{p} + \beta(1-\lambda)p_b}{1-\beta\lambda} \tag{15}$$

and

$$[1-\beta\lambda][1-\beta(1-\lambda)]p_b = \beta\lambda[v - (1-\beta)\gamma v - \beta v + (1-\beta)\gamma\bar{p} + \beta(1-\lambda)p_b] \tag{16}$$

and

$$[1-\beta\lambda-\beta(1-\lambda)+\beta^2\lambda(1-\lambda)]p_b = \beta\lambda v - \beta\lambda(1-\beta)\gamma v - \beta^2\lambda v + \beta\lambda(1-\beta)\gamma\bar{p} + \beta^2\lambda(1-\lambda)p_b \tag{17}$$

and

$$[1-\beta\lambda-\beta(1-\lambda)]p_b = \beta\lambda v - \beta\lambda(1-\beta)\gamma v - \beta^2\lambda v + \beta\lambda(1-\beta)\gamma\bar{p} \tag{18}$$

and

$$p_b = \frac{\beta\lambda v - \beta\lambda(1-\beta)\gamma v - \beta^2\lambda v + \beta\lambda(1-\beta)\gamma\bar{p}}{1-\beta} \tag{19}$$

It follows from (9) that

$$\begin{aligned}
p_s &= \tfrac{\beta\lambda v-\beta\lambda(1-\beta)\gamma v-\beta^2\lambda v+\beta\lambda(1-\beta)\gamma\bar{p}}{1-\beta}\left(\tfrac{1-\beta(1-\lambda)}{\beta\lambda}\right)\\
&= \tfrac{[1-\beta(1-\lambda)][\beta\lambda v-\beta\lambda(1-\beta)\gamma v-\beta^2\lambda v+\beta\lambda(1-\beta)\gamma\bar{p}]}{\beta\lambda(1-\beta)}\\
&= \tfrac{[1-\beta(1-\lambda)][v-(1-\beta)\gamma v-\beta v+(1-\beta)\gamma\bar{p}]}{1-\beta} \qquad (20)\\
&= \tfrac{v-(1-\beta)\gamma v-\beta v+(1-\beta)\gamma\bar{p}-\beta(1-\lambda)v+\beta(1-\lambda)(1-\beta)\gamma v+\beta^2(1-\lambda)v-\beta(1-\lambda)(1-\beta)\gamma\bar{p}}{1-\beta}\\
&= \tfrac{v-(1-\beta)\gamma v-\beta v-\beta(1-\lambda)v+\beta(1-\lambda)(1-\beta)\gamma v+\beta^2(1-\lambda)v+(1-\beta)\gamma\bar{p}-\beta(1-\lambda)(1-\beta)\gamma\bar{p}}{1-\beta}
\end{aligned}$$

We are interested in what happens in the limit, as the probability β of breakdown goes to one. This represents what happens as time periods get very short. Both the numerator and denominator of (19) and (20) approach zero, so we need to use L'Hopital's rule, which gives us:

$$\begin{aligned}
\beta \overset{p_b}{\longrightarrow} 1 &= \lim_{\beta\to 1}\tfrac{\lambda v-\lambda\gamma v+2\beta\lambda\gamma v-2\beta\lambda v+\lambda\gamma\bar{p}-2\beta\lambda\gamma\bar{p}}{-1}\\
&= -\lambda v+\lambda\gamma v-2\lambda\gamma v+2\lambda v-\lambda\gamma\bar{p}+2\lambda\gamma\bar{p} \qquad (21)\\
&= \boxed{\lambda(1-\gamma)v+\lambda\gamma\bar{p}}
\end{aligned}$$

and

$$\begin{aligned}
\beta \overset{p_s}{\longrightarrow} 1 &= \tfrac{\gamma v-v-(1-\lambda)v+\gamma(1-\lambda)v-2\beta\gamma(1-\lambda)v+2\beta(1-\lambda)v-\gamma\bar{p}-\gamma(1-\lambda)\bar{p}}{-1}\\
&= -\gamma v+v+(1-\lambda)v-\gamma(1-\lambda)v+2\gamma(1-\lambda)v -
\end{aligned}$$

$$-\gamma v+v+\gamma(1-\lambda)v-(1-\lambda)v+\gamma\bar{p}-\gamma(1-\lambda)\bar{p}$$

$$(1-\gamma)v-(1-\gamma)(1-\lambda)v+\gamma\lambda\bar{p}$$

$$\lambda(1-\gamma)v+\gamma\lambda\bar{p} \tag{22}$$

These are the same values, because if the time periods are very short there is infinitesimal advantage to offering first.

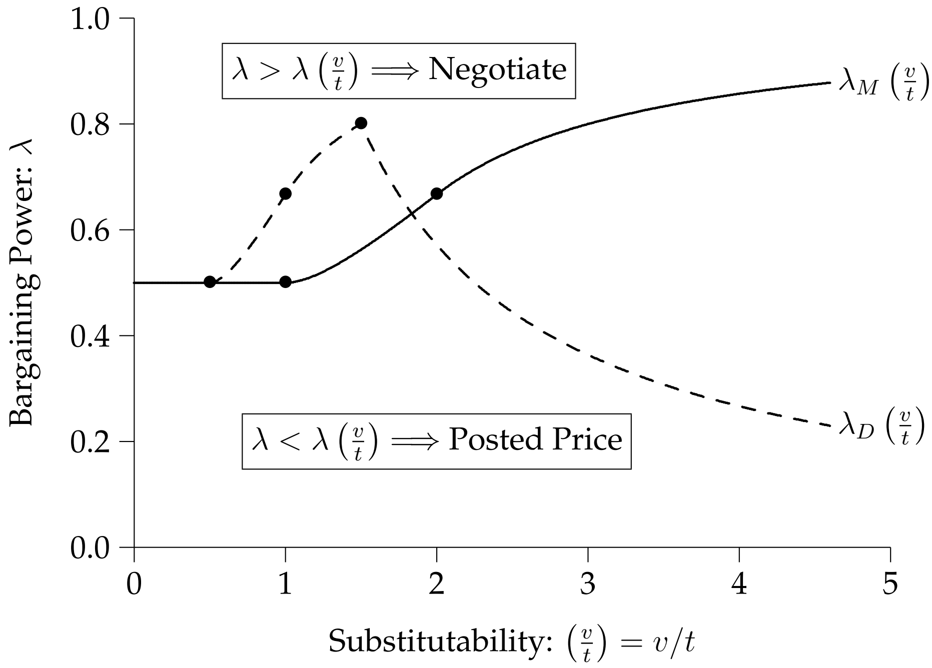

FIGURE 1. Sales Strategies for a Hotelling Monopolist

Suppose the buyer is offering a price and the time period is short. If breakdown is sudden-death ($\gamma = 0$), then $p_b = \lambda v$; the buyer pays a share of v proportional to the seller's bargaining power, and the competitive fringe doesn't matter. $p_s = \lambda v$ also.

If breakdown is bargain-specific ($\gamma = 1$), then $p_b = \lambda \bar{p}$; the buyer pays a share of $\bar{p}$ proportional to the seller's bargaining power, so the competitive fringe certainly does matter to the price.

These two polar situations illustrate the ambiguity of the term "bargaining surplus". One way to view it is as v, the gains from trade between buyer and seller. The other way to view it is as $v - \bar{p}$, the increase in social surplus from having the transaction be between this buyer and seller rather than this buyer and some other seller.

In the context of bilateral monopoly with a competitive fringe, surely bargain-specific breakdown is the correct model. [argue that here xxx] Buyer and monopolist are haggling over gains from the buyer buying from the monopolist rather than the competitive fringe.

Duopoly

If we have two firms located at 0 and 1 and the marginal cost is zero, the efficient outputs are those that would result when both firms' prices equal zero. A consumer located at x would be willing to buy from Firm 1 if $v - tx \geq 0$ and from Firm 2 if $v - t(1-x) \geq 0$. The consumer at $x = .5$ will buy if $v - .5t \geq 0$; that is, if $t \leq 2v$. In that case, the two firms split the market and $z_1 = .5, z_2 = .5$. Otherwise, $v - tz_1 = 0$ and $v - t(1 - z_2) = 0$, so $z_1 = \frac{v}{t}$, $z_1 = 1 - \frac{v}{t}$, and $z = z_1 + z_2 < 1$.

We must specify the order of play carefully:

1. The firms simultaneously choose policies. A firm can either choose posted pricing with a particular price p_1 or p_2, or price discrimination.

2. Consumers choose firms. Switching firms incurs a small cost ε.

3. Consumer purchase or haggle. A haggling consumer may switch to the other firm.

4. The game ends when all consumers have agreed to a price or decided not to buy.

Two Firms Using Posted Pricing

Suppose both firms use posted pricing. A consumer located at x has payoff $(v - tx - p_1)$ buying from Firm 1, $(v - t(1-x) - p_2)$ from Firm 2, and 0 not buying at all (ignoring the switching cost ε). If he is willing to buy, Consumer x is indifferent between the suppliers if and only if

$$v - tx - p_1 = v - t(1-x) - p_2, \tag{23}$$

which is equivalent to

$$x = .5 + \frac{p_2 - p_1}{2t}. \tag{24}$$

For t small enough, Firms 1 and 2 sell amounts x and $1-x$. For larger t, there will be consumers in a "no man's land" unserved by either firm. There are thus three cases to consider.

(3) $t \in (0, \frac{2}{3}v)$. From (23), if there is a consumer x indifferent between the two firms and willing to buy, Firm 1's profit of xp_1 equals $p_1/2 + \frac{p_1(p_2-p_1)}{2t}$. Maximizing this with respect to p_1 and solving with the analogous condition for Firm 2 yields $p_1 = p_2 = t$. Consumer $x = .5$ is willing to buy if $v - t/2 - t \geq 0$, which requires that $t \leq \frac{2}{3}v$. Hence:

$$z^{pp} = \frac{1}{2},\ p^{pp} = t \text{ and } \pi_1(pp, pp) = \frac{t}{2}. \tag{25}$$

(2) $t \in (\frac{2}{3}v, v)$. The duopolists split the market evenly and the central customer is indifferent about buying. With transportation costs below v, a duopolist does not have as much incentive as a monopolist to reduce price, because competition constrains his ability to increase quantity by selling to distant consumers. Instead, his best strategy may be to use a high price to extract surplus from nearby consumers, where he faces less competition. If $p_1 = p_2 = v - t/2$, then each firm will sell to exactly half the market and Consumer $x = .5$ will be indifferent about buying from Firm 1, buying from Firm 2, and not buying at all. Hence:

$$z^{pp} = .5,\ p^{pp} = v - .5t \text{ and } \pi_1(pp, pp) = .5\,[v - .5t]\,. \tag{26}$$

(3) $t \in [v, \infty]$. In this case, $z^{pp} \leq .5$ and there is a no man's land; the two firms act as local monopolists, using the monopoly solution from (2),

$$z^{pp} = \frac{v}{2t}, p^{pp} = \frac{v}{2} \text{ and } \pi_1(pp, pp) = \frac{v^2}{4t}. \tag{27}$$

FIGURE 2. Both Firms Use Isoperfect Price Discrimination and $t < 2v$

In summary, a posted-pricing duopolist enjoys profits of

$$\pi_1(pp, pp) = \begin{cases} \frac{v^2}{4t} & t \in [v, \infty] \\ .5v - .25t & t \in (\frac{2}{3}v, v) \\ .5t & t \in [0, \frac{2}{3}v) \end{cases} \tag{28}$$

Two Firms Using Price Discrimination

Now let both firms use price discrimination (we are not yet permitting deviation by one to posted pricing).

(1) $t \leq 2v$. Now $z^* > .5$ and efficiency dictates that all consumers be served. The duopolists split the market evenly.

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(2) $t \geq 2v$. In this case, $z^* < .5$, and efficiency dictates that centrally located consumers will not be served. Hence each duopolist earns the profits of a monopolist:

$$\pi_1(pd, pd) = \pi_1(pp, pp) = \frac{\lambda v^2}{2t}. \tag{29}$$

Mixed Duopoly: A Price Poster and a Price Discriminator

Now let us consider mixed duopoly. Firm 1 uses a posted price, p_1. Firm 2 uses price discrimination, so a consumer buying from Firm 2 enjoys a fraction $1-\lambda$ of the surplus from their transaction. In our model this is a Rubinstein bargaining game between Firm 2 and the consumer, where the consumer may purchase from Firm 1 at price p_1 as an outside option.

(1) $t \in [1.5v, \infty]$**. The Small Outputs/Dual Monopoly Case**

Ideally Firm 1 would supply z^{pp} consumers and Firm 2 would haggle with z^{pd}. If $z^{pp} + z^{pd} < 1$, then these ideals are mutually compatible. That is,

$$z^{pp} + z^{pd} \leq 1 \quad \Leftrightarrow \quad \frac{v}{2t} + \frac{v}{t} \leq 1 \quad \Leftrightarrow \quad t \geq 1.5v. \tag{30}$$

Hence in this circumstance it is straightforward to calculate profits:

$$\pi_1(pp, pd) = \frac{v^2}{4t} \quad \text{and} \quad \pi_2(pp, pd) = \frac{\lambda v^2}{2t}. \tag{31}$$

Large Outputs: Potential Conflict between the Price Poster and the Price Discriminator

When $t < 1.5v$, the positions of the two duopolists may conflict, since the "target" outputs of the price poster and the price discriminator might add to more than 1. Suppose the price poster serves z consumers in equilibrium. If z does not equal 0 or 1, the consumer located at z must be indifferent, so

$$CS(z, pd) = (1-\lambda)[v - t(1-z)] = CS(z, pp) = v - tz - p \tag{32}$$

which implies that

$$p = \lambda v + (1-\lambda)t - (2-\lambda)tz. \tag{33}$$

The revenue $zp(z)$ is strictly concave in z, and maximized by

$$\begin{aligned} \tilde{z} &= \tfrac{\lambda v + (1-\lambda)t}{2t(2-\lambda)} \\ &= \tfrac{\lambda\left(\frac{v}{t}\right) + (1-\lambda)}{4-2\lambda}. \end{aligned} \tag{34}$$

There are are various cases to consider, depending on whether the price poster captures the entire market, none of the market, part of the market where some consumers go unserved, or part of the market where all consumers are served. Let us figure out the best responses of the price discriminator in these various cases.

(2) $t \in [\frac{(4-\lambda)v}{3-\lambda}, 1.5v]$**. Constrained Monopoly**

This case is when the price setter chooses a low enough p that the price discriminator can have sales of v/t. This requires $t > v$. The price poster's sales will be $z_{pp} = 1 - v/t$. Thus, it must be that the solution to the price poster's maximization problem in (**??**) is

$$\frac{\lambda\left(\frac{v}{t}\right) + (1-\lambda)}{4-2\lambda} \leq 1 - \frac{v}{t}, which\ implies\ t \geq \frac{(4-\lambda)v}{3-\lambda}. \tag{35}$$

A necessary condition for this last inequality to hold is $t > (4/3)v$, and hence the "constrained monopolist" case can apply only when

$$\frac{4v}{3} < t \leq 1.5v. \tag{36}$$

tho it might not apply for some of that range, depending on λ's value.

For this case, the price-discriminating duopolist avoids any competition and hence

$$\pi_1(pd, pp) = \frac{\lambda v^2}{2t}. \tag{37}$$

The price-posting duopolist sells $z^{pp} = 1 - \frac{v}{t}$ by setting the price $p = v - tz^{pp}$. Hence

$$\pi_1(pp, pd) = 2(v-t)\left(1 - \frac{v}{t}\right)) = 4v - 2t - \frac{2v^2}{t}. \tag{38}$$

(3) $t \in (\frac{\lambda v}{3-\lambda}, \frac{(4-\lambda)v}{3-\lambda})$**. The True Mixed-Duopoly Case**

(4) $t \in [0, \frac{\lambda v}{3-\lambda}]$**. The Price Poster Captures the Entire Market**

FIGURE 3. Firm 1 Posts a Price; Firm 2 Uses Isoperfect Price Discrimination, Low Transportation Cost

The solution in (33) for the price-posting firm is $z^{pp} = 1$, and hence the price-discriminating firm makes no sales and earns no profits.

This is the second place where the assumption of $\varepsilon > 0$ comes into play. Suppose a consumer deviated by going to the price- discriminating Firm 2 instead of the price-posting Firm 1. Consumer x's payoff from Firm 1 is $(v - tx - p_1)$. If he visits Firm 2, then his payoff is the bargaining solution giving him either $(1-\lambda)$ of the surplus $(v-t(1-x))$ or the outside option of $[(v - tx - p_1) - \varepsilon]$, whichever is greater.

The consumer with the greatest surplus from trade with Firm 2 is at $x = 1$. If he visits Firm 1 his payoff is $v - t - p_1$. If he visits Firm 2, his payoff is $Max(1 - \lambda)v, v - t - p_1 - \varepsilon)$. In this parameter range, the outside option is binding, so his payoff from choosing Firm 2 is $v - t - p_1 - \varepsilon$, which is lower than from visiting Firm 1.

Figure 3 illustrates this for the case of $\lambda = .5$. Consumer $x = 1$ could get payoff of s_1 from visiting Firm 1. Visiting Firm 2, his Rubinstein payoff would be $v/2$, a smaller amount. His outside option would be a payoff of $s_1 - \varepsilon$, more than $v/2$ so the bargaining game would yield him a payoff of $s_1 - \varepsilon$, but that is still worse than buying from Firm 1.

$$\pi_1(pp, pd) = \lambda v - t \text{ and } \pi_2(pp, pd) = 0. \tag{39}$$

Since the maximum value that λ can take is 1, this case requires that $t < .5v$.

THE EQUILIBRIUM.

We will now assume balanced isoperfect price discrimination: $\lambda = .5$

If $t > 2v$, then the industry is in effect two separate monopolies, not a duopoly, since it is unprofitable for either firm to serve the consumer at $x = .5$. Similarly, if $t \in (1.5v, 2v]$ and one firm decides to use posted pricing, some consumers will go unserved whom that firm decides not to serve and whom it would be unprofitable for the price-discriminating firm to serve. Both cases lack true competition, so the question of how the presence of another firm affects the choice of pricing policy is vacuous.

What if $t < 1.5v$? Then there is at least one consumer whose custom is solicited by both firms, and competition does constrain a firm's pricing.

First, note that there will exist no mixed equilibrium in which one firm uses price discrimination and the other uses a posted price. For that to happen would require that there be a consumer indifferent between the two firm, given the prices he would be offered at each. If, however, that consumer is indifferent, then if he chooses to go to the price-discriminating firm, the price-discriminating firm would deviate from its equilibrium price. It would raise it by $\varepsilon/2$, which, given the switching cost of ε, would not repulse the consumer but would yield higher profit. Foreseeing this, however, the consumer would not go to the price- discriminating firm in the first place. Thus, there cannot exist a consumer indifferent between the two firms, which contradicts our original supposition.

This leaves as possible equilibria both firms using posted prices or both firms using price discrimination.

Profits with Both Using Posted Pricing versus Both Using Price Discrimination

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8. Concluding Remarks

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