Coarse Grades: Informing the Public when Certification Is Voluntary

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Abstract

Certifiers of quality often report only coarse grades to the public despite having measured quality more finely, e.g., "Pass" or "Certified" instead of "73". Why? We show that coarse grades result in more information being provided to the public because the coarseness encourages those of middling quality to apply for certification. Dropping exact grading in favor of the best coarse grading scheme always reduces public uncertainty because the extra participation outweighs the coarser reporting. In some circumstances, pass-fail grading— the coarsest meaningful grading scheme– is the most informative.

The Model

Firms vary in quality type q, distributed according to atomless $F(q)$ with support $[0, 1]$ and density $f(q) > 0$.

Consumers are willing to pay a price up to their estimate $\hat{q}$ of quality.

Each firm knows its own quality and has zero marginal cost of production. Firms maximize profits by the choice of price p and whether or not to pay the cost c of having their quality measured and reported by a certifier.

If a firm applies for certification, the certifier measures its quality perfectly. In advance, the certifier's chooses a grading scheme $\overline{q}(q)$, a mapping of measured quality q to certified quality $\overline{q}$.

The certifier's objective is to maximize the value of the public's information: specifically, to minimize the mean squared error (MSE) of the public's estimate of quality.

The General Idea

We will look at which firms volunteer to be certified under different grading schemes and distributions of firm quality.

The general force at work is that the best firms would like to go through the grading scheme so the public estimate of their value will rise, but the worst firms will prefer not to be graded and to instead be valued at the average of the ungraded pool.

This kind of game "unravels" if firms at the top of the ungraded pool would switch to being graded rather than be pooled with the rest.

The assumption that firms bear a cost to being certified stops the unravelling.

Exact Grading Schemes

(1) In an *exact* grading scheme, the exact quality of the product is revealed if it is certified:

$$\overline{q} = q. \tag{1}$$

If a scheme is not exact, it is *coarse*, and the exact quality of at least some product types is not revealed.

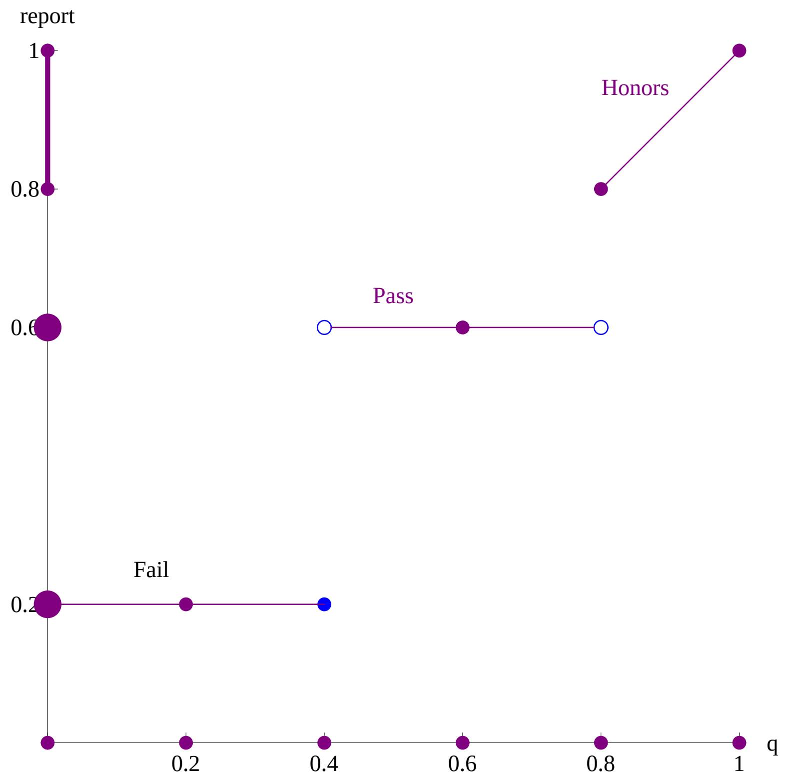
It will turn out that in equilibrium not all firms will apply to be certified under this scheme, so we will denote by $\overline{\overline{q}}$ the minimum quality that does apply.

Pass-Fail Schemes

$$\begin{aligned} \overline{q} &= fail\ if\ q < \overline{q} \\ &= pass\ if\ q \geq \overline{q} \end{aligned} \tag{2}$$

Honors Schemes

(a) reports the exact quality of all types above some cutoff $\overline{\overline{q}} < 1$ (the "plus" part);
(b) reports that types in the range $[\overline{q}, \overline{\overline{q}})$ are certified, with a "pass" grade, but without giving their exact quality; and
(c) reports nothing about types below $\overline{q}$. They fail, and remain uncertified.



Feasible Schemes

A grading scheme would be useless if it passed every firm or failed every firm. It would also be useless if no type of firm would ever apply for certification. We focus on those perfect Bayesian equilibria in which at least some types of firms apply for certification (in which case no out-of-equilibrium beliefs need be defined). If under a given grading scheme there exists no equilibrium in which any type of firm would apply for certification, we call that scheme *infeasible*.

It is, of course, possible to set up such a scheme (in fact, it would be costless!)— just infeasible to use it to increase consumer information, since no firm will ever show up to be tested.

The Mean Squared Error

The mean squared error of the public's estimate of q under exact grading consists of the square of the error in estimating the quality of the uncertified firms in the quality interval $[0, \overline{\overline{q}}]$, with zero contribution from firms in the interval $[\overline{\overline{q}}.1]$, about whom the public becomes perfectly informed. For the quality density $f(q)$,

$$MSE_{exact} = \int_0^{\overline{\overline{q}}} \left(q - Eq|(q < \overline{\overline{q}})\right)^2 f(q)dq + \int_{\overline{\overline{q}}}^1 (0)^2 f(q)dq \qquad (3)$$

The Mean Squared Error of Pass-Fail Grading

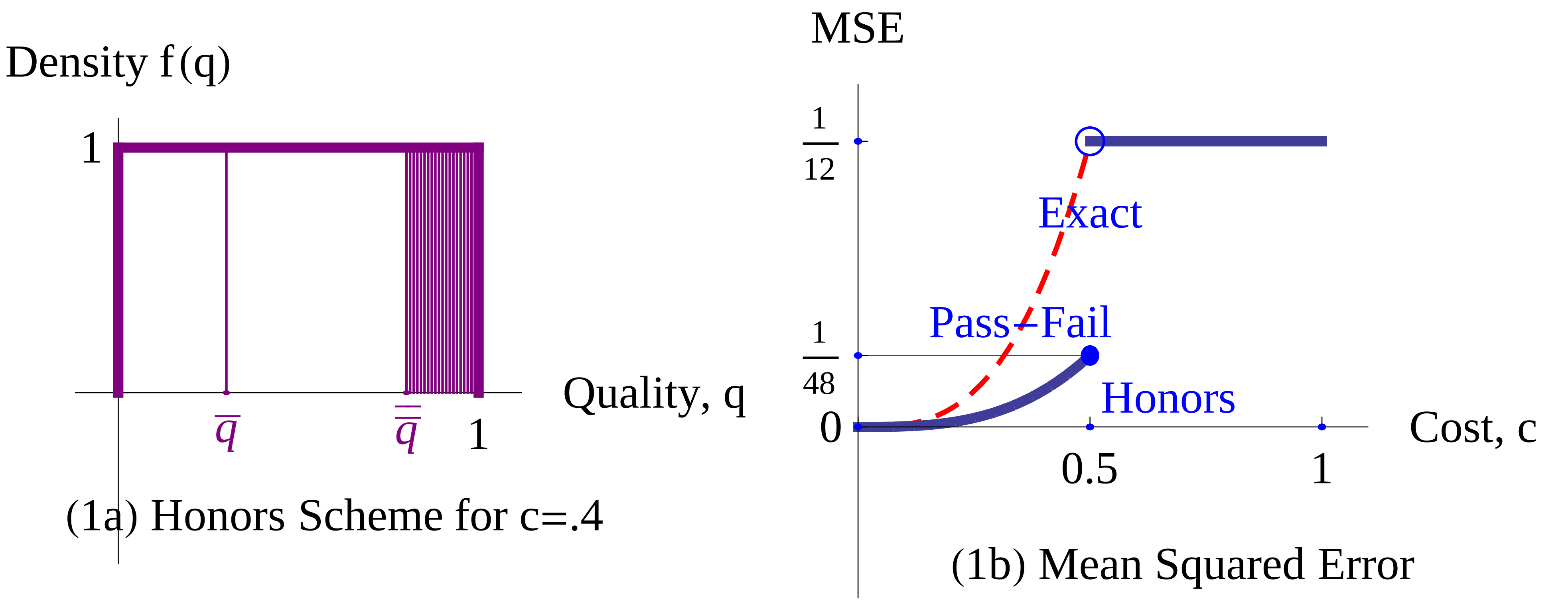
Pass-fail grading divides firms into two groups, both of whose qualities are inexactly estimated by the public: The mean squared error of the pass-fail grading scheme with a cutoff of $\overline{q}$ is:

$$MSE_{pass-fail} = \int_0^{\overline{q}} (q - Eq|(q < \overline{q}))^2 f(q)dq + \int_{\overline{q}}^1 (q - Eq|(q > \overline{q}))^2 f(q)dq \quad (4)$$

THE MEAN SQUARED ERROR OF HONORS GRADING

$$MSE_{honors} = \int_0^{\overline{q}} (q - Eq|(q < \overline{q}))^2 f(q)dq + \int_{\overline{q}}^{\overline{\overline{q}}} \left(q - Eq|(q \in [\overline{q}, \overline{\overline{q}}])\right)^2 f(q)dq + \int_{\overline{\overline{q}}}^{1} (0)^2 f(q)dq \tag{5}$$

Point 1: Coarse grading can be more informative than exact grading—for example, when the quality density is uniform.



(1a) Honors Scheme for c=.4

(1b) Mean Squared Error

THE MSE FROM EXACT GRADING, UNIFORM QUALITY DISTRIBUTION

A firm that applies for certification has payoff $q - c$. A firm that does not apply has a payoff equal to the average quality of all types that do not apply.

Firms with high quality q have the most incentive to apply, since $(q - c)$ is highest for them. The lowest-quality firm that applies, with quality $q = x$, is indifferent about applying. If it applies, its payoff is $(x - c)$. If it does not, its payoff is $Eq|(q \in [0, x])$, which is $x/2$. Equating those payoffs yields $x = 2c$.

The minimum quality that will show up to be graded is $2c$. Under exact grading, for $q \in [0, 2c)$ the estimated quality of a firm with quality q is c, for $q \in [2c, 1]$ it is q, and $f(q) = 1$, so exact grading has mean squared error of

$$MSE_{exact} = \int_0^{2c} (q - c)^2 (1) dq + \int_{2c}^{1} (q - q)^2 (1) dq = \boxed{\frac{2}{3}c^3}. \qquad (6)$$

HONORS GRADING WITH A UNIFORM QUALITY DISTRIBUTION

The most informative scheme has the smallest feasible $\overline{\overline{q}}$, so $\overline{\overline{q}} = 2c$. Any choice of $\overline{q}$ is feasible, but the most informative $\overline{q}$ divides the region below $\overline{\overline{q}}$ equally: $\overline{q} = \overline{\overline{q}}/2$ so $\overline{q} = c$.

$$MSE_{honors} = \int_0^c (q - c/2)^2\, dq + \int_c^{2c} (q - (c + 2c)/2)^2\, dq + 0 = \boxed{\tfrac{1}{6}c^3}. \qquad (7)$$

The MSE from Pass-Fail Grading, Uniform Quality Distribution

For any cutoff $\overline{q}$ note that $Eq|(q \in [0, \overline{q}]) = \overline{q}/2$ and $Eq|(q \in [\overline{q}, 1]) = (1 + \overline{q})/2$, so the difference $Eq|(q \in [\overline{q}, 1]) - Eq|(q \in [0, \overline{q}])$ is always $1/2$. In equilibrium, the firm at quality $q = \overline{q}$ must be indifferent about applying for certification, so $\overline{q}/2 = (1+\overline{q})/2 - c$. The biggest c that can support pass-fail grading is then $c = .5$.

Any value of $\overline{q}$ can support pass-fail grading, but the most informative scheme under the uniform density has $\overline{q} = 1/2$.

$$MSE_{pass-fail} = \int_0^{1/2} (q - 1/4)^2 \, dq + \int_{1/2}^{1} (q - 3/4)^2 \, dq = \boxed{\frac{1}{48}} \tag{8}$$

Why Honors Beats Exact

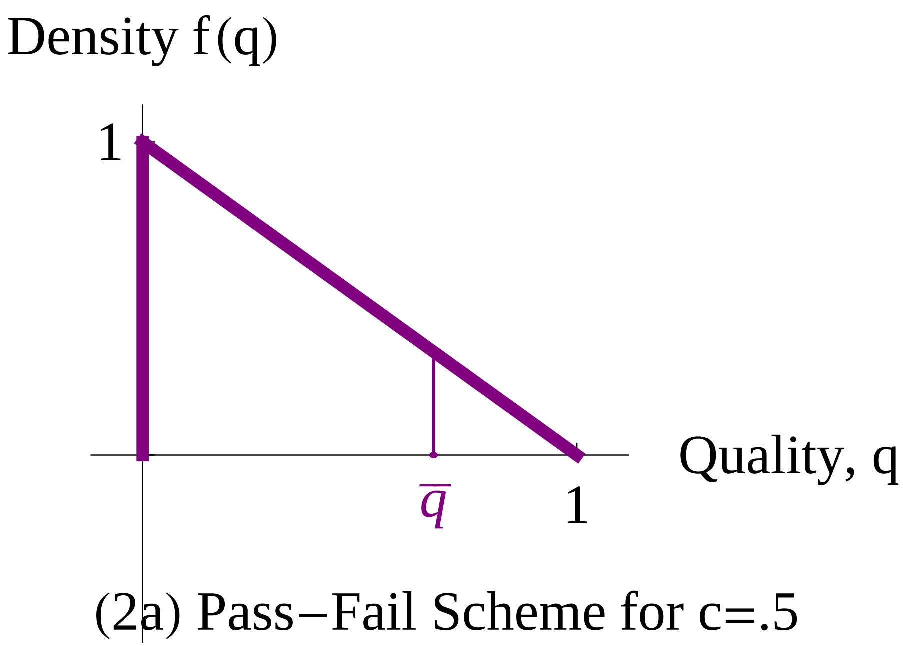
Exact grading deters too many firms from applying for certification, since it reveals the mediocre quality of the lowest firm that applies. Moving to a scheme that pools mediocre firms with superlative ones induces more firms to apply.

Think about starting from the exact scheme and suppose costs are high—say, .4. Every firm in $[0, .8]$ remains uncertified, while every firm in [.8,1] is certified.

Suppose we instead added that firms in [.6, .8] receive a grade of Pass. The firm at $q = .8$ is still willing to be certified. How about the firm at .6? If it doesn't apply, its payoff is .3. If it does apply, its payoff is .7-.4 = .3. Thus, it is willing to be certified. We have added information by moving from exact grading to honors grading.

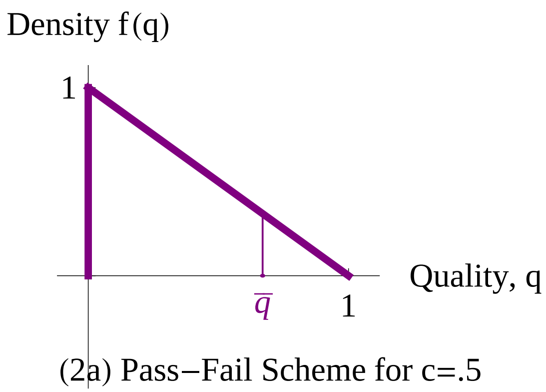
The logic works not just for the uniform density, but for any strictly positive density.

Point 2: Pass-fail grading can be either better or worse than honors grading— worse if quality has a uniform density; better if quality has a falling triangle density.



(2a) Pass−Fail Scheme for c=.5

THE FALLING TRIANGLE DENSITY

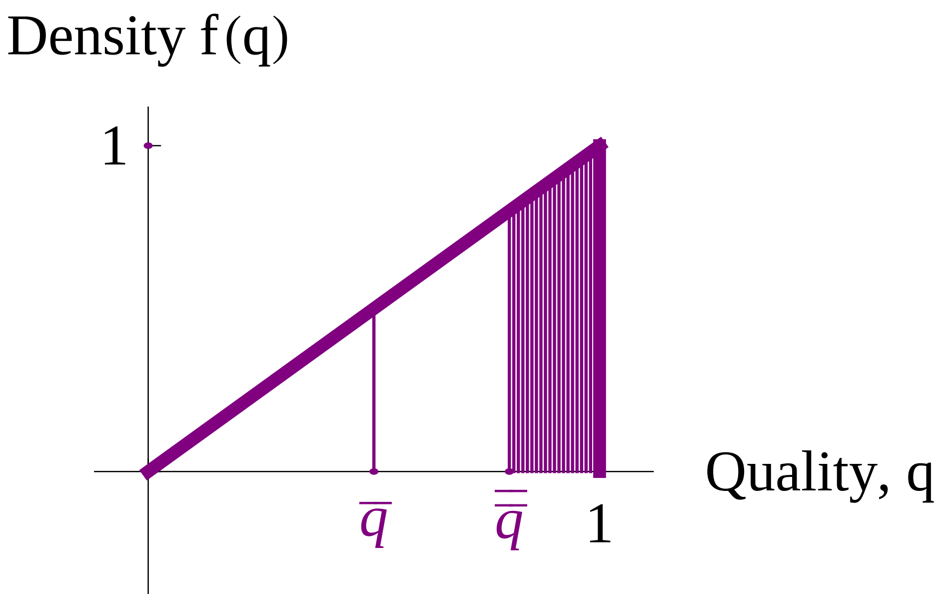


(2a) Pass–Fail Scheme for c=.5

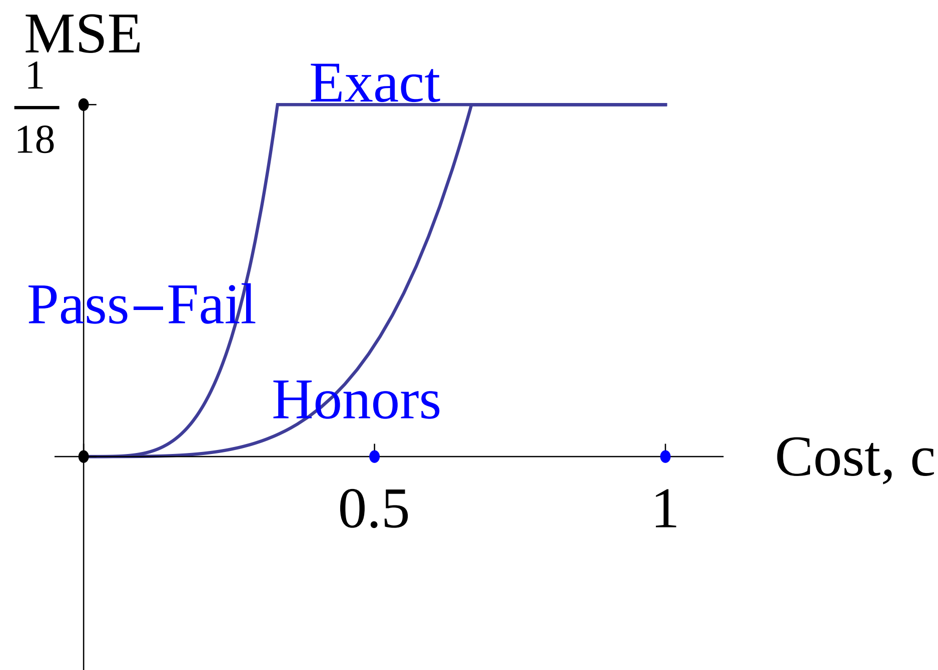
If we start exact grading at the top then the *Pass* region shrinks from $[\overline{q}, 1]$ to $[\overline{q}, \overline{\overline{q}}]$ where $\overline{\overline{q}} < 1$, which means the gap in expected values from passing and failing must shrink, so the scheme will not be feasible unless $\overline{q}$ also adjusts. Because the density is declining, $\overline{q}$ must rise to restore the gap.

The question is whether the resulting loss of information about types $q < \overline{q}$ is outweighed by the extra information about types $q \in [\overline{q}, \overline{\overline{q}}]$ and $q > \overline{\overline{q}}$. The answer is no. Because there is so little information already about types $q < \overline{q}$, any further reduction in that information leads to a large increase in mean squared error. Hence simple pass-fail grading is best in this region.

Point 3: For some quality densities and grading cost levels, exact grading is not even feasible, though coarse grading is feasible and improves consumer information



(3a) Honors Scheme for c=1/3



(3b) Mean Squared Error

The fact that lower types are partially differentiated makes it easier to fully differentiate high types.

Proposition 1: For any quality density $f(q)$, the most informative grading scheme is either pass-fail or honors.

Of all possible grading schemes– not just the three we highlight— the best will be either pass-fail or honors.

Proposition 1 and 2

Proposition 1: For any quality density $f(q)$, the most informative grading scheme is either pass-fail or honors.

Proposition 2: For c sufficiently low and any quality density $f(q)$, honors grading is most informative, exact grading is next, and pass-fail is least informative:

$$MSE_{honors} < MSE_{exact} < MSE_{pass-fail}.$$

For c sufficiently high and f decreasing, pass-fail grading is most informative, honors is next, and exact grading is least informative:

$$MSE_{pass-fail} < MSE_{honors} < MSE_{exact}.$$

PROPOSITION 3: FOR ANY QUALITY DENSITY $f(q)$, IF EXACT GRADING IS FEASIBLE THEN SO ARE PASS-FAIL AND HONORS GRADING. IF EITHER f IS INCREASING, OR F IS LOG-CONCAVE AND $Eq > .5$, THEN PASS-FAIL AND HONORS GRADING ARE STILL FEASIBLE FOR A RANGE OF GRADING COSTS SO HIGH THAT EXACT GRADING IS NOT.