Some Common Confusions about Hyperbolic Discounting

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Abstract

There is a lot of confusion over what "hyperbolic discounting" means. I try to clear up that confusion.

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What Hyperbolic Discounting Is

$$U_{2008} = C_{2008}+f(2009)C_{2009}+f(2010)C_{2010}+f(2011)C_{2011}, \tag{1}$$

where the "discount function" is $f(t) < 1$ and f is declining in t. Or, we could write the discounting in terms of per-period "discount factors" $\delta_t < 1$, as in this example:

$$U_{2008} = C_{2008}+\delta_{2009}C_{2009}+\delta_{2009}\delta_{2010}C_{2010}+\delta_{2009}\delta_{2010}\delta_{2011}C_{2011}. \tag{2}$$

t= absolute years ($\delta_t < 1$)

τ = relativistic "years in the future"

(relativistic because they depend on the year in which the person starts). As some would put it, the difference is between the *date* t and the *delay* τ. Because of using relativistic discounting, if we view our person's decisions starting one year in the future, at 2001 instead of 20001, his utility function will be:

$$U_{2001} = C_{2001} + \delta_{2002}C_{2002} + \delta_{2002}\delta_{2003}C_{2003}. \tag{3}$$

$$U_0 = C_0 + \delta_1 C_1 + \delta_1\delta_2 C_2 + \delta_1\delta_2\delta_3 C_3 \tag{4}$$

At time 1 (year 2001) the person would maximize $U_1 = C_1 + \delta_1 C_2 + \delta_1\delta_2 C_3$, not $U_1' = C_1 + \delta_2 C_2 + \delta_2\delta_3 C_3$.

$$U_{2009} = C_{2009} + \delta_{2010}C_{2010} + \delta_{2010}\delta_{2011}C_{2011}. \quad (5)$$

where $\delta_\tau < 1$, using τ now instead of t because time is relativistic rather than absolute.

At time 1 (year 2009) however, the person would maximize $U_1 = C_1 + \delta_1 C_2 + \delta_1\delta_2 C_3$, not $U_1' = C_1 + \delta_2 C_2 + \delta_2\delta_3 C_3$.

For example, it may be that the person is expecting a big income bonus in 2002. In year 2000, he might want to spread that income's consumption between 2002 and 2003 because though he highly values year 0 consumption, he is relatively indifferent between years 2 and 3. By the time 2002 arrives, however, year 2002 *is* year 0, and he would want to consume the entire bonus immediately.

Hyperbolic discounting is a useful idea.

First, it can explain revealed preferences that are inconsistent with exponential discounting.

Second, it can explain certain observed behaviors such as people's commitments to future actions when other explanations such as strategic positioning fail to apply, e. g., a person's joining a bank's saving plan which penalizes him for failing to persist in his saving.

(a) Hyperbolic discounting is not about the discount rate changing over time. A constant discount rate is not essential for time consistency, nor does a varying discount rate create time inconsistency.

(b) “Hyperbolic discounting” does not, as commonly used, mean discounting using a hyperbolic function.

(c) Hyperbolic discounting really isn’t about the shape of the discount function anyway.

(d) Hyperbolic discounting is not about someone being very impatient.

(e) Hyperbolic discounting is not necessarily about lack of self- control, or irrationality.

(f) Hyperbolic discounting does not depend delicately on the length of the time period.

(a) Hyperbolic discounting is not about the discount rate changing over time. A constant discount rate is not essential for time consistency, nor does a varying discount rate create time inconsistency.

(1) it makes the per-period discount rate change over time.

(2) it bases discounting on relativistic time rather than absolute time.

If I am planning for the consumption of my 8-year-old daughter in 2008 I might use $\rho_t = 10\%$ for each year in the interval [2008, 2015] and then use $\rho_t = 5\%$ for the interval [2015, 2025] because I expect her degree of impatience to change. That's still absolute-time discounting. It's relativistic-time discounting only if in each of those 17 years I followed a policy of using 10% for whatever years were the 7 next years from the present and 5% for whatever years were the 8th to 17th year from the present.

(b) "Hyperbolic discounting" does not, as commonly used, mean discounting using a hyperbolic function.

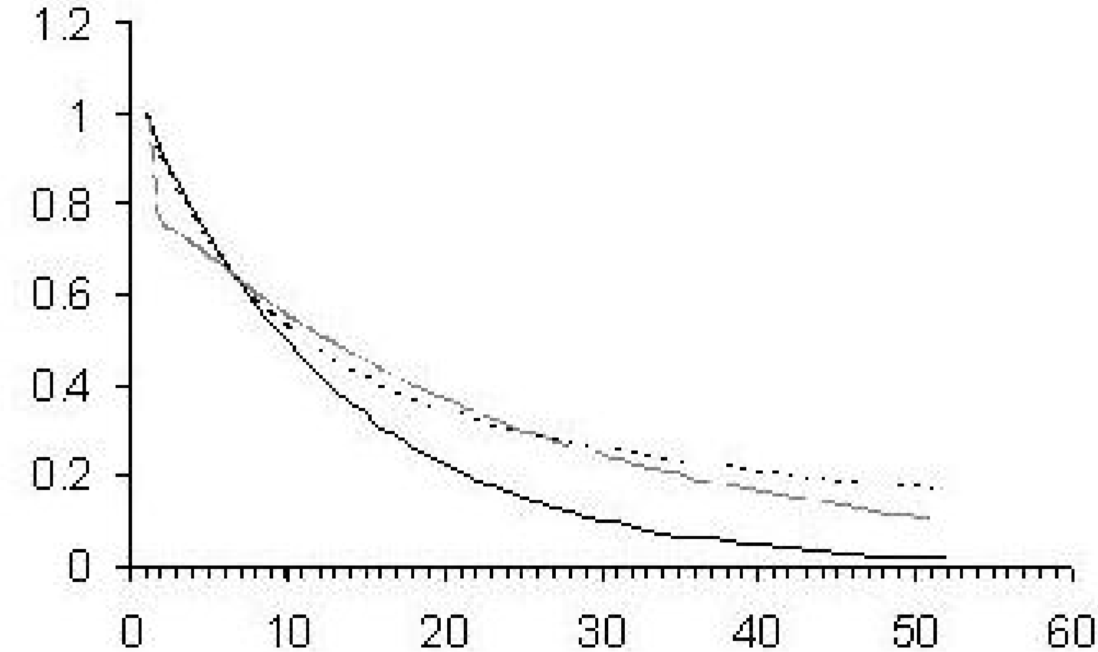

Figure 1
The Shapes of Exponential (solid), Hyperbolic (dotted), and Quasi-Hyperbolic (dashed) Discounting
($\delta_{exp} = .92$, $\frac{1}{1+.1\tau}$, $\beta = .8$ or $H = 1.25$ and $\delta_{qh} = .96$)

Exponential utility with a constant discount factor δ has the form $f(t) = \delta^t$:

$$U_0 = C_0 + \delta C_1 + \delta^2 C_1 + \delta^3 C_2 + ..., \qquad (6)$$

Quasi-hyperbolic utility (also called "Beta-Delta Utility") has the form $f(\tau) = \beta\delta^\tau$:

$$U_0 = C_0 + \beta\delta C_1 + \beta\delta^2 C_1 + \beta\delta^3 C_2 + ... \qquad (7)$$

True hyperbolic utility has the form $f(\tau) = \frac{1}{1+\alpha\tau}$:

$$U_0 = C_0 + \left(\frac{1}{1+\alpha}\right) C_1 + \left(\frac{1}{1+2\alpha}\right) C_2 + ... \qquad (8)$$

The supply side, the budget constraint must use absolute time– t, not τ.

$$C_0 + \left(\frac{1}{1+r_t}\right) C_1 + \left(\frac{1}{1+r_1}\right)\left(\frac{1}{1+r_2}\right) C_2 + ... + \left(\frac{1}{1+r_1}\right)\left(\frac{1}{1+r_2}\right) \cdots \left(\frac{1}{1+r_t}\right) C_t \qquad (9)$$

where W_0 is the present value of wealth at time 0.

Quasi-hyperbolic utility (also called "Beta-Delta Utility") has the form:

$$U_0 = C_0 + \beta\delta C_1 + \beta\delta^2 C_1 + \beta\delta^3 C_2 + ... \quad (10)$$

Quasi-hyperbolic discounting's functional form can also be written as:

$$U_0 = H * C_0 + \delta C_1 + \delta^2 C_1 + \delta^3 C_2 + ..., \quad (11)$$

where $H > 0$, and where $H > 1$ for a person who distinguishes sharply between current consumption and future consumption. The marginal rate of substitution between consumption at time 0 and time τ' is H/δ^τ, as opposed to the $(1/\beta)/\delta^\tau$ from equation (**??**), but they represent exactly the same consumer preferences.

(c) Hyperbolic discounting really isn't about the shape of the discount function anyway.

Example. We will use the hyperbolic discounting function from Figure 1:

$$\delta_\tau = \frac{1}{(1 + .1\tau)}. \tag{12}$$

The Time: t or τ	0	1	2	3	4
The Discount Function for 0 to t: $f(t)$ or $f(\tau)$	1	.91	.83	.77	.71
The Discount Rate for $t-1$ to t: ρ	—	10%	9%	8%	8%
The Discount Factor for $t-1$ to t, δ_t	—	.91	.92	.92	.93
The Quasi-Hyperbolic Delta Parameter for $t-1$ to t: δ_τ	—	.96	.92	.92	.93

Table 2: Exponential Discounting with a Hyperbolic Shape (listed to 2 decimal places)

An exponential utility function with the same shape can be derived from $f(1) = \delta_1$, $f(2) = \delta_1\delta_2$, $f(3) = \delta_1\delta_2\delta_3$, ... $f(t) = \delta_1\delta_2\delta_3 \cdots \delta_t$. so we can calculate

$$\delta_t = \frac{f(t)}{f(t-1)}, \tag{13}$$

and since $\delta_t = \frac{1}{1+\rho_t}$, we can calculate $\rho_t = \frac{1-\delta_t}{\delta_t}$.

Similarly, we can find a quasi-hyperbolic utility function with the same shape if we are allowed to vary the δ parameter. Let's set $\beta = .95$ We have $f(1) = \beta\delta_1$, $f(2) = \beta\delta_1\delta_2$, $f(3) = \beta\delta_1\delta_2\delta_3$, ... $f(\tau) = \beta\delta_1\delta_2\delta_3 \cdots \delta_\tau$. so we can calculate

$$\delta_1 = \frac{f(1)}{\beta} \tag{14}$$

and

$$\delta_\tau = \frac{f(\tau)}{f(\tau-1)}. \tag{15}$$

Time: t or τ	0	1	2	3	4	5	6
Discount Function for 0 to t: $f(t)$ or $f(\tau)$	1	.91	.83	.77	.71	.67	.63
Discount Rate for $t-1$ to t: ρ_t or ρ_τ	—	10%	9%	8%	8%	7%	7%
Discount Factor for $t-1$ to t, δ_t	—	.91	.92	.92	.93	.93	.94
Quasi-Hyperbolic Delta Parameter for $t-1$ to t: δ_τ	—	.96	.92	.92	.93	.93	.94

Table 2: Exponential Discounting with a Hyperbolic Shape (to 2 decimal places)

Note in Table 2 how the exponential discount rates are declining as time passes. This is a general feature of the hyperbolic discounting function with constant α. It is not a characteristic of the quasi-hyperbolic discounting function with constant δ, for which, of course, the discount factor is constant at δ after the first period so the discount rate is also constant.

(d) Hyperbolic discounting is not about someone being very impatient.

A person can have high time preference even under standard exponential discounting.

In theory, hyperbolic discounting could result in negative time preference, preferring future to present consumption.

Someone might always care little about the present year, but a lot about future years. This would be one way to model a person who derives much of his utility from anticipation of future consumption. Patience of this kind would introduce time inconsistency too. In 2010 the person would want to consume a lot in 2015, but in 2015 he would prefer to defer consumption. Thus, the essence of hyperbolic discounting is not excessive impatience.

In addition, see Figure 1.

(e) Hyperbolic discounting is not necessarily about lack of self- control, or irrationality.

It is one way to model lack of self-control, to be sure, by having $0 < \beta < 1$ in the quasi-hyperbolic model.

The question of whether in a particular setting hyperbolic discounting is being used to model (a) preferences that we usually don't assume in economics, or (b) mistakes such as lack of self-control, is important, especially for normative analysis.

See Bernheim & Rangel (2008) or my own Rasmusen (2008) for two attempts to grapple with welfare analysis when discounting is hyperbolic.

(f) Hyperbolic discounting does not depend delicately on the length of the time period.

If we mean "discounting using a hyperbolic function" then it is actually true that the model depends heavily on the length of the time period. Double the units in which you measure time, and you change the shape of the discounting function. But if we mean "quasi-hyperbolic discounting" that criticism fails to apply. Recall:

$$U_0 = H * C_0 + \delta C_1 + \delta^2 C_1 + \delta^3 C_2 + ...,$$

There is a big difference between the present and consumption at any future time, but the units in which time is measured do not affect tradeoffs between future time periods (though of course δ has to be written in the new time units too, so its value will change).

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