Price Discrimination between Retailers with and without Market Power

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Abstract

This paper studies the pricing strategies and ontract choice of an upstream manufacturer who sells to both competitive and monopolistic retailers in different locations. To prevent arbitrage, he sets the wholesale price difference between locations smaller than the transportation cost. This magnifies the problem of double marginalization. When some retail markets become more competitive, the manufacturer reacts in monopolized markets by (i) charging a higher wholesale price, (ii) using linear contracts instead of two-part tariffs, and (iii) closing down markets. Banning price discrimination can cause the manufacturer to switch to use an inferior type of contracts and to close markets, reducing welfare.

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1 Introduction

We study retailer arbitrage and a seller's incentive to price discriminate because of market structure differences. In our model, a manufacturer sells to retailers in towns, where retailers are monopolies, and cities, where they are competitive. To pre-empt arbitrage, he sets the wholesale price difference between locations smaller than the transportation cost, a constraint on his profit maximization problem. This magnifies the problem of double marginalization in towns. Even with two-part tariffs, the manufacturer cannot capture the vertically integrated profit.

We will show that as competition in city retail markets becomes more intense, city consumers are helped but town consumers are hurt. The manufacturer tends to change his behavior in towns by (i) charging a higher wholesale price, (ii) using linear contrac s instead of two-part tariffs, and (iii) closing down markets. Banning price discrimination causes the manufacturer to switch inferior contractse and to close down markets.

Price discrimination, market structure and vertical externalities are important areas in industrial organization that are usually studied separately.

Existing theory explains third-degree price discrimination using cost and demand variables. Westudies a third explanation – market structure differences between markets. Differences in cost and demand can lead to opposing welfare effects of price discrimination (see Villas-Boas 2009). How about market structure differences?

Concern is increasing about about the parallel trade of prescription drugs in the European Union. The cost of developing a successful new drug was over \$500 million in 1995 and as high as \$800 million in 2002 (Grabowski et al. 2002; Schweitzer 2007, p. 29). Pharmaceutical R&D is a global joint cost of serving consumers worldwide. This cost is covered by monopoly profit and price discrimination from patents. When a drug comes to the market, the R&D costs are sunk. This provides an opportunity for countries to free ride on the R&D costs by setting price regulation (see Danzon 1998). Due to different competitive conditions in each country in the EU and different bargaining power of purchasing authorities, drug manufacturers price discriminate and retailers have incentive to import drugs from low- price countries to high-price ones.

Parallel trade brings down drug prices in some countries (Ganslandt and Maskus 2004). Nevertheless, it reduces the manufacturer's ability to price discriminate and hence R&D incentive. Hausman and MacKie-Mason (1988) point out that even ignoring the long run effects on incentives for innovation, price discrimination can raise social welfare because it encourages patent holders to open new markets. Our model has a result of a similar flavor. Banning price discrimination can cause the manufacturer to close down town markets. The contrary, however, is a bit more complicated. If the manufacturer does not initially sell to towns, the allowance of price discrimination does not cause the manufacturer to open up town markets. To induce the manufacturer to sell to towns, the government needs to allow both price discrimination and exclusive territories. Moreover, price discrimination can affect the manufacturer's choice of contracts, also affecting welfare.

A second purpose of our paper is to study retailer arbitrage. Two-part tariffs enlarge the wholesale price difference between locations and provide additional incentive for retailers to arbitrage. Thus, arbitrage comes to play a role in the manufacturer's choice of contracts. With linear contracts the problem of double marginalization in towns is magnified. Even with two-part tariffs, the manufacturer cannot realize the vertically integrated profit. The result helps to explain why grocery prices are often more expensive in rural areas (Chung and Myers, 1999; Kaufman, 1998; Kaufman et. al., 1997), even though rural households are poorer.

2 Literature review

Our paper is related to the literatures on price discrimination, vertical integration and parallel trade.

Pigou (1920) and Robinson (1933) conjecture that a necessary condition for social welfare to increase under third-degree price discrimination is that total output increase. Schmalensee (1981), Varian (1985) and Schwartz (1990) prove the conjecture under various cost and demand assumptions.

Hausman and MacKie-Mason (1988) point out that these authors' assumption that outputs are positive in all markets under both price discrimination and uniform pricing does not always hold, something that will be important in the present model.

Katz (1987) opened a discussion of the welfare effect of price discrimination for an intermediate good. Intermediate and final goods' price discrimination have different effects. First, downstream firms' demands and profits are interdependent. Second, wholesale price discrimination often affects downstream buyers' nonprice decisions, which in turn, affect social welfare. Such nonprice decisions include choice of technology (DeGraba 1990; Yoshida 2000), whether to integrate backward to the supply of the intermediate good (Katz 1987), and whether to search for a lower-cost supplier (Inderst and Valletti 2009a, b). These decisions complicate the welfare analysis. Our paper studies another nonprice decisions – retailer arbitrage. Since arbitrage activities involve transportation that leads to losses, price discrimination tends to reduce social welfare.

Spengler (1950) first identified the problem of double marginalization. Cook (1955) and Hirshleifer (1956) argue that the two-part tariffs can eliminate the problem . Making a take-it-or-leave-it offer, the upstream manufacturer can gain the vertically integrated profit (see Tirole 1988). We show that if the arbitrage constraint is binding, the problem of double marginalization is magnified. Even with two-part tariffs, the manufacturer cannot capture the vertically integrated profit.

There is some overlap between the literature of third-degree price discrimination and parallel trade. Parallel trade is a kind of arbitrage. Perfect arbitrage forces the upstream seller to price uniformly. Hausman and MacKie-Mason (1988) show that if price discrimination leads to opening new markets, the welfare change can yield Pareto improvement. Malueg and Schwartz (1994) extend Hausman and MacKie-Mason's work and study the role of demand dispersion in opening new markets. They find that if demand dispersion is large, many markets go unserved under uniform pricing.

Ganslandt and Maskus (2004) provide a model of parallel trade. In their model, a drug manufacturer sells to both home and foreign markets. The government in the foreign country sets a cap on the drug price. In the first stage of the game, parallel importing firms make entry decisions. In the second, they each choose a quantity of parallel import. In the third, the manufacturer chooses a price in the home country to maximize profit. They find that the drug price in home market falls in the number of parallel importing firms. Moreover, the number of parallel importing firms falls in the foreign price and in the trade cost of parallel imports. In our model, by contrast, the move sequence of the players is different, demands are non-linear, there is no price cap set by governments, and retailers use Bertrand equilibrium for the intermediate good.

3 The Model

A monopolist manufacturer supplies a homogeneous product to retailers in n_a identical small towns and n_b identical large cities. Consumers must buy in their own city or town, but retailers can buy anywhere. Because there is only one retailer in each town, town retailers are monopolists with respect to consumers. City retailers, however, compete intensely using Bertrand pricing, choosing prices simultaneously, and earn zero profits. If retailers sell to each other, they compete using Bertrand pricing.

Denote wholesale prices by w_a for towns and w_b for cities, franchise fees by F_a and F_b, retail prices by p_a and p_b, and consumer demands by $D_a(p_a)$ and $D_b(p_b)$. We assume downward sloping and concave demands, with $D_a''' \geq 0$. Because demand is concave, there exist unique retail monopoly prices that maximize vertically integrated profits, which we will denote p_a^m and p_b^m . Denote the wholesale price and the retail price in towns under standard double marginalization by $\widetilde{w}_a$ and $\widetilde{p}_a$.

Let the manufacturer have constant marginal cost $c > 0$ and let retailers have zero distribution costs. If resale occurs between towns and cities, retailers incur a per-unit arbitrage cost (the "transportation cost") $t > 0$.

We assume that demand is stronger in cities than in towns— enough stronger that if arbitrage were impossible, the wholesale prices would be higher in the cities, i.e., $p_b^m > \widetilde{w}_a$ (since the manufacturer would set $w_a = \widetilde{w}_a$, with linear contracts in towns, and $w_b = p_b^m$). This assumption simplifies some technical difficulties in this paper and will be used in Proposition 1. If this assumption is relaxed, the main results of the paper are unaffected.

The manufacturer can choose from two types of contracts, linear pricing or a two-part tariff. Let I_a, I_b be indicator variables, where $I = 1$ if the manufacturer chooses a two-part tariff. Under linear pricing, the retailer pays amount w_a, w_b per unit for as many units as he wishes to buy. Under a two-part tariff, the retailer pays the lump sum $F_a, F_b > 0$ if he buys any units at all, plus w_a, w_b per unit. Linear pricing has zero transactions cost, but the two-part tariff, being more complicated, incurs an extra transaction or negotiation cost of $g \geq 0$[1] ($g = 0$ is allowed, and is itself an interesting case). The manufacturer chooses the contracts first. Retailers react by choosing the quantities to purchase, so the manufacturer makes take-it-or-leave-it offers to the retailers.

We will denote a retailer's derived demand for the good as a function of the wholesale price by $D_a^r(w_a)$ or $D_b^r(w_b)$. Lemma 1 establishes that these demands are concave and strictly decreasing.

Lemma 1. $D_a^r(w_a)$ and $D_b^r(w_b)$, are strictly decreasing and concave in wholesale prices w_a and w_b .

Proof: In the Appendix.

4 The Equilibrium

We seek a subgame-perfect Nash equilibrium in which the manufacturer chooses pricing schemes for cities and for towns and what prices and franchise fees to charge, and the retailers choose purchase amounts and wholesale and retail sales.

[1] Another interpretation is that retailers have outside options so that the manufacturer splits the vertically integrated profit with the retailers and takes a share $F_i - g$ under two-part tariffs. With this interpretation, the value of g is exogenously determined by the outside options.

First, consider what would happen if arbitrage could be blocked by contract— that is, if a retailer could be deterred from reselling in a forbidden market by the threat of a suit for breach of contract.

The manufacturer would choose a linear contract for cities and set the wholesale price equal to the monopoly retail price: $w_b = p_b^m$. City retailers would jointly buy the quantity $D_b\left(p_b^m\right)$ and choose the retail price p_b^m to maximize their profits. The manufacturer would do worse with a two-part tariff, because the retailers, knowing that they would compete their profit margin to zero, would be unwilling to pay $F_b > 0$ to buy a positive quantity.

The manufacturer would choose a two-part tariff for towns, with a wholesale price equal to marginal cost, so $w_a = c$. A town retailer would buy the quantity $D_a\left(p_a^m\right)$ and choose the retail price p_a^m to maximize its profits, and the franchise fee F_a would equal the gross profits.

If the manufacturer used linear pricing in towns, his profits would fall because of double marginalization. Denote his optimal linear price in this no-resale case by $\widetilde{w}_a$, which would be greater than c if he is to obtain positive profits. As a result, when town retailers react with their optimal retail price, $\widetilde{p}_a$, it will exceed the optimal monopoly price, p_a^m. More precisely,

$$\widetilde{w}_a \equiv \overset{argmax}{w_a}\left\{(w_a - c) \cdot D_a\left(p_a^*(w_a)\right)\right\} \tag{1}$$

and:

$$\widetilde{p}_a \equiv p_a^*(\widetilde{w}_a) \tag{2}$$

where $p_a^*(w_a)$ is the reaction function of the town retailers and satisfies the first order condition of the town retailers' profit maximization.

Next, let us assume that contracts cannot punish intermarket arbitrage. Since each retailer sells both in towns and cities, if the wholesale price difference,

$$d \equiv (w_b - w_a),$$

is larger than the retailer transportation cost t, a retailer will arbitrage the product among its stores across locations, or one retailer will arbitrage by buying from another.

Let us define two critical values of transportation costs:

$$t_0 \equiv p_b^m - \widetilde{w}_a \text{ and } t_1 \equiv p_b^m - c$$

.

Recall that in separate maximization problems p_b^m is the monopoly wholesale price in cities, and $\widetilde{w}_a$ and c are the wholesale prices for linear pricing and two-part tariffs in towns. Thus, t_0 is the optimal wholesale price difference between towns and cities if the retailers do not arbitrage and the manufacturer chooses linear pricing for towns. Similarly, t_1 is the optimal wholesale price difference between towns and cities if the retailers do not arbitrage and the manufacturer chooses a two-part tariff for towns. The manufacturer's profit depends on what kind of arbitrage arises from its city-town pricing. The three possibilities are: (i) the retailers do not arbitrage, (ii) the retailers arbitrage goods from towns to cities, or (iii) the retailers arbitrage goods from cities to towns. Let us examine these possibilities in turn.

Case (i) No arbitrage. If the manufacturer chooses $|(w_b - w_a)| \leq t$, then the retailers do not arbitrage the goods. The manufacturer's profit is the sum of its profits from all towns and cities:

$$\begin{aligned} \underset{w_a, w_b}{Maximum} \; \pi|_{|(w_b - w_a)| \leq t} \; &= n_a \cdot [(w_a - c) \cdot D_a^r(w_a) \\ &\quad + I_a \cdot (F_a(w_a) - g)] \\ &\quad + n_b \cdot [(w_b - c) \cdot D_b^r(w_b)] \end{aligned} \tag{3}$$

The first square bracket in (3) is the manufacturer's profit from a town, and the second square bracket is his profit from a city. The first round brackets in each square bracket are the profit margins. I_a is the indicator variable whether the manufacturer adopts two-part tariff contracts for towns, and its gain from a two-part tariff contract from a town is the franchise fee, F_a, net of the negotiation fee, g.

Case (ii) Town retailers sell to city retailers. If the manufacturer chooses $(w_b - w_a) > t$, city retailers will buy in towns. We have assumed that town retailers compete against each other in prices for sales to other retailers, so the resulting price paid by the city retailers will be driven down to

w_a. To this the city retailers must add the transportation cost t to get their marginal cost for selling to consumers, $w_a + t$. Since the retailers compete in Bertrand equilibrium in cities, the market price in cities is also $w_a + t$, and correspondingly, the effective demand is $D_b^r(w_a + t)$. The manufacturer's maximization problem becomes:

$$\underset{w_a, w_b}{Max}\ \pi|_{(w_b - w_a) > t} \begin{aligned}[t] &= n_a \cdot [(w_a - c) \cdot D_a^r(w_a) \\ &\quad + I_a \cdot (F_a(w_a) - g)] \\ &\quad + n_b \cdot [(w_a - c) \cdot D_b^r(w_a + t)] \end{aligned} \tag{4}$$

Case (iii) City retailers sell to town retailers. If the manufacturer chooses $(w_b - w_a) < -t$, retailers will arbitrage goods from cities to towns. The effective wholesale price for towns will be w_b, to which t must be added to get the town retailers' marginal cost, and the manufacturer's profit function will be:

$$\underset{w_a, w_b}{Max}\ \pi|_{(w_b - w_a) < -t} \begin{aligned}[t] &= n_a \cdot [(w_b - c) \cdot D_a^r(w_b + t) \\ &\quad + I_a \cdot (F_a(w_b + t) - g)] \\ &\quad + n_b \cdot [(w_b - c) \cdot D_b^r(w_b)] \end{aligned} \tag{5}$$

Proposition 1 tells us that the manufacturer prefers Case (i): no arbitrage.

Proposition 1. For interior solution, the manufacturer will prevent arbitrage in equilibrium. Wholesale prices will be higher in cities, but by no more than the transportation cost, so $0 < (w_b - w_a) \leq t$.

Proof: See the Appendix.

If the transportation cost were smaller than the wholesale price difference, retailers would arbitrage. To prevent this, the manufacturer sets a smaller wholesale price difference. Alternatively, the manufacturer could allow arbitrage to occur, but Proposition 1 says he will not, regardless of the choice of contracts in towns. This is because transportation is costly, so the social loss from arbitrage that also hurts the manufacturer. City retailers are earning zero profits anyway, and their higher transportation costs would result in less demand from consumers and lower sales by the manufacturer. If the manufacturer could impose vertical control over the actions of downstream retailers, he could prevent arbitrage and realize the vertically integrated profit. But he cannot, so profit is less.

Recall that t_0 (under linear pricing) and t_1 (under two-part tariff contracts in towns) are the optimal wholesale price differences between towns and cities if there is no arbitrage. If t is smaller than these critical values, t_0 and t_1, the transportation cost is "small" and arbitrage starts to be attractive to retailers. To prevent arbitrage, the manufacturer must choose $(w_b - w_a) \leq t$. Under this constraint, the manufacturer's profit is increasing in the wholesale price difference. Thus, for t smaller than the critical transportation costs the manufacturer chooses $(w_b - w_a) = t$ and the arbitrage constraint is binding. For t larger than the critical values, the retailers do not arbitrage and therefore the manufacturer chooses the unconstrained optimum.

5 The Equilibrium Contracts

Proposition 1 says that the manufacturer will prevent retailer arbitrage. Thus, the manufacturer's piecewise maximization problem in (3) to (5) can be reduced to the following:

$$\begin{aligned} \overset{Max}{w_a, w_b}\ \pi \quad &= n_a \cdot [(w_a - c) \cdot D_a^r(w_a) + I_a \cdot (F_a(w_a) - g)] \\ &\quad + n_b \cdot [(w_b - c) \cdot D_b^r(w_b)] \text{ such that } 0 < (w_b - w_a) \leq t \end{aligned} \tag{6}$$

The Lagrangian function is:

$$\begin{aligned} L = n_a \cdot [(w_a - c) \cdot D_a^r(w_a) + I_a \cdot (F_a(w_a) - g)] \\ + n_b \cdot [(w_b - c) \cdot D_b^r(w_b)] + \lambda \cdot [t - (w_b - w_a)] \end{aligned} \tag{7}$$

To determine the manufacturer's choice of contracts for towns, we will compare the manufacturer's profits under linear pricing, $\pi^* (I_a = 0, t)$, and under two-part tariffs, $\pi^* (I_a = 1, g, t)$. Recall that $g \geq 0$ is the contract cost of the manufacturer if he adopts a two-part tariff. Let $\overline{g}(t)$, a function of transportation cost t, be the contract cost such that the manufacturer is indifferent between the two types of contracts, i.e., $\overline{g}(t) : \pi^* (I_a = 1, \overline{g}, t) \equiv \pi^* (I_a = 0, t)$.

Proposition 2.

(a) There exists a continuous function $\overline{g}(t) > 0$ such that if $g < \overline{g}(t)$ a two-part tariff is used.

(b) Suppose t is small enough for the possibility of arbitrage to constrain the manufacturer ($t < t_0$ if $g \geq \overline{g}(t)$ or $t < t_1$ if $g < \overline{g}(t)$, as in Figure 1's Regions I and II). In Figure 1's Region I, with linear pricing, the retail price in towns is higher than under standard double marginalization. In Region II, with two-part tariffs, the wholesale price for town retailers will exceed marginal cost and the retail price will be higher than the integrated monopoly price.

(c) $\overline{g}'(t) > 0$ for $t < t_1$, and $\overline{g}'(t) = 0$ otherwise.

Proof: See the Appendix.

At the beginning of last section, we showed that to the left of the critical values the transportation costs are "small", and therefore in Regions I and II the manufacturer chooses $(w_b - w_a)^* = t$ to prevent arbitrage. To achieve this target, the manufacturer can adjust the wholesale price for only towns, only cities, or both. Proposition 2 says that the manufacturer will adjust both. While decreasing the wholesale price for cities, the manufacturer increases the wholesale price for towns. Preventing arbitrage magnifies the double marginalization problem for towns. Even with a two-part tariff, the wholesale price will be above marginal cost and the

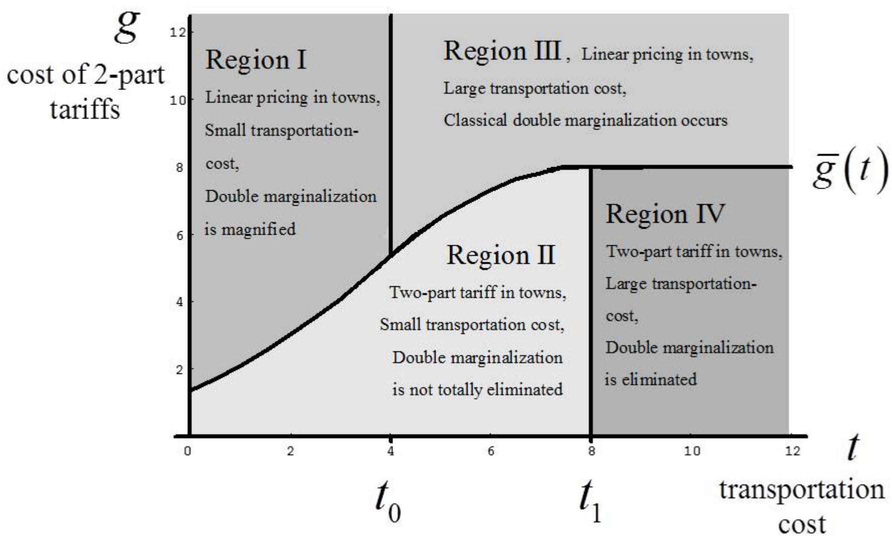

Figure 1(a): Double Marginalization in the Four Regions

magnified double marginalization cannot be totally eliminated.

We have not and will not assume linear demands in our model, but for illustration we use linear demands in Figure 1 to generate the $\overline{g}(t)$ function (the upward sloping curve in the middle of the figure). Figure 1 shows the manufacturer's choice of contracts and pricing decisions in equilibrium (with $n_a = 2$, $n_b = 1$, $c = 2$, $Q_a = 10 - p_a$, and $Q_b = 18 - p_b$).

In Figure 1(a), the curve $\overline{g}(t)$ and the two critical values of the transportation costs partition the $t - g$ space into four pieces, Regions I to IV. Above $\overline{g}(t)$, the negotiation cost is "large". The manufacturer uses linear pricing and thus t_0 is the critical value of transportation cost, and below $\overline{g}(t)$, the negotiation cost is "small" and t_1 is the critical value. To the left of the critical values (i.e., Regions I and II) the transportation costs are "small", and the manufacturer chooses $(w_b - w_a)^* = t$ to prevent arbitrage. To the right of the critical values (i.e., Regions III and IV) the transportation costs are "large", and the manufacturer chooses the unconstrained optimal wholesale prices.

$\overline{g}(t)$ is non-decreasing in t because $\overline{g}(t)$ is the manufacturer's benefit from using two-part tariff contracts in a town. Using two-part tariffs leads to a lower wholesale price in towns. An increase in the transportation cost can relax the manufacturer's constraint and increase the manufacturer's profit. This effect is larger with two-part tariffs than with linear contracts. A prediction of this paper is that the manufacturer tends to use linear contracts in towns when transportation or arbitrage costs are lowered.

The four regions in Figure 1(a) exist for non-linear concave demands as long as t_0 and t_1 are strictly positive and finite and $\overline{g}(t)$ is continuous, positive and finite. The value $t_0 \equiv p_b^m - \widetilde{w}_a$ is positive because $p_b^m > \widetilde{w}_a$ and is finite because p_b^m is bounded. Similarly, t_1 is strictly positive and finite. $\overline{g}(t)$ is generally continuous because $w_a^*(t)$ and $w_b^*(t)$ are continuous at the critical values. The function is non-negative because the two-part tariff is more flexible than linear pricing. For t larger than the critical values, retailers do not arbitrage and the manufacturer's benefit from using a two-part tariff is strictly positive. Thus, $\overline{g}(t)$ is strictly positive at the critical values, and since $\overline{g}(t)$ is continuous at these values, the four regions are not empty.

For some large enough difference in the demand parameters between towns and cities, if the transportation cost is smaller than some critical value, $\bar{t}(g)$, the prevention of arbitrage causes so much distortion that the manufacturer prefers not to sell to towns at all.[2]. In Figure 1(b), we change the demand in towns into $Q_a = 6 - p_a$ (other parameters remain unchanged). A fifth region — Region $\varnothing$, in which the manufacturer's profit selling to both towns and cities, with the constraint, is less than its profit selling only to cities, without the constraint, arises. We define $\bar{t}(g)$ as the transportation cost such that the manufacturer is indifferent between selling to both towns and cities and selling to only cities.

If Region $\varnothing$ exists, it is to the left of Regions I and II. Moreover, $\bar{t}(g)$ is upward sloping between Regions $\varnothing$ and II and is a vertical line between Regions $\varnothing$ and I. In Region $\varnothing$, the manufacturer sells only to cities, at a wholesale price equal to the monopoly price, and his profit is a constant

[2] The contrary does not happen: the manufacturer would never sell only to towns. If the manufacturer sells only to towns at price p'_a, it can always gain a higher profit by selling to both towns and cities at p'_a.

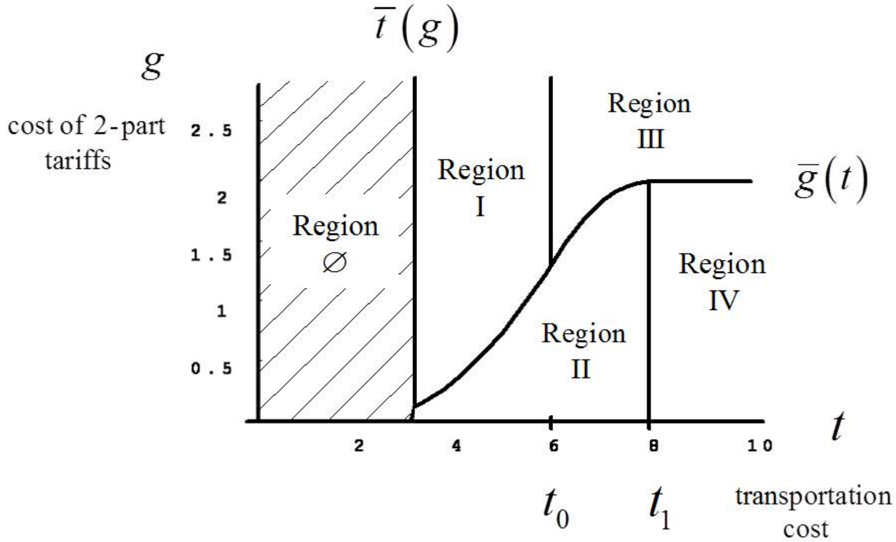

Figure 1(b): The Fifth Region – Region $\varnothing$

independent of g and t. In Regions I and II, the manufacturer sells to both towns and cities, and by the Envelope Theorem, the manufacturer's profit falls in the transportation cost. Thus, if Region $\varnothing$ appears it is to the left of Regions I and II. In Region I, the manufacturer uses linear contracts, and his profit is independent of g. Thus, $\bar{t}(g)$ is a vertical line between Regions $\varnothing$ and I. In Region II, the manufacturer uses two-part tariffs, and the smaller the transportation cost, the smaller is the contract cost for the manufacturer to have equal profit. Thus, $\bar{t}(g)$ is upward sloping between Regions $\varnothing$ and II.

Some prescription drugs and electronic products are sold at extraordinarily high prices for a period of time when they are newly introduced. These products are light and easy to transport and consumers in particular locations have high willingness to pay for early consumption. Region $\varnothing$ can explain why these products are sold in some locations and not in the others. If the government prohibits price discrimination, the effect of the prohibition is equivalent to bringing a zero transportation cost into our model. If the manufacturer initially sells to both towns and cities, banning price discrimination will cause the manufacturer to close down the markets in towns and the welfare change will be Pareto inferior (see discussions in the Social Welfare section).

6 Comparative Statics

In this section, we study the effect of the intensity of competition in city retail markets on the manufacturer's pricing decisions and choice of contracts. We fix the market structure in towns to monopoly, and vary the market structure in cities between monopoly and perfect competition. For simplicity, we fix the contract types in cities to linear pricing contracts. We assume retailers compete with each other in quantity, q, where total quantity (in a city) is $Q_b \equiv \sum_{i=1}^{k} q_b$. Let k be the number of retailers in each city, where $1 \leq k \leq \infty$, and $\overline{g}\,(t,k)$ and $\overline{t}\,(g,k)$ be the contract cost and the transportation cost defined in the ways as in the above section.

Proposition 3. As long as t is smaller than the critical values, the greater the intensity of competition in cities (i.e., the larger is k):

(a) the larger the manufacturer's profit (i.e., $\frac{\partial \pi^*(g,t,k)}{\partial k} > 0$),

(b) the higher the wholesale price in cities (i.e., $\frac{\partial w_b^*}{\partial k} \geq 0$),

(c) the higher the wholesale and retail prices in towns (i.e., $\frac{\partial w_a^*}{\partial k} \geq 0$ and $\frac{\partial p_a^*}{\partial k} \geq 0$),

(d) the more likely the manufacturer uses linear contracts in towns (i.e., $\frac{\partial \overline{g}(t,k)}{\partial k} \leq 0$), and

(e) the more likely the manufacturer refuses to sell to towns (i.e., $\frac{\partial \overline{t}(g,k)}{\partial k} \geq 0$, if Region $\varnothing$ exists).

Proof: See the Appendix.

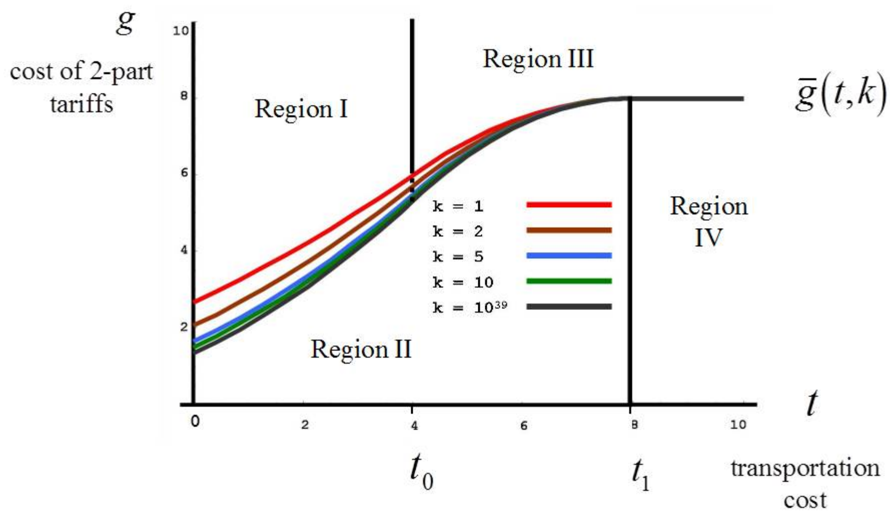

Figure 2(a): As k Becomes Larger, $\overline{g}\,(t,k)$ Shifts Downwards

Figure 2(a) illustrates the effects of a change in the value of k on the manufacturer's contracts and pricings. We have set five values of k, i.e., $k = 1$, 2, 5, 10, and 10^{39}, and retain all other parameter values used in Figure 1(a) (i.e., $n_a = 2$, $n_b = 1$, $c = 2$, $Q_a = 10 - p_a$, and $Q_b = 18 - p_b$). When $k = 1$, the retailers in each town and each city are monopoly, and the red color $\overline{g}\,(t,k)$ indicates the case. As k gets larger, $\overline{g}\,(t,k)$ shifts downwards. When k becomes very large (i.e., $k = 10^{39}$), $\overline{g}\,(t,k)$ converges to the grey color curve. Figure 2(a) says, when k is getting larger, for a greater set of parameter values $\{g,t\}$, the manufacturer chooses to use linear contracts in towns.

In Figure 2(b), we assume a weaker demand in towns (i.e., $Q_a = 6 - p_a$), but the other parameters are the same as in Figure 1(a). Now the difference in the demand parameters is large enough so that if $t < \overline{t}\,(g,k)$, the manufacturer will sell only to cities. If Region $\varnothing$ exists, the value of k affects not only the position of curve $\overline{g}\,(t,k)$, but also curve $\overline{t}\,(g,k)$. Figure 2(b) says, when k is getting larger, for a greater set of parameters $\{g,t\}$, the manufacturer refuses to sell to towns.

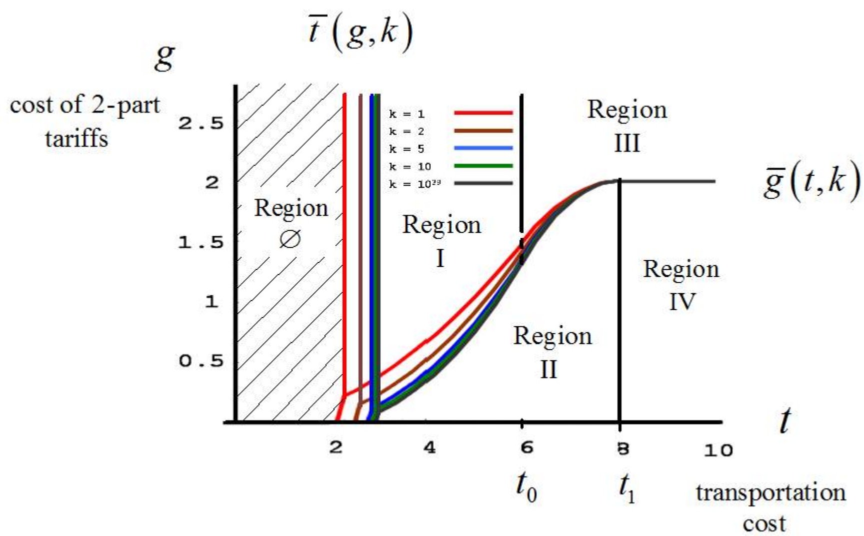

Figure 2(b): As k Becomes Larger, $\bar{t}\,(g,k)$ Shifts Rightward

The intensity of competition in the retail markets in cities affects the welfare of consumers in towns, hurting them. We identify three sources of such adverse effects: With a greater value of k: (i) the manufacturer charges a higher wholesale price in towns, (ii) the manufacturer is more likely to use linear contracts in towns, and (iii) the manufacturer is more likely to refuse to sell to towns.

7 Social Welfare

If price discrimination is prohibited, consumers in cities enjoy a lower price but those in towns face a higher price. Exclusive territories hurt city consumers but help town consumers and the manufacturer. Neither overall welfare's change nor total output is determinate.

Suppose the government restricts the degree of wholesale price discrimination such that $|w_b - w_a| \leq s$, where $s \geq 0$ is a constant. If $s = 0$, the government bans wholesale price discrimination altogether. We notice that the constraint, $|w_b - w_a| \leq s$, is of the same form as the arbitrage constraint, $|w_b - w_a| \leq t$, and thus the discussion of the prevention of arbitrage in this paper provides some insight to the discussion of the welfare effect of third-degree wholesale price discrimination. If both constraints, $|w_b - w_a| \leq t$ and $|w_b - w_a| \leq s$, are present, the one with a greater value of the constants on the right hand side of the inequalities is not effective.

Proposition 4.

(a) (i) If Region $\varnothing$ exists (i.e., the region in which the manufacturer's profit selling to both towns and cities is less than his profit selling only to cities is non-empty) and the manufacturer sells to both towns and cities (i.e., $t \geq \bar{t}(g,k)$), the prohibition of price discrimination causes the manufacturer to close down the markets in towns. The welfare change is Pareto inferior.
(ii) If Region $\varnothing$ exists and the manufacturer sells only to cities (i.e., $t < \bar{t}(g,k)$), the allowance of price discrimination does not cause the manufacturer to open up the markets in towns. To induce the manufacturer to sell to towns, the government should allow both price discrimination and exclusive territories. The welfare change yields a Pareto improvement.

(b) If Region $\varnothing$ does not exists and the manufacturer uses two-part tariffs in towns, with $g > \bar{g}(t = 0)$, the prohibition of price discrimination causes the manufacturer to switch to use linear pricing contracts in towns. As a result, the manufacturer and consumers in both towns and cities are worse off.

Proof: See the Appendix.

Proposition 4(a) (i) is a variation of one of the findings by Hausman and MacKie-Mason (1988). Part 4(a) (ii), however, says that if the arbitrage constraint binds, the allowance of price discrimination is no longer the

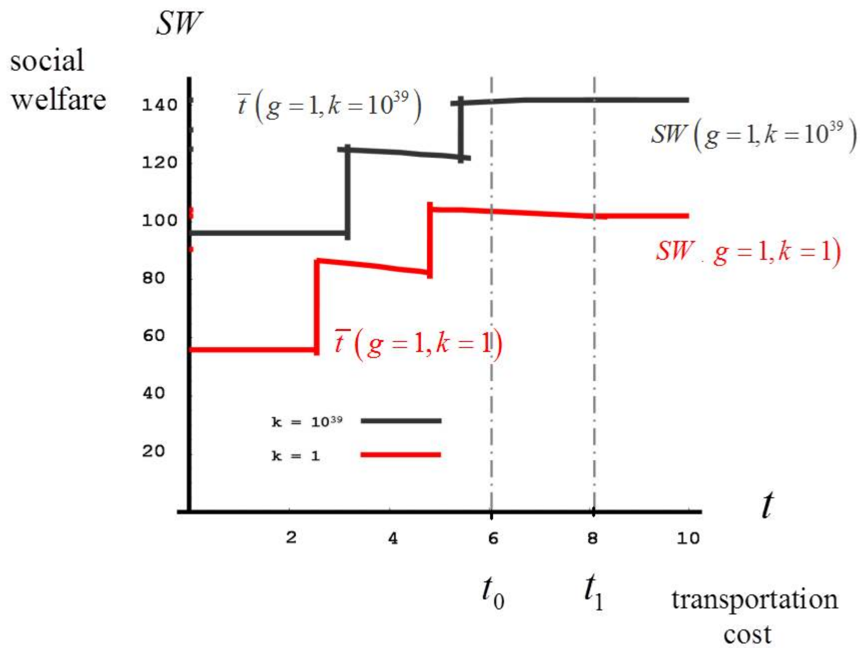

Figure 3: Social Welfare Changes with k and t

opposite to the prohibition of price discrimination. Even though banning wholesale price discrimination can cause the manufacturer to close down the markets in towns, in some situations the allowance of price discrimination does not sufficiently cause the manufacturer to open up the markets in towns. Instead, the government should allow both price discrimination and exclusive territories. Proposition 4(b) tells us that price discrimination affects not only the manufacturer's decision whether to open new markets but also the manufacturer's choice of contracts. Opening markets, like two-part tariffs, can yield Pareto improvement.

Figure 3 shows how opening new markets and the choice of contracts are important in the welfare change between uniform pricing and price discrimination. We simulate the change of social welfare with respect to the transportation cost, along the cross section plane, X–X, in Figure 2(b), assuming $g = 1$ and other parameter values used in Figure 2(b). The top curve indicates the social welfare for $k = 1$ and the bottom curve indicates the social welfare for $k = 10^{39}$.

To the left of $\bar{t}(g,k)$, the manufacturer does not sell to towns and charges the monopoly price in cities, and the social welfare is unchanged throughout Region $\varnothing$. At $\bar{t}(g,k)$, the manufacturer opens up the markets in towns and causes the first jump of the social welfare. To the right of $\bar{t}(g,k)$, the social welfare can be increasing or decreasing in t. Given the parameters used here, social welfare is decreasing. The second jump happens at the border between Regions I and II. In Region II, the manufacturer uses two-part tariff contracts in towns that cause the second jump of the social welfare. With two-part-tariff contracts, the critical value of the transportation cost is t_1. When $t \geq t_1$, the constraint becomes non-binding, and the social welfare remains unchanged in Region IV.

Recall that $SW(t = 0)$ is the social welfare if the government bans price discrimination and $SW(t \geq t_1)$ is the social welfare if the government allows both price discrimination and exclusive territories. The two jumps in Figure 3 dominate the change of the social welfare between $SW(t = 0)$ and $SW(t \geq t_1)$, for the two given values of k. Thus, the figure reminds us the followings: (i) Opening new markets and contract types are important in the discussion of welfare effects of third-degree wholesale price discrimination, (ii) consumer welfare in towns is important in the discussion of welfare effects of third-degree wholesale price discrimination, and (iii) competition in general improves social welfare.

8 Conclusion

This paper studies the choice of contracts and pricing strategies of a manufacturer who sells to retailers in various geographic locations (towns and cities). We have shown that the manufacturer will prevent arbitrage setting the wholesale price difference between locations smaller than the transportation cost. If the transportation cost is less than some critical values, the constraint becomes binding and the manufacturer will not price discriminate to the full extent. While lowering the wholesale price in cities, the manufacturer raises the wholesale price in towns. As a result, with linear contracts double marginalization is magnified. That is, the retail price in towns will be higher than that under standard double marginalization. This explains why groceries in rural areas are often more expensive than in cities, even though households in rural areas can less likely to afford a higher price.

So long as the constraint is binding, the larger the transportation cost, the larger the incentive for the manufacturer to use two-part tariffs in towns. A prediction of this paper is thus, other things equal, the manufacturer tends to use linear contracts in towns when costs of arbitrage activities (e.g. transportation costs) are lowered. We have also shown a situation in which the manufacturer refuses to sell to towns. When demand parameters differ a lot between locations and the transportation costs are small, the prevention of arbitrage causes so much distortion that the manufacturer is better to sell only to cities. This explains why some prescription drugs are sold in some countries and not others.

We have also studied how the intensity of competition in retail markets in cities affects the prices and consumer welfare in towns. In the comparative statics section, we modify our model that retailers in each city compete in Cournot equilibrium and then vary the number of these retailers to examine the effects on the manufacturer's pricing and choice of contracts. While competition in city retail markets benefits consumers in cities, it harms consumers and retailers in towns. We have identified three sources of such across location effects: As competition in city retail markets gets more intense, the manufacturer (i) charges a higher wholesale price in towns, (ii) tends to use linear contracts in towns, instead of two-part tariffs, and (iii) tends to close down the markets in towns.

Finally, we have studied the welfare effects of third-degree wholesale price discrimination. If the government bans price discrimination, the effect is equivalent to bringing a zero transportation cost in to our model, and the manufacturer will close down the markets in towns. Conversely, if the manufacturer does not sell to towns, the allowance of price discrimination does not help to open up the markets in towns. In that situation, the government should allow both price discrimination and exclusive territories. We have also shown that banning price discrimination may affect the manufacturer's choice of contracts. The prohibition of price discrimination can cause the manufacturer to switch to use linear pricing contracts in towns. The welfare change due to using linear contracts and closing down the markets in towns is inferior. In our simulation, the choice of contracts, as well as whether to sell to towns, explains much of the welfare change due to banning price discrimination.

9 Appendix

Lemma 1. The retailers' derived demands for the good, $D_a^r(w_a)$ and $D_b^r(w_b)$, are strictly decreasing and concave in wholesale prices w_a and w_b.

Proof: First, we show that $D_b^r(w_b)$ is strictly decreasing and concave in w_b. Differentiating the derived demand function $D_b^r(w_b) \equiv D_b(p_b^*(w_b))$ two times with respect to w_b we obtain $\frac{dD_b^r(w_b)}{dw_b} = D_b' \cdot \frac{dp_b^*}{dw_b}$ and $\frac{d^2D_b^r(w_b)}{dw_b^2} = D_b'' \cdot \left(\frac{dp_b^*}{dw_b}\right)^2 + D_b' \cdot \frac{d^2p_b^*}{dw_b^2}$. Since in cities the retailers compete in prices, $p_b^*(w_b) = w_b$, and therefore $\frac{dp_b^*(w_b)}{dw_b} = 1$ and $\frac{d^2p_b^*(w_b)}{dw_b^2} = 0$. Thus, we obtain $\frac{dD_b^r(w_b)}{dw_b} < 0$ and $\frac{d^2D_b^r(w_b)}{dw_b^2} \leq 0$.

Second, differentiating two times $D_a^r(w_a) \equiv D_a(p_a^*(w_a))$ with respect to w_a we obtain $\frac{dD_a^r(w_a)}{dw_a} = D_a' \cdot \frac{dp_a^*}{dw_a}$ and $\frac{d^2D_a^r(w_a)}{dw_a^2} = D_a'' \cdot \left(\frac{dp_a^*}{dw_a}\right)^2 + D_a' \cdot \frac{d^2p_a^*}{dw_a^2}$. Then we differentiate the retailer's first order condition with respect to w_a and obtain $\frac{dp_a^*(w_a)}{dw_a} = \frac{D_a'}{2D_a' + (p_a^* - w_a)\cdot D_a''} > 0$ and therefore $\frac{dD_a^r(w_a)}{dw_a} < 0$. Differentiating the retailer's first order condition again with respect to w_a, we obtain $\frac{d^2p_a^*(w_a)}{dw_a^2} = \frac{p_a^{*\prime}\cdot D_a''\cdot(2-3p_a^{*\prime}) - (p_a^{*\prime})^2\cdot(p_a^* - w_a)\cdot D_a'''}{2D_a' + (p_a^* - w_a)\cdot D_a''}$. Since $\frac{dp_a^*(w_a)}{dw_a}$ is smaller than $\frac{1}{2}$, non-negative D_a''' is a sufficient condition for $\frac{d^2p_a^*(w_a)}{dw_a^2} \geq 0$ and hence $\frac{d^2D_a^r(w_a)}{dw_a^2} \leq 0$. $\square$

Proposition 1. In equilibrium, the manufacturer will prevent arbitrage. Wholesale prices will be higher in the city, but by no more than the transportation cost, so $0 < (w_b - w_a) \leq t$.

Proof: To show that there is no arbitrage we need to show that the manufacturer's maximum profit in (3) is strictly greater than in (4) or (5). Let w_a' solve the maximization problem in (4). Choose $w_a = w_a'$ and $w_b = w_a'$ in (3) and compare the resulting profit functions in (3) and (4). By Lemma 1, $D_b^r(w_b)$ is strictly decreasing in w_b, so the manufacturer's maximum profit in (3) is strictly greater than in (4), regardless of the choice of I_a.

Let w_b' solve the maximization problem in (5). Choose $w_a = w_b'$ and $w_b = w_b'$ in (3) and compare the resulting profit functions in (3) and (5). Since $D_a^r(w_a)$ is strictly decreasing in w_a by Lemma 1 and $F_a(w_a)$ is decreasing in w_a, the manufacturer's maximum profit in (3) is strictly greater than in (5), regardless of the choice of I_a.

Thus, if arbitrage occurred in any equilibrium, the manufacturer would decrease the absolute value of the wholesale price difference until it did not, which rules out equilibria with a wholesale price difference of $|(w_b - w_a)| > t$ with arbitrage occuring.

Next let us show that $(w_b - w_a) > 0$. First remove the constraint $|(w_b - w_a)| \leq t$ from maximization problem (3) and let (w_a', w_b') be the solution. Without the constraint $|(w_b - w_a)| \leq t$, the manufacturer's problem is separable into maximization problems for towns and for cities. If the manufacturer chooses linear pricing, it chooses wholesale prices $\widetilde{w}_a$ for towns and p_b^m for cities. Since we have $p_b^m > \widetilde{w}_a$, the unconstrained optimal $(w_b - w_a)' = p_b^m - \widetilde{w}_a$ is positive. If the manufacturer chooses a two-part tariff, the wholesale price is even lower for towns and the sign of $(w_b - w_a)'$ does not change.

Now, put back the constraint $|w_b - w_a| \leq t$ into maximization problem (3). If $(w_b - w_a)'$ satisfies the constraint $|(w_b - w_a)| \leq t$, the manufacturer will choose (w_a', w_b') and thus $(w_b - w_a)^* > 0$, though possibly that difference is strictly less than t.

If $(w_b - w_a)'$ does not satisfy $|(w_b - w_a)| \leq t$, positive $(w_b - w_a)'$ implies $(w_b - w_a)' > t$. Since the manufacturer's profit in (3) is twice differentiable and $\frac{\partial^2 \pi}{\partial (w_b - w_a)^2} = n_b \cdot \left[2D_b^{r\prime} + (w_b - c) \cdot D_b^{r\prime\prime}\right] < 0$, the profit function is concave in $(w_b - w_a)$. Because of the concavity, $(w_b - w_a)' > t$ implies π is increasing over $-t \leq (w_b - w_a) \leq t$ and the manufacturer will choose $(w_b - w_a)^* = t$. Thus, the price difference is positive and might be as great as t. $\square$

Proposition 2. (a) There exists a continuous function $\overline{g}(t) > 0$ such that if $g < \overline{g}(t)$ a two-part tariff is used. (b) Suppose t is small enough for the possibility of arbitrage to constrain the manufacturer ($t < t_0$ if $g \geq \overline{g}(t)$ or $t < t_1$ if $g < \overline{g}(t)$, as in Figure 1's Regions I and II). In Figure 1's Region I, with linear pricing, the retail price in towns is higher than under standard double marginalization. In Region II, with two-part tariffs, the wholesale price for town retailers will exceed marginal cost and the retail price will be higher than the integrated monopoly price. (c) $\overline{g}'(t) > 0$ for $t < t_1$, and $\overline{g}'(t) = 0$ otherwise.

Proof: (a) In the Lagrangian function in (7), we regard the wholesale price difference, $(w_b - w_a)$, as a choice variable of the manufacturer. Instead of choosing both wholesale prices, the manufacturer chooses the wholesale price for towns, w_a, and the wholesale price difference, $(w_b - w_a)$. Since the wholesale price difference $(w_b - w_a) > 0$ (Proposition 1) and $w_a \geq c > 0$, the Kuhn-Tucker conditions are:

$$\begin{aligned} \frac{\partial L}{\partial w_a} &= n_a \cdot [D_a^r(w_a^*) + (w_a^* - c) \cdot D_a^{r\prime}(w_a^*) + I_a \cdot F_a'(w_a^*)] \\ &\quad + n_b \cdot [D_b^r(w_b^*) + (w_b^* - c) \cdot D_b^{r\prime}(w_b^*)] = 0 \end{aligned} \tag{8}$$

$$\frac{\partial L}{\partial (w_b - w_a)} = n_b \cdot [D_b^r(w_b^*) + (w_b^* - c) \cdot D_b^{r\prime}(w_b^*)] - \lambda^* = 0 \tag{9}$$

and

$$t \geq (w_b^* - w_a^*) \quad (\text{with } t = (w_b^* - w_a^*) \text{ if } \lambda^* > 0) \tag{10}$$

Notice that variables I_a for whether a two-part tariff is used and λ for whether the arbitrage constraint is binding each have values either positive or zero. There are in total four different combinations of I_a and λ. Let us relate the four combinations of policies in Figure 1 to the two critical values of transportation costs, t_0 and t_1. We will need to show three things about this:

(i) If the manufacturer chooses linear pricing for towns, then $\lambda^* > 0$ and the arbitrage constraint is binding if and only if $t < t_0$.

(ii) If the manufacturer chooses a two-part tariff for towns, then $\lambda^* > 0$ and the arbitrage constraint is binding if and only if $t < t_1$.

(iii) The manufacturer's optimal choices of wholesale prices, $w_a^*(t)$ and $w_b^*(t)$, are continuous at the two critical values of transportation costs.

(i) Consider linear pricing contracts ($I_a = 0$). If $\lambda^* = 0$, the arbitrage constraint is not binding and the Kuhn-Tucker conditions (8) and (9) give the same first order conditions as unconstrained maximization. Thus, the manufacturer chooses $\widetilde{w}_a$ for towns and p_b^m for cities. This implies $(w_b - w_a)^* = p_b^m - \widetilde{w}_a$ and according to the definition of t_0 we obtain $(w_b - w_a)^* = t_0$. On the other hand, since $\lambda^* = 0$, (10) implies $t \geq (w_b - w_a)^*$. Combining these two conditions we obtain that $t \geq t_0$. Conversely, if $\lambda^* > 0$, then (9) implies that $n_b \cdot [D_b^r(w_b^*) + (w_b^* - c) \cdot D_b^{r\prime}(w_b^*)] > 0$. The left hand side of this inequality is $\frac{\partial \pi}{\partial (w_b - w_a)}$. Since π is concave in $(w_b - w_a)$, the inequality $\frac{\partial \pi}{\partial (w_b - w_a)} > 0$ implies $(w_b - w_a)^* < p_b^m - \widetilde{w}_a \equiv t_0$. On the other hand, since $\lambda^* > 0$, (10) implies that the constraint is binding and $(w_b - w_a)^* = t$. Combining the two conditions we obtain that $t < t_0$.

(ii) The proof is parallel to that for (i), with t_1 being the critical value instead.

(iii) By parts (i) and (ii), for t greater than the critical values, $\lambda^* = 0$ and the constraint is not necessarily binding. If $I_a = 0$, wholesale prices $\widetilde{w}_a$ and p_b^m solve the Kuhn-Tucker conditions (8) and (9), and if $I_a = 1$, then marginal cost c and p_b^m solve both (8) and (9). For t smaller than the critical values, $\lambda^* > 0$ and by (10) $(w_b - w_a)^* = t$. Kuhn-Tucker conditions (8) and (9) become:

$$n_a \cdot [D_a^r(w_a^*) + (w_a^* - c) \cdot D_a^{r\prime}(w_a^*) + I_a \cdot F_a'(w_a^*)] \\ + n_b \cdot \left[D_b^r(w_a^* + t) + (w_a^* + t - c) \cdot D_b^{r\prime}(w_a^* + t)\right] = 0 \tag{8′}$$

and

$$n_b \cdot \left[D_b^r(w_a^* + t) + (w_a^* + t - c) \cdot D_b^{r\prime}(w_a^* + t)\right] - \lambda^* = 0 \tag{9′}$$

As the transportation cost t approaches the critical values, $\lambda^* \to 0$. We can substitute $t = t_0 \equiv p_b^m - \widetilde{w}_a$ into (8′) and (9′) and find that $\widetilde{w}_a$ and p_b^m solve these equations when $I_a = 0$. Similarly, substitute $t = t_1 \equiv p_b^m - c$ into (8′) and (9′) to find that c and p_b^m solve these equations when $I_a = 1$. Thus, the manufacturer's optimal wholesale prices are continuous at the critical values of transportation costs.

The manufacturer's profit $\pi^* (I_a = 1, g = 0, t)$ cannot be strictly less than $\pi^* (I_a = 0, t)$. Moreover, $\pi^* (I_a = 1, g, t)$ is strictly decreasing in g, and therefore we can always find $\overline{g}(t) \geq 0$ satisfying the identity $\pi^*(I_a = 1, \overline{g}, t) \equiv \pi^*(I_a = 0, t)$. In other words, $\overline{g}(t)$ exists for all $t > 0$. Equating the manufacturer's optimal profits in (6) for $I_a = 0$ and $I_a = 1$, we can solve explicitly $\overline{g}(t)$ in terms of $w^*(I_a = 0, t)$ and $w^*(I_a = 1, t)$. By result (iii) above, $w_a^*(t)$ and $w_b^*(t)$ are continuous at the critical values for both $I_a = 0$ and $I_a = 1$. Thus, $\overline{g}(t)$ is also continuous at the critical values and is in general continuous for $t > 0$.

(b) In (i), (ii) and (iii) we have established that if transportation costs are less than the critical values (Figure 1's Regions I and II), the Lagrange multiplier is positive: $\lambda^* > 0$. From Kuhn-Tucker condition (9), $\lambda^* > 0$ implies $n_b \cdot [D_b^r(w_b^*) + (w_b^* - c) \cdot D_b^{r\prime}(w_b^*)] > 0$. The terms in the square bracket of this inequality is the first derivative of the manufacturer's profit from a city, π_b, with respect to w_b. Since π_b is concave in w_b and p_b^m solves $\frac{\partial \pi_b}{\partial w_b} = 0$, $\frac{\partial \pi_b}{\partial w_b} > 0$ implies $w_b^* < p_b^m$. Substituting $n_b \cdot [D_b^r(w_b^*) + (w_b^* - c) \cdot D_b^{r\prime}(w_b^*)] > 0$ into (8) we obtain $n_a \cdot [D_a^r(w_a^*) + (w_a^* - c) \cdot D_a^{r\prime}(w_a^*) + I_a \cdot F_a'(w_a^*)] < 0$. The terms in the square bracket of this last inequality is the first derivative of the manufacturer's profit from a town, π_a, with respect to w_a. If $I_a = 0$ (in Region I), π_a is concave in w_a and $\widetilde{w}_a$ solves $\frac{\partial \pi_a}{\partial w_a} = 0$. Thus, $\frac{\partial \pi_a}{\partial w_a} < 0$ implies that $w_a^* > \widetilde{w}_a$. Since with linear pricing the retail price is increasing in wholesale price, $w_a^* > \widetilde{w}_a$ implies $p_a^* > \widetilde{p}_a$. If $I_a = 1$ (as in Region II), then $w_a = c$ solves $\frac{\partial \pi_a}{\partial w_a} = 0$ and the stationary point locally maximizes the manufacturers profit. Then $\frac{\partial \pi_a}{\partial w_a} < 0$ implies $w_a^* > c$ and $p_a^* > p_a^m$.

(c) $\overline{g}(t)$ is defined such that $\pi^*(I_a = 1, \overline{g}(t), t) \equiv \pi^*(I_a = 0, t)$. Differentiate the identity with respect to t and obtain:

$$\frac{\partial \pi^*(I_a = 1, \overline{g}(t), t)}{\partial t} + \frac{\partial \pi^*(I_a = 1, \overline{g}(t), t)}{\partial g} \cdot \frac{\partial \overline{g}(t)}{\partial t} = \frac{\partial \pi^*(I_a = 0, t)}{\partial t} \tag{11}$$

By the Envelope Theorem, $\frac{\partial \pi^*(I_a=1,\overline{g}(t),t)}{\partial g} = \frac{\partial \pi(I_a=1,g,w_a,w_b)}{\partial g} \mid_{w_a=w_a^*(I_a=1,t),w_b=w_b^*(I_a=1,t)} = -I_a \cdot n_a = n_a$, arrange the terms in (11) and obtain:

$$\frac{\partial \overline{g}(t)}{\partial t} = \frac{1}{n_a} \cdot \left(\frac{\partial \pi^*(I_a = 1, \overline{g}(t), t)}{\partial t} - \frac{\partial \pi^*(I_a = 0, t)}{\partial t} \right) \tag{12}$$

By the Envelope Theorem, $\frac{\partial \pi^*(I_a=1,\overline{g}(t),t)}{\partial t} = \frac{\partial \pi(I_a=1,g,w_a,w_b)}{\partial t} |_{w_a=w_a^*(I_a=1,t),w_b=w_b^*(I_a=1,t)} = \lambda^*(I_a=1,t)$ and $\frac{\partial \pi^*(I_a=0,t)}{\partial t} = \frac{\partial \pi(I_a=0,w_a,w_b)}{\partial t} |_{w_a=w_a^*(I_a=0,t),w_b=w_b^*(I_a=0,t)} = \lambda^*(I_a=0,t)$. Substitute into (12) and obtain

$$\frac{\partial \overline{g}(t)}{\partial t} = \frac{1}{n_a} \cdot (\lambda^*(I_a=1,t) - \lambda^*(I_a=0,t)) \tag{13}$$

(13) implies:

$$\frac{\partial \overline{g}(t)}{\partial t} \gtreqless 0 \text{ if and only if } \lambda^*(I_a=1,t) \gtreqless \lambda^*(I_a=0,t) \tag{14}$$

Three cases of the transportation cost:

Case (i): $t \geq t_1$

The constraint is not binding (i.e., $\lambda^* = 0$), regardless of the contract types, and thus $\lambda^*(I_a=1,t) - \lambda^*(I_a=0,t) = 0 \Longleftrightarrow \overline{g}'(t) = 0$.

Case (ii): $t_0 \leq t < t_1$

With linear contracts, the constraint is not binding (i.e., $\lambda^*(I_a=0) = 0$). With two-part tariffs, the constraint is binding (i.e., $\lambda^*(I_a=1,t) > 0$). Combine the conditions and obtain $\lambda^*(I_a=1,t) - \lambda^*(I_a=0,t) > 0 \Longleftrightarrow \overline{g}'(t) > 0$.

Case (iii): $t < t_0$

The constraint is binding (i.e., $\lambda^* > 0$) regardless of the contract type (no matter whether $I_a = 0$ or $I_a = 1$). We claim that $\lambda^*(I_a=1) > \lambda^*(I_a=0)$ and prove by contradiction. Suppose the opposite is true, and thus either (i) $\lambda^*(I_a=1) < \lambda^*(I_a=0)$ or (ii) $\lambda^*(I_a=1) = \lambda^*(I_a=0)$. Let $\lambda^1 \equiv \lambda^*(I_a=1)$ and w_a^1 and w_b^1 solve the Kuhn Tucker conditions in (8) and (9) when $I_a = 1$. Similarly, let $\lambda^0 \equiv \lambda^*(I_a=0)$ and w_a^0 and w_b^0 solve the Kuhn Tucker conditions when $I_a = 0$. The Kuhn Tucker conditions become:

$$\begin{aligned}
\left\{ \begin{array}{c} n_a \cdot [D_a^r(w_a^0) + (w_a^0 - c) \cdot D_a^{r\prime}(w_a^0) + 0 \cdot F_a'(w_a^0)] \\ +n_b \cdot [D_b^r(w_b^0) + (w_b^0 - c) \cdot D_b^{r\prime}(w_b^0)] \end{array} \right\} &= 0 && (8'') \\
n_b \cdot [D_b^r(w_b^0) + (w_b^0 - c) \cdot D_b^{r\prime}(w_b^0)] - \lambda^0 &= 0 && (9'') \\
\left(w_b^0 - w_a^0\right) &= t && (10'')
\end{aligned}$$

and

$$\left\{ \begin{array}{c} n_a \cdot [D^r_a(w^1_a) + (w^1_a - c) \cdot D^{r\prime}_a(w^1_a) + 1 \cdot F^\prime_a(w^1_a)] \\ +n_b \cdot [D^r_b(w^1_b) + (w^1_b - c) \cdot D^{r\prime}_b(w^1_b)] \end{array} \right\} = 0 \qquad (8''')(15)$$

$$n_b \cdot [D^r_b(w^1_b) + (w^1_b - c) \cdot D^{r\prime}_b(w^1_b)] - \lambda^1 = 0 \qquad (9''')(16)$$

$$\left(w^1_b - w^1_a\right) = t \qquad (10''')(17)$$

Case (i): $\lambda^0 > \lambda^1$

(9) implies that $D^r_b(w^0_b) + (w^0_b - c) \cdot D^{r\prime}_b(w^0_b) > D^r_b(w^1_b) + (w^1_b - c) \cdot D^{r\prime}_b(w^1_b)$. Since $D^r_b(w^*_b) + (w^*_b - c) \cdot D^{r\prime}_b(w^*_b)$ is the first derivative of the manufacturer's profit from a city, i.e., $\frac{\partial \pi_b}{\partial w_b}$, the inequality implies that $\frac{\partial \pi_b(w^0_b)}{\partial w_b} > \frac{\partial \pi_b(w^1_b)}{\partial w_b}$ and the concavity of π_b in w_b implies that $w^1_b > w^0_b$. Given t, (10) implies $w^1_b - w^1_a = t = w^0_b - w^0_a$ and thus $w^1_a > w^0_a$. Similarly, $D^r_a(w^*_a) + (w^*_a - c) \cdot D^{r\prime}_a(w^*_a) + I_a \cdot F^\prime_a(w^*_a)$ is the first derivative of the manufacturer's profit from a town, i.e., $\frac{\partial \pi_a}{\partial w_a}$, $w^1_a > w^0_a$ implies $\frac{\partial \pi_a(w^1_a)}{\partial w_a} < \frac{\partial \pi_a(w^0_a)}{\partial w_a}$. Combine (8) and (9) and obtain $n_a \cdot \frac{\partial \pi_a(w^0_a)}{\partial w_a} + \lambda^0 = 0 = n_a \cdot \frac{\partial \pi_a(w^1_a)}{\partial w_a} + \lambda^1 \iff n_a \cdot \frac{\partial \pi_a(w^1_a)}{\partial w_a} > n_a \cdot \frac{\partial \pi_a(w^0_a)}{\partial w_a} \iff \frac{\partial \pi_a(w^1_a)}{\partial w_a} > \frac{\partial \pi_a(w^0_a)}{\partial w_a}$. Obtain a contradiction and thus λ^0 cannot be strictly greater than λ^1.

Case (ii): $\lambda^0 = \lambda^1$

(9) implies that $D^r_b(w^0_b) + (w^0_b - c) \cdot D^{r\prime}_b(w^0_b) = D^r_b(w^1_b) + (w^1_b - c) \cdot D^{r\prime}_b(w^1_b) \iff \frac{\partial \pi_b(w^0_b)}{\partial w_b} = \frac{\partial \pi_b(w^1_b)}{\partial w_b}$, and the concavity of π_b implies $w^0_b = w^1_b$. Given t, (10) implies $w^1_b - w^1_a = t = w^0_b - w^0_a$ and thus $w^1_a = w^0_a$. This says that the manufacturer charges the same wholesale price in towns under two-part tariffs and under linear contracts, which is not true unless retail markets in towns are competitive. □

Proposition 3. As long as t is smaller than the critical values, the larger the intensity of competition in city retail markets (i.e., a larger value of k): (a) the larger the manufacturer's profit (i.e., $\frac{\partial \pi^*(g,t,k)}{\partial k} > 0$), (b) the higher the wholesale price in cities (i.e., $\frac{\partial w^*_b}{\partial k} \geq 0$), (c) the higher the wholesale and retail prices in towns (i.e., $\frac{\partial w_a^*}{\partial k} \geq 0$ and $\frac{\partial p_a^*}{\partial k} \geq 0$), (d) the more likely the manufacturer uses linear contracts in towns (i.e., $\frac{\partial \bar{g}(t,k)}{\partial k} \leq 0$), and (e) the more likely the manufacturer refuses to sell to towns (i.e., $\frac{\partial \bar{t}(g,k)}{\partial k} \geq 0$, if Region $\varnothing$ exists).

Proof: Parallel to the proof of Proposition 1, we can show that the manufacturer will prevent arbitrage. The Lagrangian function is:

$$\begin{aligned} L = n_a \cdot [(w_a - c) \cdot Q_a^r(w_a) + I_a \cdot (F_a(w_a) - g)] \\ + n_b \cdot \left[(w_b - c) \cdot Q_b^r(w_b, k)\right] + \lambda \cdot [t - (w_b - w_a)] \end{aligned} \tag{18}$$

where $Q_a^r(w_a) = q_a^*(w_a)$ and $Q_b^r(w_b, k) = k \cdot q_b^*(w_b, k)$, and q_a^* and q_b^* satisfy the town and city retailers' reaction functions: $-w_a + P_a(Q_a(q_a^*)) + q_a^* \cdot P_a'(Q_a(q_a^*)) \cdot Q_a'(q_a^*) = 0$ and $-w_b + P_b\left(Q_b\left(q_b^*\right)\right) + q_b^* \cdot P_b'\left(Q_b\left(q_b^*\right)\right) \cdot Q_b'\left(q_b^*\right) = 0$.

$$\begin{aligned} \tfrac{\partial L}{\partial w_a} &= n_a \cdot [Q_a^r(w_a^*) + (w_a^* - c) \cdot Q_a^{r\prime}(w_a^*) + I_a \cdot F_a'(w_a^*)] \\ &\quad + n_b \cdot \left[Q_b^r(w_b^*, k) + (w_b^* - c) \cdot \tfrac{\partial Q_b^r}{\partial w_b}(w_b^*, k)\right] = 0 \end{aligned} \tag{19}$$

$$\frac{\partial L}{\partial (w_b - w_a)} = n_b \cdot \left[Q_b^r(w_b^*, k) + (w_b^* - c) \cdot \frac{\partial Q_b^r}{\partial w_b}(w_b^*, k)\right] - \lambda^* = 0 \tag{20}$$

and

$$t \geq (w_b^* - w_a^*) \quad (\text{with } t = (w_b^* - w_a^*) \text{ if } \lambda^* > 0) \tag{21}$$

(a) By the Envelope Theorem, $\frac{\partial \pi^*(I_a,g,t,k)}{\partial k} = \frac{\partial \pi(I_a,w_a,w_b)}{\partial k} \mid_{w_a = w_a^*(I_a,t,k), w_b = w_b^*(I_a,t,k)}$. Differentiate (18) with respect to k and evaluate the resulting derivative at $w_a^*(I_a, t, k)$ and $w_b^*(I_a, t, k)$. Obtain:

$$\frac{\partial \pi(I_a, w_a, w_b)}{\partial k} \mid_{w_a = w_a^*(I_a=1,t,k), w_b = w_b^*(I_a=1,t,k)} = \left[n_b \cdot (w_b - c) \cdot \frac{\partial Q_b^r(w_b, k)}{\partial k}\right] \mid_{w_b = w_b^*(I_a,t,k)} \tag{22}$$

We notice the right hand side of (22) is $\frac{\partial \pi_b}{\partial k} |_{w_b=w_b^*(I_a,t,k)}$, where π_b is the manufacturer's profit from a city. Since in a Cournot model $\frac{\partial Q_b^r(w_b,k)}{\partial k} > 0$ for each w_b, and thus $\frac{\partial \pi_b(w_b,k)}{\partial k} > 0 \Longleftrightarrow \frac{\partial \pi(I_a,w_a,w_b)}{\partial k} |_{w_a=w_a^*(I_a,t,k),w_b=w_b^*(I_a,t,k)} > 0 \Longleftrightarrow \frac{\partial \pi^*(I_a,g,t,k)}{\partial k} > 0$.

(b) Let w_a^{**} and w_b^{**} be the unconstrained optimal wholesale prices in towns and in cities. Recall that $Q_a^r(w_a^*) + (w_a^* - c) \cdot Q_a^{r\prime}(w_a^*) + I_a \cdot F_a'(w_a^*)$ in (19) is the first derivative of the manufacturer's profit in a town with respect to w_a^* and $Q_b^r(w_b^*,k) + (w_b^* - c) \cdot \frac{\partial Q_b^r}{\partial w_b}(w_b^*,k)$ in (20) is the first derivative of the manufacturer's profit in a city with respect to w_b^*. Combine (19) and (20) and obtain:

$$-n_a \cdot \frac{\partial \pi_a\left(w_a^*\right)}{\partial w_a} = n_b \cdot \frac{\partial \pi_b\left(w_b^*,k\right)}{\partial w_b} \tag{23}$$

Since t is smaller than the critical values, $\lambda^* > 0$. By Proposition 2, $w_a^* > w_a^{**}$ and $\frac{\partial \pi_a(w_a^*)}{\partial w_a} < 0$, and $w_b^* < w_b^{**}$ and $\frac{\partial \pi_b\left(w_b^*,k\right)}{\partial w_b} > 0$. (23) says that the manufacturer will choose w_a^* and w_b^* so that the rates of change of profits across towns and cities with respect to the wholesale prices, multiplied by n_a and n_b, are equal in magnitude. Suppose that k is changed to k', where $k' > k$. By results in part (a), $\pi_b\left(w_b,k'\right) > \pi_b\left(w_b,k\right)$ for each $w_b > c$. Since $w_b^* < w_b^{**}$ and $\pi_b\left(w_b = c,k\right) = \pi_b\left(w_b = c,k'\right) = 0$, the concavity of $\pi_b\left(w_b,k\right)$ in w_b generally implies $\frac{\partial \pi_b\left(w_b^*,k'\right)}{\partial w_b} > \frac{\partial \pi_b\left(w_b^*,k\right)}{\partial w_b}$. (23) becomes:

$$-n_a \cdot \frac{\partial \pi_a\left(w_a^*,k'\right)}{\partial w_a} < n_b \cdot \frac{\partial \pi_b\left(w_b^*,k'\right)}{\partial w_b} \tag{24}$$

(24) says that the wholesale prices, w_a^* and w_b^*, are no longer optimal with k'. To restore the equality sign, suppose that the manufacturer chooses wholesale prices $\left\{w_a', w_b'\right\}$. Since $\lambda^* > 0$, (21) implies $t = \left(w_b^* - w_a^*\right)$, and similarly, $t = \left(w_b' - w_a'\right)$. Combine these two constraints and obtain: (i) $w_a' < w_a^*$ and $w_b' < w_b^*$, or (ii) $w_a' > w_a^*$ and $w_b' > w_b^*$. If the manufacturer chooses $w_a' < w_a^*$ and $w_b' < w_b^*$, concavity of the profit functions imply that the left hand side of (24) becomes smaller while the right hand side becomes larger, and the equality sign is not restored. On the other hand,

concavity implies that some $\{w_a', w_b'\}$, where $w_a' > w_a^*$ and $w_b' > w_b^*$, restores the equality sign. We obtain $\frac{\partial w_b^*}{\partial k} > 0$.

(c) From the result in part (b), $\lambda^* > 0 \Longrightarrow \frac{\partial w_b^*}{\partial k} > 0$. From (21), $\lambda^* > 0 \Longrightarrow t = \left(w_b^* - w_a^*\right)$. Differentiate $t = \left(w_b^* - w_a^*\right)$ with respect to k and obtain $\frac{\partial w_a^*}{\partial k} = \frac{\partial w_b^*}{\partial k} > 0$. A higher wholesale price in towns implies a higher retail price in towns.

(d) Recall that $\overline{g}(t,k)$ is defined such that $\pi^*\left(I_a = 1, \overline{g}(t,k), t, k\right) \equiv \pi^*\left(I_a = 0, t, k\right)$. Differentiate the identity with respect to k and obtain: $\frac{\partial \pi^*(I_a=1,\overline{g}(t,k),t,k)}{\partial k} + \frac{\partial \pi^*(I_a=1,\overline{g}(t,k),t,k)}{\partial g} \cdot \frac{\partial \overline{g}(t,k)}{\partial k} = \frac{\partial \pi^*(I_a=0,t,k)}{\partial k}$. Since $\frac{\partial \pi^*(I_a=1,\overline{g}(t,k),t,k)}{\partial g} = -n_a$, arrange the terms and obtain:

$$\frac{\partial \overline{g}(t,k)}{\partial k} = \frac{1}{n_a} \cdot \left(\frac{\partial \pi^*\left(I_a = 1, \overline{g}(t,k), t, k\right)}{\partial k} - \frac{\partial \pi^*\left(I_a = 0, t, k\right)}{\partial k} \right) \tag{25}$$

(25) implies:

$$\frac{\partial \overline{g}(t,k)}{\partial k} \leq 0 \text{ if and only if } \frac{\partial \pi^*\left(I_a = 1, \overline{g}(t,k), t, k\right)}{\partial k} \leq \frac{\partial \pi^*\left(I_a = 0, t, k\right)}{\partial k} \tag{26}$$

By the Envelope Theorem, we obtain: $\frac{\partial \pi^*(I_a=1,\overline{g}(t,k),t,k)}{\partial k} = \frac{\partial \pi(I_a=1,w_a,w_b,\overline{g}(t,k),t,k)}{\partial k} \mid_{w_a=w_a^*(I_a=1,t,k),w_b=}$ $\frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b=w_b^*(I_a=1,t,k)}$ and $\frac{\partial \pi^*(I_a=0,t,k)}{\partial k} = \frac{\partial \pi(I_a=0,w_a,w_b,t,k)}{\partial k} \mid_{w_a=w_a^*(I_a=0,t,k),w_b=w_b^*(I_a=0,t,k)} = \frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b=w_b^*(I_a=0,t,k)}$, where π_b is the manufacturer's profit from a city. Thus, (25) becomes:

$$\frac{\partial \overline{g}(t,k)}{\partial k} \leq 0 \text{ if and only if } \frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b=w_b^*(I_a=1,t,k)} \leq \frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b=w_b^*(I_a=0,t,k)} \tag{27}$$

From the proof of part (b), we know that $\frac{\partial \pi_b(w_b,k')}{\partial w_b} > \frac{\partial \pi_b(w_b,k)}{\partial w_b}$ evaluated at $w_b = w_b^*$. Let $k' \equiv k + \Delta k$ and take limit $\Delta k \to 0$, and obtain $\frac{\partial}{\partial k}\left(\frac{\partial \pi_b(w_b^*,k)}{\partial w_b}\right) > 0 \Longleftrightarrow \frac{\partial}{\partial w_b}\left(\frac{\partial \pi_b(w_b^*,k)}{\partial k}\right) > 0$. This says that $\frac{\partial \pi_b(w_b,k)}{\partial k}$ is increasing in w_b at the neighborhood of w_b^*. On the other hand, the Kuhn Tucker conditions in (19)–(21) imply that $w_b^*(I_a = 1) \leq w_b^*(I_a = 0)$. Thus, obtain in general $\frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b = w_b^*(I_a=1,t,k)} \leq \frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b = w_b^*(I_a=0,t,k)} \Longleftrightarrow \frac{\partial \bar{g}(t,k)}{\partial k} \leq 0$.

(e) Recall that $\bar{t}(g,k)$ is the transportation cost such that the manufacturer is indifferent between selling to both towns and cities, with the constraint, and selling only to cities, without the constraint. Let $\Pi_b(w_b,k)$ be the manufacturer's profit selling only to cities, and $\widehat{w}_b(k)$ solves $\max_{w_b} \Pi_b(w_b,k) \equiv n_b \cdot (w_b - c) \cdot Q_b^*(w_b,k)$. Let $\pi(g,t,k)$ be the manufacturer's profit selling to both towns and cities, and w_a^* and w_b^* solve the manufacturer's problem in (18). Thus, $\bar{t}(g,k)$ is defined such that $\pi^*\left(g,\bar{t}(g,k),k\right) \equiv \Pi_b^*(k)$, where $\Pi_b^*(k) \equiv \Pi_b(\widehat{w}_b(k),k)$.

Differentiate the identity and obtain: $\frac{\partial \pi^*\left(g,\bar{t}(g,k),k\right)}{\partial k} + \frac{\partial \pi^*\left(g,\bar{t}(g,k),k\right)}{\partial t} \cdot \frac{\partial \bar{t}(g,k)}{\partial k} = \frac{\partial \Pi_b^*(k)}{\partial k}$. By the Envelope theorem, $\frac{\partial \pi^*\left(g,\bar{t}(g,k),k\right)}{\partial t} = \lambda^*\left(g,\bar{t}(g,k),k\right)$, and thus, for $\lambda^* > 0$:

$$\frac{\partial \bar{t}(g,k)}{\partial k} = \frac{1}{\lambda^*\left(g,\bar{t}(g,k),k\right)} \cdot \left(\frac{\partial \Pi_b^*(k)}{\partial k} - \frac{\partial \pi^*\left(g,\bar{t}(g,k),k\right)}{\partial k} \right) \tag{28}$$

(28) implies that for $\lambda^* > 0$:

$$\frac{\partial \bar{t}(g,k)}{\partial k} \geq 0 \text{ if and only if } \frac{\partial \Pi_b^*(k)}{\partial k} \geq \frac{\partial \pi^*\left(g,\bar{t}(g,k),k\right)}{\partial k} \tag{29}$$

By the Envelope Theorem, obtain that $\frac{\partial \Pi_b^*(k)}{\partial k} = \frac{\partial \Pi_b(w_b,k)}{\partial k} \mid_{w_b = \widehat{w}_b(k)}$, and also that $\frac{\partial \pi^*\left(I_a,g,\bar{t}(g,k),k\right)}{\partial k} = \frac{\partial \pi\left(I_a,w_a,w_b,g,\bar{t}(g,k),k\right)}{\partial k} \mid_{w_a = w_a^*(I_a,t,k), w_b = w_b^*\left(I_a,\bar{t},k\right)}$

$= \left[n_b \cdot (w_b - c) \cdot \frac{\partial Q_b^*(w_b,k)}{\partial k}\right] \mid_{w_b = w_b^*\left(I_a,\bar{t},k\right)} = \frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b = w_b^*\left(I_a,\bar{t},k\right)}$. Thus, (29) becomes:

$$\frac{\partial \bar{t}(g,k)}{\partial k} \geq 0 \text{ if and only if } \frac{\partial \Pi_b(w_b,k)}{\partial k} \mid_{w_b = \widehat{w}_b(k)} \geq \frac{\partial \pi_b(w_b,k)}{\partial k} \mid_{w_b = w_b^*\left(I_a,\bar{t},k\right)} \tag{30}$$

Notice that $\pi_b(w_b, k) = \Pi_b(w_b, k)$, for all w_b and k, and thus $\frac{\partial \pi_b(w_b,k)}{\partial k} = \frac{\partial \Pi_b(w_b,k)}{\partial k}$. From the proof of part (d), we know that $\frac{\partial \pi_b(w_b,k)}{\partial k}$ is increasing in w_b at the neighborhood of w_b^*. The Kuhn Tucker conditions in (19)–(21) imply $w_b^*(I_a = 1) \leq w_b^*(I_a = 0) < \widehat{w}_b$. Thus, obtain in general $\frac{\partial \pi_b(w_b,k)}{\partial k} |_{w_b=w_b^*(I_a=1,\bar{t},k)} \leq \frac{\partial \pi_b(w_b,k)}{\partial k} |_{w_b=w_b^*(I_a=0,\bar{t},k)} \leq \frac{\partial \Pi_b(w_b,k)}{\partial k} |_{w_b=\widehat{w}_b(k)} \Longleftrightarrow \frac{\partial \bar{t}(g,k)}{\partial k} \geq 0$. □

P roposition 4. (a) (i) If Region $\varnothing$ exists (i.e., the region in which the manufacturer's profit selling to both towns and cities is less than his profit selling only to cities is non-empty) and the manufacturer sells to both towns and cities (i.e., $t \geq \bar{t}(g, k)$), the prohibition of price discrimination causes the manufacturer to close down the markets in towns. The welfare change is Pareto inferior. (ii) If Region $\varnothing$ exists and the manufacturer sells only to cities (i.e., $t < \bar{t}(g, k)$), the allowance of price discrimination does not cause the manufacturer to open up the markets in towns. To induce the manufacturer to sell to towns, the government should allow both price discrimination and exclusive territories. The welfare change yields a Pareto improvement. (b) If Region $\varnothing$ does not exists and the manufacturer uses two-part tariffs in towns, with $g > \overline{g}(t = 0)$, the prohibition of price discrimination causes the manufacturer to switch to use linear pricing contracts in towns. As a result, the manufacturer and consumers in both towns and cities are worse off.

Proof: (a) (i) If price discrimination is prohibited, $s = 0$ and the manufacturer faces constraint $0 = (w_b^* - w_a^*)$. By the Envelope theorem, the manufacturer's profit is decreasing in the transportation cost, and thus the manufacturer is strictly worse off with $t = 0$ or $s = 0$. Since Region $\varnothing$ exists, the manufacturer does not sell to towns with $s = 0$. Town retailers and town consumers are strictly worse off because the market disappears. Since the manufacturer does not sell to towns, it charges the monopoly price in cities. A higher wholesale price in cities reduces the retailers' profits and consumer surplus in cities. (a) (ii) Since Region $\varnothing$ exists and the manufacturer sells only to cities, $t < \bar{t}(g, k)$. Even though the government allows price discrimination, the arbitrage constraint is still binding. If the government allows both price discrimination and exclusive territories, the manufacturer can use the vertical restraint to prevent arbitrage, and gains additional profit if he sells also to towns. (b) Since the manufacturer uses two-part tariff contracts, the manufacturer's choice of contracts falls in Region II or IV. If the government prohibits price discrimination, the manufacturer's choice of contracts falls into Region I, because $g > \overline{g}(t = 0, k)$. By the Envelope theorem, the manufacturer is strictly worse off. With linear contracts, town consumers are worse off. Since the constraint is binding in Region I, the wholesale price in cities is higher with linear contracts. Retailers and consumers in cities are worse off. $\square$

10 Reference

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