SOMETHING HILBERT GOT WRONG AND EUCLID GOT RIGHT: THE METHOD OF SUPERPOSITION AND THE SIDE-ANGLE-SIDE AXIOM IN PROPOSITIONS 1 TO 4 OF BOOK I OF THE ELEMENTS

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Abstract

Add a new postulate saying we can draw a line of length pi times the reference line, adn we get interestnig new geometrya dn can square the circle.

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This paper: http://www.rasmusen.org/papers/euclid-plus.pdf.

Keywords: xxx

I would like to thank xxx

NOTES

I could bring in an arc definition based on pi, but I would have to define pi rather than derive it. I should probably do that anyway. $\pi \equiv \frac{circumference}{2*radius}$.Can I prove that this is the same for all circles?

I can have another postulate:

Postulate 5C: To draw a segment equal to a circle's circumference divided by twice its radius.

Then I can square the circle and so forth. Probably I can trisect the angle too.

I need a definition of circumference.

Add pi to get Euclid+. Add the cube root of two to get Euclid++ and double the cube.

Proposition I-45B (quadrature) To construct a rectangle equal to a given rectilinear figure in a given rectilinear angle and with one side equal to the reference segment.

Proposition I-45C (area) To construct a segment equal to a given rectilinear figure. (maybe make this a corollary)

Proof:

Use I-45B. The side not equal to the reference segment will equal the given rectilinear figure.

With I-45C and Postulate 5C (the pi postulate) I can find the area of a circle. It is easy then to square the circle by constructing a square of that area.

The Pi Postulate is reasonable. It is like pos 1, 3. We cannot draw true segments or circles, so we postualte them. We cannot draw a true pi-segement, so we postualte it. It is no stronger a postulate. If we draw a segement at random, almost surely t will be irrational,a nd almost surely transcndental. We can ddeduce hwo to draw a square root of two segement in Euclid.

We can certainly square the circle now. I bet we can trisect the angle too, because pi is related to trigonometry.

1. Two distinct points A and B always completely determine a straight line a. We write $AB = a$ or $BA = a$.

Instead of "determine," we may also employ other forms of expression; for example, we may say "A lies upon a", "A is a point of a", "a goes through A and through B", "a joins A to B", etc. If A lies upon a and at the same time upon another straight line b, we make use also of the expression: "The straight lines a and b have the point A in common," etc.

2. Any two distinct points of a straight line completely determine that line; that is, if $AB = a$ and $AC = a$, where $B \neq C$, then also $BC = a$.

6. If two planes α, β have a point A in common, then they have at least a second point B in common.

7. Upon every straight line there exist at least two points, in every plane at least three points not lying in the same straight line, and in space there exist at least four points not lying in a plane.

II. Order

1. If a point B is between points A and C, B is also between C and A, and there exists a line containing the points A, B, C.

2. If A and C are two points of a straight line, then there exists at least one point B lying between A and C and at least one point D so situated that C lies between A and D.

3. Of any three points situated on a straight line, there is always one and only one which lies between the other two.