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Players 1 and 2 are splitting a pie of size 1. Each simultaneously chooses a toughness level x_i in $[0, \infty)$. With probability $p(x_1, x_2)$, bargaining fails and each ends up with a payoff of zero. Otherwise, player 1 receives $\pi(x_1, x_2)$ and Player 2 receives $1 - \pi(x_1, x_2)$.

Example 1: The Basics. Let $p(x_1, x_2) = \frac{x_1+x_2}{12}$ and $\pi(x_1, x_2) = \frac{x_1}{x_1+x_2}$. Equilibrium: $x_1 = x_2 = 3$, $p = 50\%$. Each player's expected payoff is .25.

General Model. Effort costs $c(x_i)$ for player i, with $c \geq 0$, $c' \geq 0, c'' \geq 0$.
Utility is quasinlinear: $u_1(\pi) - c(x_1)$ and $u_2(\pi) - c(x_2)$ with $u_1' > 0, u_1'' \leq 0$ and $u_2' < 0, u_2'' \leq 0$.
Breakdown: $p_1 > 0$ $p_2 > 0$, $p_{11} \geq 0$, $p_{22} \geq 0$, and $p_{12} \geq 0$ for all values of x_1, x_2 such that $p < 1$, and $p_1 = p_2 = 0$ for greater values. $p(a, b) = p(b, a)$.
Player 1's share: $\pi \in [0, 1]$, $\pi_1 > 0$, $\pi_{11} \leq 0$, and $\pi_{12} \geq 0$. $\pi(a, b) = 1 - \pi(b, a)$.
These assumptions on π imply that $lim_{x_1 \to \infty} \pi_1 \to 0$, since $\pi_1 > 0$, $\pi_{11} \leq 0$, and $\pi \leq 1$.

Proposition 1. *The general model has a unique Nash equilibrium, and that equilibrium is in pure strategies with a 50-50 split of the surplus:* $x_1^* = x_2^*$ *and* $\pi(x_1^*, x_2^*) = .5$.

Example 2: A Vanishingly Small Probability of Breakdown. Keep $\pi(x_1, x_2) = \frac{x_1}{x_1+x_2}$ but let the breakdown probability be $p(x_1, x_2) = \frac{(x_1+x_2)^k}{12k^2}$ for k to be chosen. In equilibrium, each player's expected payoff is close to .5.

N Players. Player i's payoff function is $Payoff(i) = (1 - \frac{\sum_{i=1}^N x_i}{12}) \frac{x_i}{\sum_{i=1}^N x_i}$. In equilibrium, $\pi = 1/N$ and $p(x, \ldots, x) = \frac{(N-1)}{N}$.

Example 3: Unequal Bargaining Power. Let the probability of breakdown be $p(x_1, x_2) = Min\{e^{(1-\theta)\beta x_1 + \theta \beta x_2} - 1, 1\}$, where $\theta \in [0, 1]$ is player 1's bargaining power and $\beta > 0$ is a parameter for breakdown risk. Let player 1's share of the pie be $\pi(x_1, x_2) = .5 + (x_1 - x_2)$. In equilibrium, player 1's share is θ and player 2's is $1 - \theta$.

Proposition 2: *If player 1 is more risk averse than player 2, his share is smaller in equilibrium.*

Proposition 3. *In the multiperiod bargaining game, a player's toughness and equilibrium share falls in his discount rate.*

Example 6: Player 1 Has an Outside Option of z. As in Example 1, let the breakdown probability be $p(x_1, x_2) = \frac{x_1+x_2}{12}$ and player 1's share be $\pi(x_1, x_2) = \frac{x_1}{x_1+x_2}$. Player 1 has an outside option of z, a payoff he receives if bargaining breaks down. In equilibrium, player 1's share is $\frac{1}{2-z}$.

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