The Relation between Quasiconvexity and Convexity

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Abstract

What is the relation between concavity and quasi-concavity? We show that if and only if function $f(x)$ is *strictly* quasi- concave except possibly for a flat interval at its maximum, there exists a strictly monotonically increasing function $g(x)$ such that $g(f(x))$ is concave.

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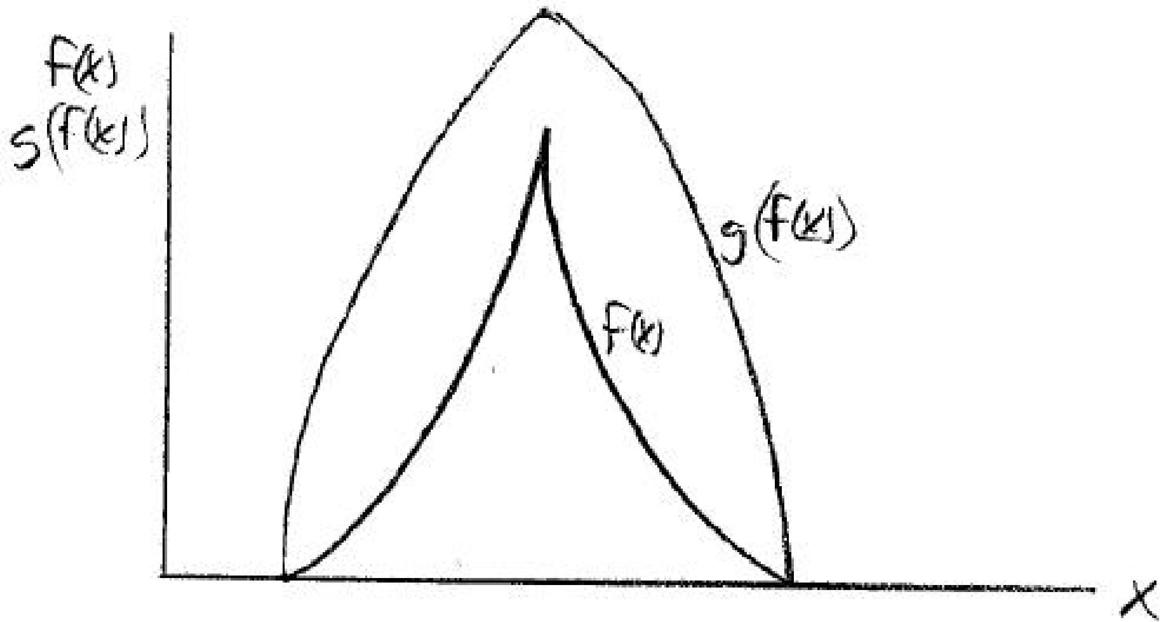

Figure 1: A Quasi-Concave Function $f(x)$ That Is Not Concave

Discontinuous Functions

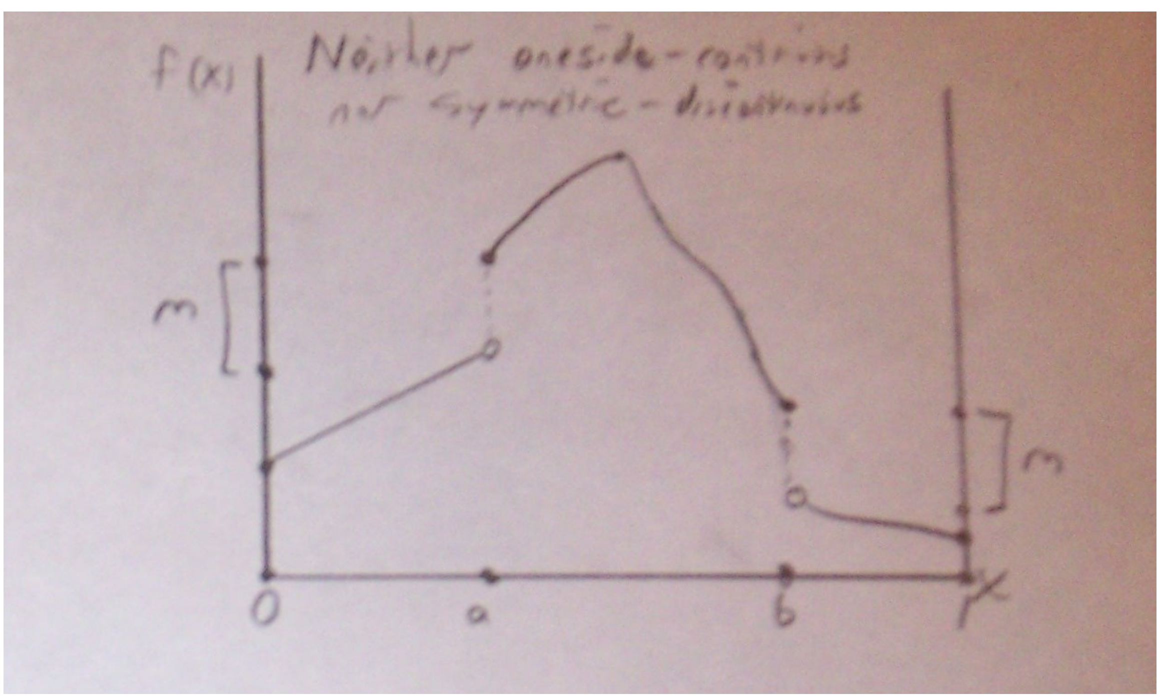

Figure Q2: BAD

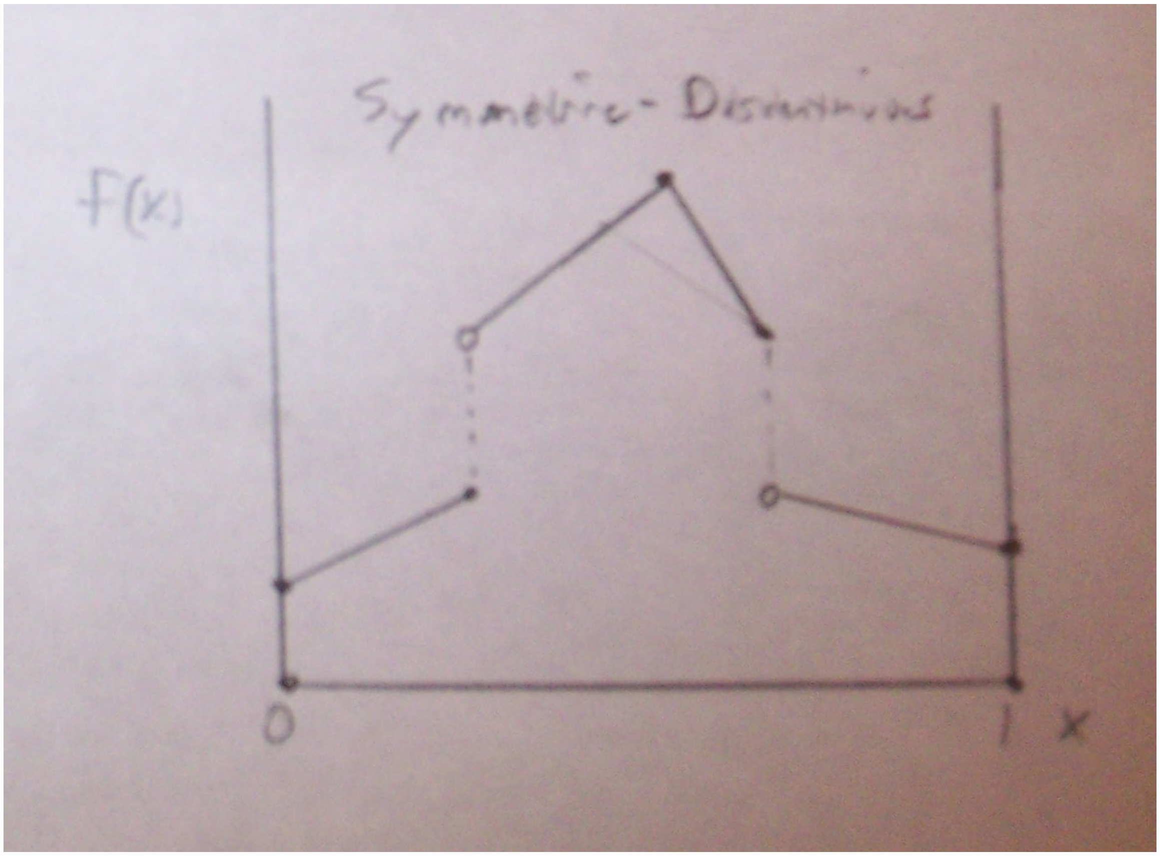

Figure Q3: OKAY

We can map the function in Q3 to a concave function using a monotonic g, but not the function in Q2.

Weakly Quasi-Concave Functions

(1) A weakly quasi-concave function f cannot be transformed ot a concave function by a monotonically increasing g.

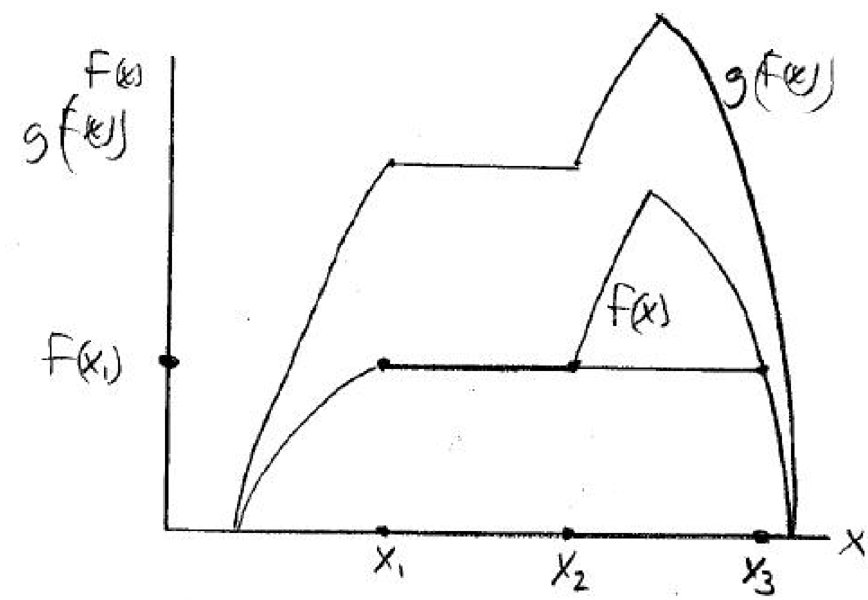

Figure 3: A Weakly Quasi-Concave Function

(2) If function $f(x)$ is weakly quasi-concave, there exists a "monotonizing function" $h(x)$ and a monotonically increasing function $g(x)$ such that $g(h(f(x)))$ is quasi-concave.