The Rubinstein Bargaining Model with Both Discounting and Fixed Per-Period Costs


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Abstract

Rubinstein (1982) describes two bargaining models which reach opposite conclusions. In Model I has discounting and has become the workhorse model of bargaining. In Model I, the split is about 50-50 when discount rates are small and almost equal. In Model II, which has a fixed cost for each period of bargaining, the split is 100-0 when bargaining costs are small and almost equal. Rubinstein does not say what happens in a model with both discounting and bargaining costs. If the mixed model were to behave more like Model II, the Rubinstein model would be an exceptionally poor fit to reality. In fact, it behaves like Model I, so conventional usage is safe from this criticism.


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1. Introduction

Rubinstein’s 1982 *Econometrica* article on bargaining has been exceptionally influential, receiving 2003 cites in Google Scholar as of June 2008. It has been so influential because it constructs a simple and elegant model of bargaining in which agreement is reached immediately, the split is close to 50-50 between identical players, and the split diverges from 50-50 depending on a simple measure of bargaining power, the discount rate. The model is useful not only in direct application, but as a component of more complex models of interactions in which players split a surplus at some point in the game they are playing. Rather than simply assume that the bargaining split is 50-50, or is \( \lambda, (1 - \lambda) \) for some reduced-form bargaining parameter, the modeller can build a model from primitive assumptions about payoffs and actions.

A complete reading of Rubinstein (1982), however, raises a nagging doubt about the model. Rubinstein (1982) actually contains two simple models of bargaining, and they reach opposite conclusions. In Model I, the much-cited model based on time discounting, the split is about 50-50 when discount rates are small and almost equal. In Model II, which has a fixed cost for each period of bargaining, the split is 100-0 when bargaining costs are small and almost—but not quite—equal. Rubinstein does not say what happens in a model with both discounting and bargaining costs. If that model were to behave more like Model II, the Rubinstein model would be an exceptionally poor fit to reality. In fact, the mixed model is like Model I, as I will show below.

2. The Model

Ann and Bob are splitting a pie of size 1. First Ann makes an offer of a split of \( x_A \) for herself and \( (1 - x_A) \) for Bob. If Bob accepts, the game is over and the payoffs are

\[
\pi_A = x_A, \quad \pi_B = 1 - x_A.
\]

(1)

If Bob rejects, a period of time passes, at the end of which he pays \( c_B \) and Ann pays \( c_A \).\(^1\) He then makes an offer of a split of \( x_B \) for herself and \( (1 - x_B) \) for Ann. If Ann accepts, the game is over and the payoffs are (viewed from the start of the game)

\[
\pi_A = \delta_A[-c_A + (1 - x_B)], \quad \pi_B = \delta_B[-c_B + x_B]
\]

(2)

because second-period payoffs are discounted by discount factors of \( \delta_A = \frac{1}{1 + \rho_A} < 1 \) and \( \delta_B = \frac{1}{1 + \rho_B} < 1 \). Our main interest is in what happens when the periods are short, so the discount rates approach \( \rho_A = \rho_B = 0 \) and the discount factors approach \( \delta_A = \delta_B = 1 \).

If Ann rejects Bob’s offer, another period of time elapses and she makes the next offer. The two players make alternating offers until agreement is reached, or forever if agreement is not reached.\(^2\)

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\(^1\)We could also modify the model to make the bargaining cost an “offer cost” paid only by the offeror. The outcome would be the same.

\(^2\)Unlike in some games, it makes little difference whether the game is infinitely repeated or just repeated a large finite number of times.
3. The Equilibrium

In any subgame perfect equilibrium, Ann’s first offer will be accepted immediately. The only reason to wait is that Ann can get a bigger share $x'_a$ later. She would prefer to offer $x'_a - c_a + \epsilon$ now.

It follows that the equilibrium is stationary. The players’ payoffs cannot get bigger or smaller, or they would make more generous offers now that would equal any bigger payoffs without having to pay the bargaining costs.

Denote Ann’s equilibrium offer by $x^*_A$ and Bob’s by $x^*_B$. In making her offer, Ann realizes that Bob could reject it and, if both players follow their equilibrium strategies, offer $x^*_B$ next period and have it accepted. Thus, rejection gives Bob a payoff of $\delta_B[-c_B + x^*_B]$, as in equation (??). Ann can make Bob willing to accept her offer by making it generous enough to give him the same payoff, i.e.

$$1 - x^*_A = \delta_B[-c_B + x^*_B]$$

(3)

In making his offer, Bob realizes that Ann could reject it and, if both players follow their equilibrium strategies, offer $x^*_A$ next period and have it accepted. Thus, rejection gives Ann a payoff of $\delta_A[-c_A + x^*_A]$. Bob can make Ann willing to accept his offer by making it generous enough to give her the same payoff, i.e.

$$1 - x^*_B = \delta_A[-c_A + x^*_A].$$

(4)

These two equations solve to

$$x^*_A = \frac{(1 - \delta_B) + \delta_B(c_B - \delta_A c_A)}{1 - \delta_B \delta_A}$$

(5)

Bob’s offer $x^*_A$ is exactly analogous.

We get the Rubinstein solution if $c_A = c_B = 0$ and either $\delta_A \neq 1$ or $\delta_B \neq 1$:

$$x^*_A(c_A = c_B = 0) = \frac{1 - \delta_B}{1 - \delta_B \delta_A}$$

(6)

As Rubinstein found, we cannot let $\delta_A = \delta_B = 1$ or the solution technique fails to work.

An important case is what happens when there is positive and equal discounting, but the two bargainers differ in their bargaining cost:

$$x^*_A(\delta_A = \delta_B = \delta < 1) = \frac{(1 - \delta) + \delta(c_B - \delta_A c_A)}{1 - \delta^2}.$$
Using L'Hopital’s Rule, if $\delta$ approaches 1 then
\[
x_A^* (\delta_A = \delta_B = \delta \to 1) = \frac{(1-\delta) + \delta(c_B - \delta c_A)}{1-\delta} = \frac{-1 + c_B - 2\delta c_A}{2\delta} = \frac{1 + c_B - 2\delta c_A}{2} = \frac{1}{2} + \frac{c_B}{2} - c_A
\]

Thus, when both bargaining costs and the discount rates are small and almost equal, the split is close to 50-50. The mixed model is more like Model I (with discounting) than like Model II (with per-period bargaining costs).

References


Appendix (not to be published)

Solving,
\[
1 = -\delta_B c_B + \delta_B x_B^* + x_A^*
\]
\[
1 + \delta_B c_B - x_A^* = \delta_B x_B^*
\]
\[
x_B^* = \frac{1 + \delta_B c_B - x_A^*}{\delta_B}
\]

So
\[
1 - \frac{1 + \delta_B c_B - x_A^*}{\delta_B} = \delta_A[-c_A + x_A^*].
\]
\[
\delta_B - 1 - \delta_B c_B + x_A^* = \delta_B \delta_A[-c_A + x_A^*].
\]
\[
x_A^* = -\delta_B \delta_A c_A + \delta_B \delta_A x_A^* - \delta_B + 1 + \delta_B c_B
\]
\[
x_A^* (1 - \delta_B \delta_A) = -\delta_B \delta_A c_A - \delta_B + 1 + \delta_B c_B
\]
\[ x_A^* = \frac{-\delta_B \delta_A c_A - \delta_B + 1 + \delta_B c_B}{(1 - \delta_B \delta_A)} \]