Common Confusions over Hyperbolic Discounting

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Abstract

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Strotz, etc.

Phelps & Pollack (1968) had used the quasi-hyperbolic form to discount generations of people over time. Laibson (1997) applied it to within the self and named it "quasi-hyperbolic". True "hyperbolic" discounting, proposed by Chung & Herrnstein (1961) in connection with a particular theory of behavior, has similar qualitative features but is more complicated to work with. See Angeletos, Laibson, Repetto, Tobacman & Weinberg (2001) for a brief explanation of the differences.

What is "hyperbolic discounting"?

It is easy for people to get this wrong, particularly since the term is, as I will explain below, used for a form of discounting that does I don't think any of the scholars who specialize in it are confused, but their explanations of it to other scholars are sometimes misleading. I hope this note will be useful to them for revising their explanations and for interested outsiders.

What Hyperbolic Discounting Is

This section is copied verbatim from my Internalities article. I should rewrite it for here.

People commonly have positive time preference: they prefer to consume more now rather than more later. The standard way to include this in economic analysis is by having a positive personal discount rate in the utility function so that consumption earlier adds more to utility than consumption later, e.g. $U_{2000} = C_{2000} + \delta_{2001}C_{2001} + \delta_{2001}\delta_{2002}C_{2002} + \delta_{2001}\delta_{2002}\delta_{2003}C_{2003}$, where $\delta_t < 1$ and we could but need not assume a constant discount factor $\delta_t = \delta$. This functional form is an example of exponential discounting, whose key feature is that the time subscripts for the discount factors are objective years rather than "years in the future". As a result, if we view the person's decisions starting one year in the future his utility function will be: $U_{2001} = C_{2001} + \delta_{2002}C_{2002} + \delta_{2002}\delta_{2003}C_{2003}$.

A nice property of exponential discounting is that a person's consumption path will be time consistent, meaning that if the person maximizes his utility at time 2000 by the choices $(C^*_{2000}, C^*_{2001}, C^*_{2002}, C^*_{2003})$ then he will maximize them at time 2000 by the same values $(C^*_{2001}, C^*_{2002}, C^*_{2003})$ given his reduced wealth as the result of the consumption in 2000. The person may regret consuming so much in the year 2000, so his decisions across time are not consistent in the sense of being the choices he would make ex post, but the 2000 consumption is a sunk decision by 2001 anyway.

An alternative way a person might have time preference is to have discount factors whose levels depend not on the year itself– 2000, 2001, 2002– but on how many years in the future the consumption will occur–now, one year from now, two years from now. Looking at the decision made in year 0, the functional form could be exactly the same, e.g. $U_0 = C_0 + \delta_1 C_1 + \delta_1\delta_2 C_2 + \delta_1\delta_2\delta_3 C_3$, where $\delta_t < 1$. At time 1 (year 2001) however, the person would maximize $U_1 = C_1 + \delta_1 C_2 + \delta_1\delta_2 C_3$, not $U_1' = C_1 + \delta_2 C_2 + \delta_2\delta_3 C_3$. This is just one of many ways time preference could be non-exponential, but if the form it take is not exponential, the person's decisions become time inconsistent. The optimal choices $(C^*_{2000}, C^*_{2001}, C^*_{2002}, C^*_{2003})$ from the 2000 utility function will not match the optimal choices using the 2001 utility function, $(C^{**}_{2001}, C^{**}_{2002}, C^{**}_{2003})$. For example, it may be that the person is expecting a big income bonus in 2002. In year 2000, he might want to choose to spread that income's consumption between 2002 and 2003 because though he highly values year 0 consumption, he is relatively indifferent between years 2 and 3. By the time 2002 arrives, however, year 2002 *is* year 0, and he would want to consume the entire bonus immediately.

What Hyperbolic Discounting Is Not

(a) Hyperbolic discounting is not a high level of impatience or time preference. A person can have high time preference even under standard exponential discounting. The key to hyperbolic discounting is that the person's high rate of discounting for a given future year's utility (say, utility in 2020) changes as that year approaches.

Note too that in theory hyperbolic discounting could exhibit strangely time preference for the present, negative time preference. Someone might always care little about the present year but a lot about future years. I expect that would introduce time inconsistency just like hyperbolic discounting usually does.

(b) "Hyperbolic discounting" does not, as commonly used, mean time discounting using a hyperbolic function form. In fact, the standard way to model it is with the "quasi-hyperbolic discounting" of Laibson (1997), which is simpler to use and has pretty much the same properties. Quasi-hyperbolic utility (also called "Beta-Delta Utility") would have a form something like:

$$U_0 = C_0 + \beta\delta C_1 + \beta\delta^2 C_1 + \beta\delta^3 C_2 + ...$$

for consumption over time. Hyperbolic utility would have a form something like:

$$U_0 = C_0 + \left(\frac{1}{1+k}\right) C_1 + \left(\frac{1}{1+2k}\right) C_2 + ...$$

Exponential utility would have a form something like:

$$U_0 = C_0 + \delta C_1 + \delta^2 C_1 + \delta^3 C_2 + ...$$

And in fact any functional form for discounting except exponential discounting would give rise to time inconsistency. I don't think the particular functional form matters much, except for simplicity. This is important because one might think that hyperbolic discounting was a special case, which is false– exponential discounting is the special case. It would probably be better to use the term "nonexponential discounting", but "hyperbolic" is what is commonly used.

(c) Hyperbolic discounting really isn't about the shape of the discount function anyway. It's about how a person's discount rate for a given time period changes as that time period gets closer. At any one time, knowing the shape of the person's discount function for each period's future consumption doesn't tell you whether he is using "hyperbolic discounting" or not– except for the special case in which his per- period exponential discount factor is constant. This is best seen with examples.

First let's look at the special case. xxx

That is a special case because there is no reason that delta should stay the same over time. If we were looking at the term structure of interest rates, we would not assume that. Here we are looking at parameters of a utility function, but it still not clear, except for simplicity, why we should assume a constant discount rate. People change as they get older. It is not implausible that I would use a different rate of time preference at age 70 than at age 49. But that is fully compatible with exponential discounting. The key is that exponential discounting treats the parameter as "Rasmusen's rate of time preference for when he is 70 in the year 2058" whereas non-exponential discounting treats it as "Rasmusen's rate of time preference for 21 years from the present."

It is easy to come up with examples where exponential and non- exponential discount functions are identical. Let us continue with the example above.

Let the QH form be beta, delta, with beta=.5, delta =.9. The exponential equivalent is: D1 = .45, D1D2 = .5(.81), D1D2D3 = .5 (.9) (.81).

It is that the Ds are getting bigger over time.

Let the EXP form be .9, .81, (.9)(.81) The QH form is .5D1=.9. .5D1D2= .81, .5 D1 D2D3 = (.9)(.81). It is that the D's are getting smaller over time.

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