The Parking Lot Problem

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Abstract

If property rights are not assigned over individual goods such as parking spaces, competition for them can eat up the entire surplus. We find that when drivers are homogeneous, there is a discontinuity in social welfare between "enough" and "not enough." Building slightly too small a parking lot is worse than building much too small a parking lot, since both have zero net benefit and larger lots cost more to build. More generally, the welfare losses from undercapacity and overcapacity are asymmetric, and parking lots should be "overbuilt." That is, the optimal parking lot size can be well in excess of mean demand. Uncertainty over the number of drivers, which is detrimental in the first-best, actually increases social welfare if the parking lot size is too small.

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The Model

A set of players – the drivers – is indexed by $I \equiv \{1, ..., N\}$.

The drivers are workers who must show up for work no later than time $t = T$.

Each driver demands one space in the parking lot, and his value for it is $v > 0$.

Let $K > 0$ be the size of the parking lot, and let $c < v$ be the cost of increasing capacity by one space.

If $N \leq K$, each driver is guaranteed a parking space, and all drivers choose to arrive at their preferred time, T. If $N > K$, however, the drivers compete for spaces, and they may wish to arrive early to secure spots.

A driver who arrives earlier than time T incurs a cost of $w > 0$ per unit of time, so his cost of arrival at time t is $L(t) = (T - t)w$. We will model time as discrete, with the interesting case being what happens as the time interval $\Delta > 0$ shrinks to zero. Thus, a time grid includes times $t = k\Delta \in [0, T]$ where $k \in \{0, ..., T/\Delta\}$.

We compare two alternative assumptions on whether a driver knows the number of spaces still open in the parking lot at time t. When a player making his arrival decision at time t observes all arrivals prior to t, this is called *full observability*; when he must decide without knowing if the parking lot is full, this is called *unobservability.*

For any time t, define N_t as the number of unarrived drivers and K_t as the number of unoccupied parking spaces; $N_t = N - K + K_t$.

At time t, the remaining drivers simultaneously decide whether to arrive at the parking lot at that instant. Under full observability, the decision can be conditioned on the number of parking spaces still available, K_t.

If player i arrives at t he obtains a spot with probability $p_{i,t} = \min\{K_t/n_t, 1\}$, where $n_t \leq N_t$ denotes the number of players arriving at exactly time t.

If player i arrives at time t, given that $K_t \in \{0, ..., K\}$ parking spaces are unoccupied at t and $n_t \in \{0, ..., N_t\}$ players arrive at t, his payoff is

$$u_{i,t} = p_{i,t}v - L(t). \tag{1}$$

Let us define the *indifference arrival time*, t^*, as the time at which the arrival cost, $L(t^*)$, equals the prize value, v. It follows that

$$t^* \equiv T - v/w. \tag{2}$$

A player who arrives at t^* and finds a parking space receives a payoff of zero since the disutility of the early arrival equals the value of the parking space. When looking for equilibria, we can restrict our attention to $t \geq t^*$ since arriving before t^* yields a negative payoff and is strictly dominated by arriving at T.

Define t_p^* as the time at which a player receives a zero payoff from participating in a lottery with probability p of winning a parking space; t_p^* is determined from equation $pv - L(t_p^*) = 0$. It follows that

$$t_p^* \equiv T - pv/w. \tag{3}$$

Finally, let $\underline{t}$ and $\bar{t}$ denote the earliest and the latest time any player arrives with a positive probability along the equilibrium path.

Definition 1. *A time grid is fine if*

$$\Delta < \frac{v}{Nw}. \tag{4}$$

When a time grid is fine, there are more than $N \geq K + 1$ periods between t^* and T since $T - t^* = v/w > N\Delta$. No more than K players can profitably enter at time $t = t^* + \Delta$ when the time grid is fine. Otherwise, even at the highest odds of obtaining a space, $K/(K + 1)$, the payoff of a player is negative at $t = t^* + \Delta$ for a fine time grid.

When a player is not guaranteed to obtain a parking spot, the player would choose to arrive one period earlier to secure a parking space, because for a fine time grid the cost of one-period earlier arrival, $w\Delta$, is justified by the gain associated with the higher probability of obtaining parking.

In the first-best outcome, all N players arrive at T, and K of them get to park in the parking lot.

This cannot be part of an equilibrium strategy profile when the time grid is fine, since each player would prefer to deviate by arriving just before T and increase his probability of finding a spot to 100%.

The Simplest Case: Two Drivers

Claim 1. *Consider the parking game on a fine time grid under full observability when there are two drivers and one parking spot (*$N = 2$ *and* $K = 1$*). The following arrival strategies constitute a subgame-perfect pure-strategy equilibrium: contingent on the parking lot not being full at time* t*, (i) at* $t = t^* + k\Delta \in [t^*, t^*_{1/2})$*, player* 1 *arrives if* $k = 0, 2, 4, ...$ *and player* 2 *arrives if* $k = 1, 3, 5, ...$*; (ii) at* $t \in [t^*_{1/2}, T)$*, both players arrive.*

When Claim 1's strategies are played out, player 1 arrives at t^*, player 2 arrives at T, and both players obtain zero payoffs. Thus, the parking lot yields zero social benefit; and if it is costly to construct or the land has other uses, the lot has a negative social value.

The General Case: N Drivers

Denote by $\widehat{n}_t$ the critical number of players for whom entry yields a nonnegative payoff at time t, given that K_t spaces remain unclaimed by time t.

Claim 2. *Consider the parking game on a fine time grid under full observability and with more drivers than parking spaces ($N > K$).*

A. *In any pure-strategy equilibrium to the game, the parking lot is full after $t^* + \Delta$. The equilibrium outcome is for $K_0 \in \{0, ..., K\}$ players to arrive at t^*; $K - K_0$ players to arrive at $t^* + \Delta$, and $N - K$ players to arrive at T.*

B. *All the pure-strategy equilibria have the following arrival schedule:*

(i) at $t = t^*$, $K_0 \in \{0, ..., K_t\}$ *players arrive;*

(ii) at $t = t^* + \Delta$, K_t *players arrive; if player $i \in I$ deviated by not arriving at t^*, the set of arriving players excludes player i;*

(iii) at $t \in [t^* + 2\Delta, t^*_{1/2})$, *if player $i \in I$ deviated by not arriving at $t - \Delta$, one other player arrives;*

(iv) at $t \in [t^*_{1/2}, T)$, *if player $i \in I$ deviated by not arriving at $t - \Delta$, then* $\min\{\widehat{n}_t, N_t\} \geq 2$ *players arrive at t*

(v) at $t = T$ *all players arrive who have not yet arrived.*

There are many sizes of this initial shrinkage that support equilibria.

1. For example, there is an equilibrium outcome in which one group of K players arrive at time t^* and they park in the lot, while a second group of $N - K$ players arrive at T and park elsewhere. Both groups receive the same payoff of zero.

2. Another polar equilibrium outcome is for no drivers to arrive at t^* and for K players to arrive at $t^* + \Delta$. In all pure-strategy equilibria, the parking lot fills up no later than $t^* + \Delta$ and no players arrive between $t^* + \Delta$ and T.

Corollary to Claim 2. *As the time grid becomes infinitely fine, drivers dissipate all the rents from parking in any pure-strategy equilibrium of the parking game under full observability when there are more drivers than parking spaces.*

Nonexistence of Pure-Strategy Equilibria under Unobservability

Claim 2 said that under full observability some players arrive at $t \leq t^* + \Delta$ and others arrive at T, both having nearly zero expected payoffs.

Under unobservability, Claim 2 does not apply because one of the players who is supposed to arrive at $t \leq t^* + \Delta$ could deviate and arrive at $T - \Delta$ instead, reducing his early-arrival cost.

The other players would not observe that he had failed to arrive as scheduled at t, so they would be unable to respond by taking "his" spot before $T - \Delta$.

Claim 3. *There does not exist a pure-strategy Nash equilibrium under unobservability when the time grid is fine and there are more drivers than parking spaces.*

Proof. Denote by t' the time when the parking lot becomes full in a pure-strategy equilibrium. If at t' not all arriving drivers obtain parking, then each of them would find it profitable to deviate by arriving a period earlier.

If parking is guaranteed at $t^{'}$, then there are drivers who arrive at T, because arriving between t' and T yields a negative payoff. Each of these drivers benefits from arriving shortly before t' unless $t' \leq t^* + \Delta$.

A fine time grid implies that only K drivers would arrive at $t' = t^*$ or $t' = t^* + \Delta$. Then, each of them can arrive later and still obtain the parking space. Hence, no pure- strategy equilibrium exists in this case either. Q.E.D.

This is an all-pay auction with multiple prizes. The arrival times are like bids, and the empty parking spaces are like prizes. The reason for the nonexistence of a pure-strategy Nash equilibrium under unobservability when the time grid is fine is essentially the same as in a multi-prize all-pay auction with continuous strategy space (see Barut and Kovenock, 1998, Clark and Riis, 1998, or the general characterization by Baye et al. [1996]).

Bulow-Klemperer (1999, AER) is different. It is about a war of attrition by N players with K prizes. They show that K+1 players will mix and the rest give up immediately. But in a pre-emption game like this one, "doing nothing" means staying in, and not incurring costs.

Mixed-strategy equilibria under unobservability are complicated to describe fully. In the two-driver game there is, for example, an alternating equilibrium in which for $k \in \{0, 1, 2...\}$ and $t \leq T$ player 1 arrives at $t = t_0 + 2k\Delta$ with probability $(v/(2w\Delta) - k)^{-1}$ and player 2 arrives at $t = t_0 + (2k+1)\Delta$ with the same probability.

In the case of $w = 1$, $v = 10$, $T = 10$, and $\Delta = 1$, the probabilities of player 1 arriving at $t = 1, 3, 5, 7, 9$ and of player 2 arriving at $t = 2, 4, 6, 8, 10$ are 1/5, 1/4, 1/3, 1/2, and 1. After $t = T - \Delta = 9$ the parking lot is full with probability one.

1 Full Rent Dissipation

We first establish that in any equilibrium, players arriving at $\underline{t}$ and $\bar{t}$ (the earliest and the latest times any player arrives with a positive probability along the equilibrium path) obtain nearly zero payoffs whenever there are more drivers than parking spaces. Then, we will show that all the players in between must almost fully dissipate the value from parking. Our propositions will apply under either full observability or unobservability.

Claim 4. *Consider the parking game on a fine time grid with more drivers than parking spaces. In any subgame-perfect equilibrium, the probability that the parking lot is full at time* $\bar{t} \leq T$ *is one under full observability and approaches one as the time grid becomes infinitely fine under unobservability. The equilibrium payoffs of players who arrive at* $\bar{t}$ *with a positive probability are zero under full observability and tend to zero under unobservability as the time grid becomes infinitely fine. Any player who arrives at* $\underline{t}$ *has a payoff that tends to zero as the time grid becomes infinitely fine.*

Corollary. *The parking lot is almost surely full by time* T.

Proposition 1. *If there are more drivers than parking spots ($N > K$), then under either full observability or unobservability, players fully dissipate rents from the parking lot in any equilibrium as the time grid becomes infinitely fine.*

Proof of Proposition 1. The proof is by contradiction. Suppose player i is earning a positive payoff bounded away from zero. Let $t \in (\underline{t}, \bar{t})$ be the earliest moment the player arrives with a positive probability; $t > \underline{t}$. Consider a player (player j) who arrives at $\bar{t}$ (with pure strategy) and earns an almost zero payoff (by Claim 4). If player j follows his equilibrium strategy until $t - \Delta$ and arrives with probability one at $t - \Delta$, he would obtain a positive payoff bounded away from zero. The costs of arriving at t and $t - \Delta$ differ by Δw, a difference that becomes negligible as $\Delta \to 0$. It suffices to prove that player j's odds of obtaining parking at $t - \Delta$ are no worse than player i's odds of obtaining parking at t. Note that by the definition of time t, player i has not attempted to arrive before t. Player j may have been mixing before t. Therefore, player j has the expected payoff from arriving at $t - \Delta$ that is no less than that of player i. Since there is a profitable deviation, there cannot exist an equilibrium with a player obtaining a positive payoff at a time t between $\underline{t}$ and $\bar{t}$. Hence, all players earn almost zero payoffs. Q.E.D.

Welfare and Parking Lot Size

The drivers have a value $v > 0$ for each of the parking spaces in use. Let us denote the cost of providing K parking spaces by an increasing function $C(K)$. The welfare from a parking lot of size K is

$$W(K) = \begin{cases} vN - E_{\sigma(N,K)}\left(\sum_{i=1}^{N} L(t_i)\right) - C(K) & \text{if } K \geq N \\ vK - E_{\sigma(N,K)}\left(\sum_{i=1}^{N} L(t_i)\right) - C(K) & \text{if } K < N, \end{cases} \tag{6}$$

where $E_{\sigma(N,K)}\left(\sum_{i=1}^{N} L(t_i)\right)$ denotes the expected equilibrium cost to drivers of arriving early for given K and N. Welfare can be decomposed into three parts: the *total value* of the parking lot (which is either vN or vK, depending on whether $K \geq N$ or not), the *rent-seeking loss*, and the *construction cost* $C(K)$.

The rent-seeking loss from earlier arrivals equals zero when $K \geq N$ and is vK when $K < N$. Hence, welfare is

$$W(K) = \begin{cases} vN - C(K) & \text{if } K \geq N \\ \\ -C(K) & \text{if } K < N \end{cases}. \tag{7}$$

Proposition 2. *The optimal size of the parking lot under certainty equals the number of users:* $K^* = N$. *All smaller sizes have negative welfare, with the minimum welfare being at* $K = N - 1$. *All greater sizes have the same total value from parking equal to* vN*, but increasingly high construction costs. This is true under both full observability and unobservability.*

Proof of Proposition 2. If $K < N$, then by Proposition 1, in any equilibrium almost all rents are dissipated and $W(K) = -C(K)$. Thus, the minimum welfare for $K \in \{0, 1, 2, ..., N-1\}$ is at $K = N - 1$. If $K \geq N$, each player is guaranteed a parking space and arrives at the preferred time T; therefore $W(K) = vN - C(K)$ and welfare falls gradually in K. It follows that welfare is maximized at $K = N$ as long as it is at all socially beneficial to build a parking lot, i.e. when $vN \geq C(K)$. Q.E.D.

The Optimal Parking Lot Size under Uncertainty

Assume that planners must decide on the size of the parking lot before they know the number of drivers, N.

When there is competition for parking spots, $K < N$, the welfare from the parking lot is negative, $W = -C(K)$. When the size of the lot is large enough, $K \geq N$, the welfare is $W = vN - C(K)$. Therefore, for $K \leq \overline{N}$, the expected welfare is

$$EW(K) = \sum_{N=0}^{K} (vN) f(N) - C(K) = vE(N \,|N \leq K) - C(K). \tag{8}$$

The size of the parking lot should be increased from $K-1$ to K as long as the change in the expected welfare is nonnegative: $EW(K) - EW(K-1) \geq 0$. Thus, the optimal K is the biggest K such that $K \leq \bar{N}$ and

$$vKf(K) - (C(K) - C(K-1)) \geq 0. \tag{9}$$

$$vKf(K) - (C(K) - C(K-1)) \geq 0. \tag{9}$$

Drivers benefit from the Kth parking space only if $N = K$ exactly.

If $N < K$ the marginal space is unused, and if $N > K$ the benefit is dissipated by rent-seeking so it does not matter whether there are K or $K - 1$ spaces.

Since the Kth space matters only if $N = K$, the change in the expected welfare is the probability that there are exactly K drivers multiplied by the benefit from eliminating rent-seeking behavior, vK, net of the change in construction costs.

On the one hand, at larger parking lot sizes, it is more important to have a sufficiently big parking lot because there are more people who could get benefit from it. On the other hand, it could be less likely that larger parking lots are filled up. When the first effect dominates and $Kf(K)$ increases in K, the marginal benefit increases with K too. For a constant-marginal-cost technology, $C(K) = cK$, this implies a corner solution to the problem: the parking lot should accommodate all potential drivers, even if that is much greater than the expectation of the number of drivers, or not be built at all.

Example

Consider a discrete uniform distribution for the number of drivers on the support $\{0, ..., \overline{N}\}$. For the uniform distribution, $Kf(K)$ does increase in K, so we have a corner solution. Under the uniform distribution, it is equally likely at any capacity that the parking demand will be barely met. At a larger capacity benefits accrue to more people, so at larger capacity levels, the benefit from building an additional space is higher, and planners should design the parking lot for the "peak demand," accommodating all potential drivers.

Designing for peak demand is not always the optimal choice. For example, in a case of the binomial distribution of N, if each of 100 drivers is in need of parking with probability 0.5 and $c/v = 1/5$, the optimal parking size is $K^* = 58$. Only about 8 out of 58 spaces are empty, on average. This corresponds to about 86% utilization level.

When the number of drivers is large, calculus can be used. Consider the production technology with a constant marginal cost, $C(K) = cK$ where $c < v$. The expected welfare from a parking lot of size K is then

$$EW(K) = v \int_0^K N f(N) dN - cK. \tag{10}$$

The optimal size of the parking lot is the solution to the first-order condition

$$\frac{\partial EW(K)}{\partial K} = vKf(K) - c = 0, \tag{11}$$

which can be written as

$$Kf(K) = \frac{c}{v}. \tag{11'}$$

We can use the first-order condition to find the optimal level of K as an internal solution if the maximand is concave. Expansion of the parking lot improves welfare if the probability that exactly K drivers compete for K parking spaces times the size of the parking lot exceeds the relative cost of building a parking space.

Unfortunately, having a concave problem seems unlikely. The second- order condition, $\partial^2 EW(K)/\partial K^2 < 0$, can be written as $Kf'(K) + f(K) < 0$, or $Kf'/f < -1$. To guarantee the existence of the interior solution, the density must be declining and elastic in the relevant range of K. Otherwise, a parking lot should be built for the peak demand (the highest N possible).

For example, consider a family of continuous density functions $f(N) = \alpha N^{-\beta}$ defined on the support $[\underline{N}, \overline{N}]$, where $\alpha, \beta > 0$, and $[\underline{N}, \overline{N}] \subset [0, \infty)$ are such that $\int_{\underline{N}}^{\overline{N}} \alpha N^{-\beta} dN = 1$. The first-order condition implies the optimal capacity $K^* = \left(\frac{1}{\alpha}\frac{c}{v}\right)^{-1/(\beta-1)}$, and the second-order condition is satisfied if and only if $\beta > 1$.

For $\beta \leq 1$, the parking lot should have $\overline{N}$ spaces, if it is built. The mean utilization of the parking lot of optimal size can be measured as a ratio of the mean number of parking spots taken, $E(X)$, to the optimal capacity size, K^*. If $N < K$, then all N drivers find parking; if $N \geq K$, then K out of N drivers find parking. Hence, $E(X) = \int_{\underline{N}}^{K^*} Nf(N)dN + \int_{K^*}^{\overline{N}} K^* f(N)dN$. For example, 50% of the parking spots will remain unoccupied on average when $c/v = 0.5$, $\beta = 1.5$, $K^* = 4$, and $E(X) = 2$, and the 50%-utilized parking lot is socially optimal.

Heterogeneous Drivers

The effects of heterogeneity in parking value, v, or early arrival cost, w, are the same.

Suppose drivers have heterogenous values for parking. Let N be the number of drivers with positive values and let v_i denote the value of i'th-highest-valuing driver, so $v_1 \geq v_2 \geq ... \geq v_N > 0$.

For example, in the case of linear demand, $v_i = a - bi$ for some constants a and b.

Assume that there are also drivers who have zero value for parking and hence arrive at their preferred time, T.

As before, the cost of constructing a parking space is c, and the cost per unit time of early arrival is w.

In the first-best case, both the parking lot capacity and access to parking can be regulated. The first-best policy is to choose capacity to accommodate all players with values higher (or weakly higher) than $v_i = c$ and to give only them access to the parking lot.

When access to parking cannot be restricted, planners should design a larger parking lot.

In the second- best world, the optimally designed parking lot provides parking even to drivers who value it less than the construction cost. In many cases, including that of linear demand for parking, the second-best policy is to choose $K = N$ and guarantee parking to all players with positive values.

If $K < N$, the equilibrium outcome to the parking game on an infinitely fine time grid dictates that the K players with the highest values arrive at $t^*_{K+1} \equiv T - v_{K+1}/w$ and find a space, and players with values v_{K+1} or less arrive at T.

The player with value v_{K+1} will not deviate because his benefit from arriving at t^*_{K+1} would be equal to his cost.

The strategies that sustain this equilibrium outcome are similar to those given in Claim 2, and arguments similar to those given in Proposition 1 imply that the rent-seeking losses are equal to $K \cdot v_{K+1}$, because the K players with the highest values must incur disutility v_{K+1} of arriving at t^*_{K+1} to deter players with value v_{K+1} or less from arriving early. There is always partial rent dissipation when the parking capacity cannot accommodate all drivers with positive value for parking, that is, when $v_{K+1} > 0$.

The welfare from a parking lot of size $K \geq N$ is equal to the total value minus construction costs. For any $K < N$, the welfare also includes rent-seeking losses. Thus, welfare is

$$W(K) = \begin{cases} \sum_{i=1}^{N} v_i - cK & \text{if } K \geq N \\ \\ \sum_{i=1}^{K} v_i - K \cdot v_{K+1} - cK & \text{if } K < N, \end{cases} \tag{12}$$

and the change in welfare from adding an extra parking spot for $K \leq N$ is

$$W(K) - W(K-1) = K\left(v_K - v_{K+1}\right) - c. \tag{13}$$

Expansion of capacity is warranted whenever $v_K - v_{K+1} \geq c/K$, or the value from parking does not decline at K faster than c/K. For linear demand, there is a corner solution: a planner designs the parking lot to accommodate all N players with positive values, as long as it is worthwhile to build the parking lot at all, that is, as long as $W(N) = \sum_{i=1}^{N} v_i - cN \geq 0$.

When K and N are large we could treat them as continuous variables and use calculus. Using $v = v(x)$ to denote the demand for parking, welfare can be written as

$$W(K) = \begin{cases} \int_0^N v(x)dx - cK & \text{if } \ K \geq N \\ \\ \int_0^K v(x)dx - K \cdot v(K) - cK & \text{if } \ K < N. \end{cases} \tag{14}$$

Taking the derivative, we obtain $\partial W(K)/\partial K = -K \cdot v'(K) - c$ for $K < N$. An interior solution exists when v is sufficiently convex to ensure that $v'(K) + K \cdot v''(K) > 0$. In other words, allowing all drivers with positive values to park is not optimal when $K \cdot v''(K)/\left(-v'(K)\right) > 1$, that is, when the slope of the inverse demand is elastic.

Another way to look at the problem is that if capacity is chosen at the first-best optimum of $K = 45$ there will be a discontinuous loss from allowing more than $N = 45$ players access to the parking lot. Consider the problem of choosing the number of parking permits, n, for a lot of a given size, K, which arises when capacity is chosen in the long run while permits are allocated in the short run. Assume that permits are rationed efficiently to the highest-value players. Clearly, the optimal number of permits cannot be less than capacity. For any $n > K$, the welfare is $K \cdot v_{K+1}$ lower than at $n = K$. Hence, the welfare loss from having too few or too many parking permits is not symmetric. Welfare jumps down by $K \cdot v_{K+1}$ when one too many parking permits are issued, but welfare falls gradually when too few drivers are served.

At the first-best capacity level $K = 45$, welfare drops due to rent-seeking loss by $45 \cdot 5 = 380$ when 46 permits (or more) are allocated. In contrast, welfare drops by 1.2 when $n = 44$ and one driver with value 1.2 is unserved. There is a discontinuity in welfare at the optimal number of permits, $n = K$; oversupply of permits is more dangerous than undersupply.

If there were no rentseeking, the expected welfare would be composed of the welfare from N places being used if $N \leq K$ plus K spaces being used if $N > K$:

$$EW(K) = \sum_{N=0}^{K} (vN)f(N) + \sum_{N=K+1}^{\overline{N}} (vK)f(N) - C(K). \quad (1)$$

The size of the parking lot should be increased from $K-1$ to K as long as

$$vKf(K) + \sum_{N=K+1}^{\overline{N}} (vK)f(N) - \sum_{N=K}^{\overline{N}} (v(K-1))f(N) - (C(K) - C(K - \quad (2)$$

This is equivalent to:

$$vKf(K) + \sum_{N=K+1}^{\overline{N}} vf(N) - f(K)(K-1)v - (C(K) - C(K-1)) \geq 0 \quad (3)$$

or

$$\sum_{N=K}^{\overline{N}} vf(N) - (C(K) - C(K-1)) \geq 0. \quad (4)$$

In words, for the extra space to be worthwhile, the probability the extra parking space gets used times its use value, v, must be as great as its marginal cost. In the case of the uniform distribution example, this leads to a parking lot of size $K = 80$, rather than the $K = 100$ we found as the result of rentseeking.

Whether the optimal parking lot size is greater with rentseeking than without is unclear to me now. I think it depends on the $f(N)$ function. The optimality condition for the no-rentseeking case gives a smaller marginal benefit from increasing K than for the rentseeking case if, looking at the middle terms of the intermediate step in the equation above for the no-rentseeking case:

$$vKf(K) + \sum_{N=K+1}^{\overline{N}} vf(N) - f(K)(K-1)v - (C(K) - C(K-1)) \geq 0 \tag{5}$$

versus for the rentseeking case:

$$vKf(K) - (C(K) - C(K-1)) \geq 0 \tag{6}$$

The difference is:

$$\sum_{N=K+1}^{\overline{N}} vf(N) - f(K)(K-1)v < 0. \tag{7}$$

If $f(N)$ is smaller for bigger N beyond K^*, this condition will be true.

Another case to consider is what happens if the drivers do not know the number of drivers at the beginning of the day, but they do know the distribution. That case is both better and worse than the case we just analyzed, in which the number of drivers is fixed after the parking lot size. It is better because drivers will not know that showing up early is necessary on a particular day, so they will moderate their rentseeking. It is worse because drivers will pursue the same policy every day, so they will pursue some rentseeking even when it is unnecessary.

Drivers will pursue mixed strategies, since this is a version of the unobservable arrival game. Consider the payoff from showing up at exactly time T. With probability $\sum_{N=0}^{K} f(N)$ the parking lot does not fill up. Thus, the payoff to one driver must be $\sum_{N=0}^{K} f(N)$ if the support of the mixing distribution includes arrival at T. This must be multiplied by the expected number of players, which is $\sum_{N=0}^{\bar{N}} Nf(N)$, which yields

$$\begin{aligned} EW(K) &= (\textstyle\sum_{N=0}^{\bar{N}} Nf(N))(\sum_{N=0}^{K} f(N)) - C(K) \\ &= (E(N))(\textstyle\sum_{N=0}^{K} f(N)) - C(K) \end{aligned} \tag{8}$$

The optimality condition is then

$$E(N)f(K)v - (C(K) - C(K-1)) \geq 0. \tag{9}$$

Compare that with the optimality condition when drivers know N:

$$Kf(K)v - (C(K) - C(K-1)) \geq 0. \tag{10}$$

If $K^* > E(N)$, then optimal capacity is smaller when drivers do not know N.