October 11, 2018

Notes on the Shutdown Decision

1. Teaching the Decision to Shut Down if $P < Min(AVC)$

The decision to shut down if $P < Min(AVC)$ is hard to teach and hard to learn. Below I've written up two numerical examples that might help. The first is a cubic equation. Someone could improve it by making it work out to more even numbers, perhaps. The second is a piecewise equation with a V-shaped AVC and a discontinuous MC (a kinked TC curve). The numbers work out to be even, but the piecewiseness is kludgy and the discontinuity will perturb students— though it would actually be a good thing for helping them to understand the intuition.

I think what might be even better would be a discrete numerical example that would allow more of a storytelling approach.

2. U-Shaped Marginal Costs

Figure 1:
The Total Cost Curve $TC = 144 + 32Q - 8Q^2 + Q^3$

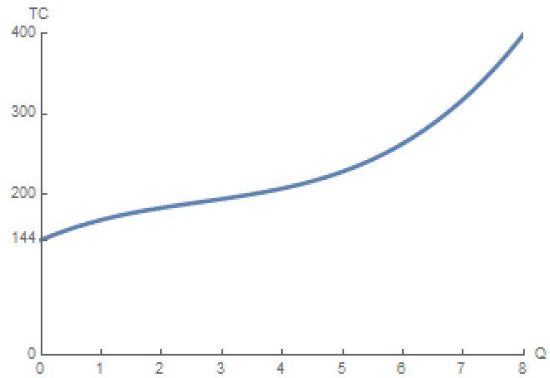
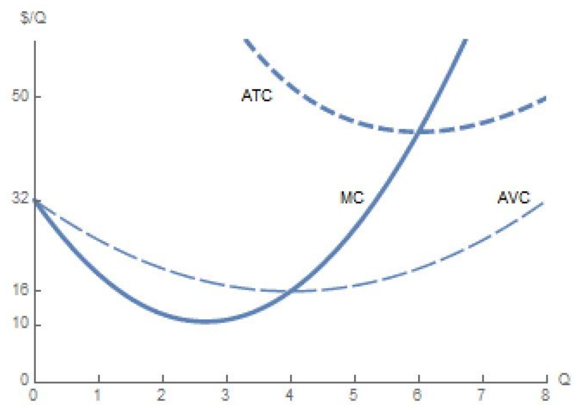


Figure 2:
The Average Total Cost, Average Variable Cost, and Marginal Cost Curves



Suppose the total cost is

$$\text{Total Cost} = 144 + 32Q - 8Q^2 + Q^3 \tag{1}$$

The average total cost is $\frac{\text{Total Cost}}{Q}$, so it is

$$\text{Average Total Cost} = \frac{144}{Q} + 32 - 8Q + Q^2 \tag{2}$$

The minimum of the average total cost is found by differentiating it with respect to Q and setting the derivative equal to zero— finding the "bottom of the valley". Doing that we get, since
$ATC = \frac{144}{Q} + 32 - 8Q + Q^2$ means that
$ATC = 144Q^{-1} + 32 - 8Q + Q^2$,

$$\begin{aligned}
\frac{d\text{ATCt}}{dQ} &= (-1)144Q^{-1-1} - 8 + 2Q = 0 \\
&= \frac{-144}{Q^2} - 8 + 2Q = 0 \\
&\rightarrow -8Q^2 + 2Q^3 = 144 \\
&\rightarrow Q^3 - 4Q^2 = 72 \\
&\rightarrow Q = 6
\end{aligned}$$

$$\text{The minimum ATC} = \frac{144}{6} + 32 - 8 \cdot 6 + 6^2 = 24 + 32 - 48 + 36 = 44 \tag{3}$$

The price will have to rise to $P = 44$ for the firm to make a long-run profit, covering its fixed costs as well as its variable costs.

The fixed cost is 144, and the variable cost is whatever is left over. Thus

$$\text{Average Variable Cost} = 32 - 8Q + Q^2 \tag{4}$$

The minimum of the average average cost is also found by differentiating it with respect to Q and setting the derivative equal to zero— finding the "bottom of the valley". Doing that we get

$$\begin{aligned}
&\frac{d\text{AVC}}{dQ} = -8 + 2Q = 0 \\
&Q = 4 \text{ has the minimum AVC} = 32 - 8 \cdot 4 + 4^2 = 16
\end{aligned} \tag{5}$$

The marginal cost is the derivative of total cost, $TC = 144 + 32Q - 8Q^2 + Q^3$, so it is

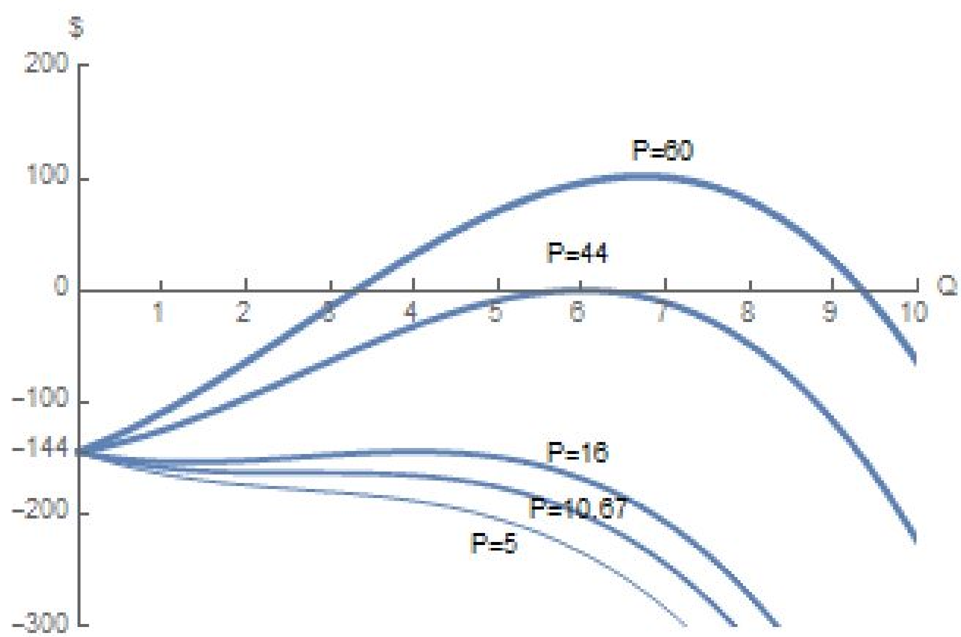
$$\text{Marginal Cost} = \frac{d\text{TC}}{dQ} = 32 - 16Q + 3Q^2 \tag{6}$$

The minimum of the marginal cost is also found by differentiating it with respect to Q and setting the derivative equal to zero— finding the "bottom of the valley". Doing that we get

$$\begin{aligned}
\tfrac{d\text{MC}}{dQ} &= -16 + 6Q = 0 \\
&\rightarrow Q = \tfrac{16}{6} = 2\tfrac{2}{3} \\
\text{with } MC &= 32 - 16(8/3) + 3 \cdot (8/3)^2 = 32 - 128/3 + 3(64/9) = 32 - 128/3 + 64/3 = 96/3 \\
&= 10\tfrac{2}{3}
\end{aligned} \tag{7}$$

What is the optimal output for the firm? That depends on the market price. If the price is zero, the firm should produce $Q = 0$, for profits of -144. It has to pay the fixed cost of 144, but the marginal cost is 16 at $Q = 0$, so it isn't worth producing anything. Clearly, if $P < 10\frac{2}{3}$, the firm should produce $Q = 0$. In Figure 3, for example, the $P = 5$ curve shows that profit is always negative, for any Q, but it's least negative at $Q = 0$, where profit is -144, and just gets lower as output is increased.

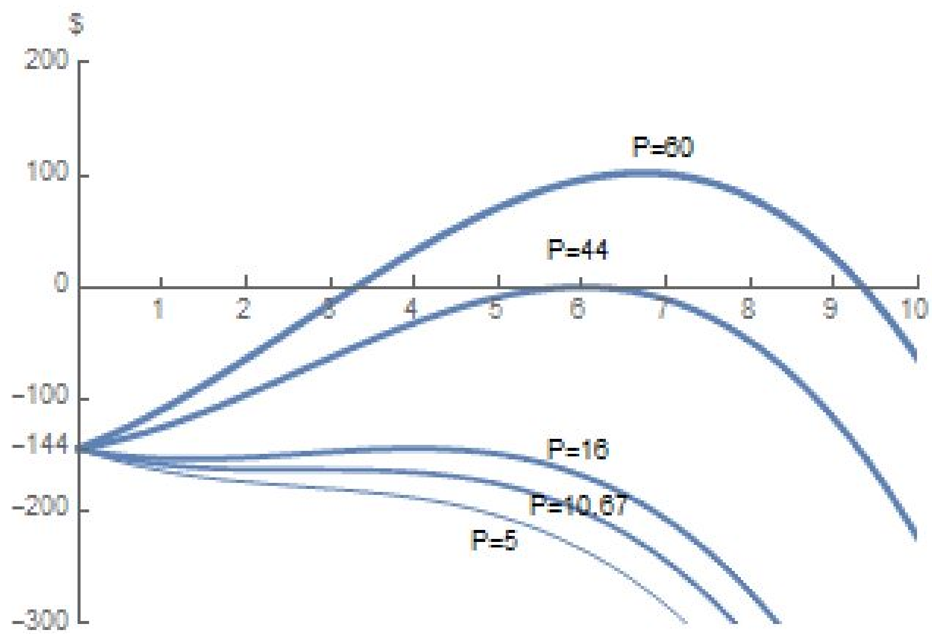
FIGURE 3:
PROFIT AS A FUNCTION OF OUTPUT IF THE FIRM USES THE MC(Q) = P RULE BLINDLY



How about if the price rises to $P = 10\frac{2}{3}$, which is $Min(MC)$? The $MC(Q) = P$ rule is still misleading, because each unit except the last has a marginal cost less than the price. We'd get $Q = 2\frac{2}{3}$, what we found earlier was the Q that minimized $MC(Q)$, and profit would be -191. It's still best to produce $Q = 0$.

How about if the price rises to $P = 16$, which is $Min(AVC)$? The $MC(Q) = P$ rule now starts to work properly. We'd get $Q = 4$, what we found earlier was the Q that minimized $AVC(Q)$, and profit would be -144, which is at least just as good as producing $Q = 0$. Notice in Figure 3 that the profit curve is slightly falling starting from $Q = 0$, but then is slightly rising up to $Q = 4$, after which it starts to fall sharply. The firm should pick $Q = 0$ or $Q = 4$, but not some output in between.

Figure 3: (repeated)
Profit As a Function of Output if the Firm Uses the MC(Q) = P Rule Blindly



How about if the price rises to $P = 44$? The $MC(Q) = P$ rule tells us to pick $Q = 6$, what we found earlier was the Q that minimized $ATC(Q)$, and profit would be 0. Hurrah! We break even. And producing $Q = 6$ is much better than producing $Q = 0$. The firm will want to keep in existence even in the long run if we can expect $P = 44$ to continue.

If $P = 44$, things get even better. The $MC(Q) = P$ rule tells us to pick $Q \approx 6.8$, looking at Figure 3, and profit would be over 100.

Thus, the supply curve should have $Q(P) = 0$ if the price is below $10\frac{2}{3}$ and then it should trace out the MC. We have $P = MC = 32 - 16Q + 3Q^2$, so we have $P(Q) = 32 - 16Q + 3Q^2$, which we need to invert to get $Q(P)$ instead of $P(Q)$. One (even"you" if you had time) can do that with high school's quadratic formula, but I used the computer language Python,[1] and arrived at $Q(P) = 2\frac{2}{3} + \frac{\sqrt{3P-32}}{3}$. The supply equation is therefore

$$\begin{aligned} Q^{supply}(P) \quad = \quad & 0 & if \;\; P \leq 10\tfrac{2}{3} \\ & 2\tfrac{2}{3} + \tfrac{\sqrt{3P-32}}{3} & if \;\; P \geq 10\tfrac{2}{3}. \end{aligned} \tag{8}$$

[1]The Python 2 code is:

```
from sympy import *
from sympy.abc import * #This makes each letter and greek letter name a symbol
answer = solve( 32 - 16*Q + 3*Q**2-p, Q)
print("answer = ", answer , "\n")
```

3. V-Shaped Marginal Costs

Suppose the total cost is

$$\text{Total Cost} = \begin{cases} 49 + 16Q - Q^2 & if \;\; Q \leq 6 \\ 49 + 4Q + Q^2 & if \;\; Q \geq 6 \end{cases} \tag{9}$$

The average total cost is $\frac{\text{Total Cost}}{Q}$, so it is

$$\text{Average Total Cost} = \begin{cases} \frac{49}{Q} + 16 - Q & if \;\; Q \leq 6 \\ \frac{49}{Q} + 4 + Q & if \;\; Q \geq 6 \end{cases} \tag{10}$$

Figure 3:
The Total Cost Curve, for V-Shaped Average Variable Cost

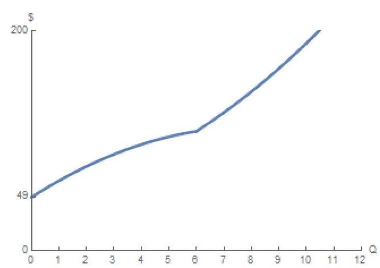
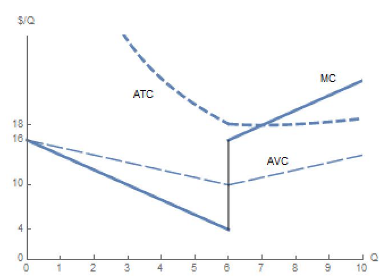


Figure 4:
The Average Total Cost, Average Variable Cost, and Marginal Cost Curves



The minimum of the average total cost is found by differentiating it with respect to Q and setting the derivative equal to zero— finding the "bottom of the valley". Doing that we get, since the minimum will be at a value greater than $Q = 6$ and $ATC = \frac{49}{Q} + 4 + Q = 49Q^{-1} + 4 + Q$,

$$\frac{d\text{Average Total Cost}}{dQ} = (-1)49Q^{-1-1} + 1 = 0$$

$$\frac{d\text{Average Total Cost}}{dQ} = -\frac{49}{Q^2} + 1 = 0 \tag{11}$$

$$49 = Q^2$$

$$Q = \sqrt{49} = 7 \text{ has the minimum ATC} = 7 + 4 + 7 = 18$$

This means the fixed cost is 49, and the variable cost is whatever is left over. Thus

$$\begin{aligned} \text{Average Variable Cost} &= 16 - Q \quad if \;\; Q \leq 6 \\ & \quad\; 4 + Q \quad\;\; if \;\; Q \geq 6 \end{aligned} \tag{12}$$

The average variable cost is V-shaped, and its minimum is 10, at $Q = 6$. Its minimum is lower than ATC's because it doesn't included the fixed cost. It's at a lower Q because it doesn't need to spread the fixed cost over more units to get a minimum.

The marginal cost is the derivative of total cost, so it is

$$\begin{aligned} \text{Marginal Cost} &= 16 - 2Q \quad if \;\; Q \leq 6 \\ & \quad\; 4 + 2Q \quad\;\; if \;\; Q > 6 \end{aligned} \tag{13}$$

The marginal cost is V-shaped, and its minimum is 4, at $Q = 6$. Its minimum is lower than AVC's because it doesn't average in all of the high values of marginal cost for $Q < 6$.

Note that the marginal cost jumps at 6, from MC=4 to MC = 16. There is a kink in the total cost curve at $Q = 6$, so the slope suddenly increases.

What is the optimal output for the firm? That depends on the market price. If the price is zero, the firm should produce $Q = 0$, for profits of -49. It has to pay the fixed cost of 49, but the marginal cost is 16 at $Q = 0$, so it isn't worth producing anything. Clearly, if $P < 4$, the firm should produce $Q = 0$.

How about if the price rises to $P = 4$? Then we could use the rule of picking Q so $MC(Q) = P$. Doing that, we'd have $MC = 16 - 2Q = 4$, so $12 = 2Q$, and $Q = 6$. But this would be a very bad idea, because $P < Min(AVC) = 10$. The profit at $Q = 6$ would be $PQ - TC(Q) = (4)(6) - (49 + 16(6) - 6^2) = 24 - 49 - 96 + 36 = -85$. That's lower than the profit of -49 from $Q = 0$. It is better if the firm continues to produce $Q = 0$.

That's continues to be true up until the price reaches $P = 10$, so $P = Min(AVC) = 10$. At that price, producing $Q = 6$ yields a profit of $PQ - TC(Q) = (10)(6) - (49 + 16(6) - 6^2) = 24 - 49 - 96 + 36 = -49$. That's the same profit as from $Q = 0$.

What if the price rises to $P = 11$? The intersection of the MC and market price curves continues to be at $Q = 6$. The firm should not increase output beyond $Q = 6$, because the marginal cost immediately jumps to $MC = 16$. So the elasticity of supply is 0 at $P = 11$. In fact, until the price rises to 16, the firm should stick with $Q = 6$.

If the price rises above 16, though, then at $Q = 6$ the marginal cost of 16 would be less than the price, so the firm should start increasing its output beyond 6. If $P = 20$, for example, the firm should pick output so $MC(Q) = 4 + 2Q = P = 20$, so $2Q = 16$ and $Q = 8$. Solving that rule for P, we get the supply curve $Q = \frac{P}{2} - 2$ for $P \geq 16$.

Thus, the supply curve is:

$$\begin{aligned} \text{Q}^{\text{supply}} \;=\;& 0 && if \;\; P \leq 10 \\ & 6 && if \;\; P \in [10, 16] \\ & \tfrac{P}{2} - 2 && if \;\; P \geq 16. \end{aligned} \tag{14}$$