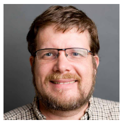 

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March 3, 2021

Splitting a Pie: Mixed Strategies in Bargaining under Complete Information
(link to seminar web page with slides and handouts and the paper or go via https://www.rasmusen.org/rasmapedia
)

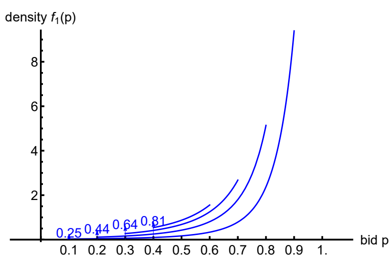
The Model Players 1 and 2 simultaneously choose bids p_1 and p_2 in the interval [0,1]. If their bids add up to more than 100%, both get zero and we say the pie "explodes". Otherwise, when the bids are $p_1 + p_2 \leq 1$, player 1 gets share $p_1/(p_1+p_2)$ as his payoff and player 2 gets share $p_2/(p_1+p_2)$.

This game has a continuum of pure strategy Nash equilibria, every permutation such that $p_1 + p_2 = 1$, and the pie never explodes.

One of the Hawk-Dove Equilibria: Each player chooses .30 with probability .6 and .70 with probability .4 The expected payoff is .42. With probability .16, they both choose .70 and the pie explodes.

We will focus on a particularly interesting kind of equilibrium, that in which the players bid by mixing over the intervals $[a, b]$ and $[c, d]$ using strictly positive probability densities $f_1(p_1)$ and $f_2(p_2)$, possibly with atoms of probability on particular bids.

Most simple is the symmetric equilibrium, in which both mix over the same interval $[a, 1-a]$ using the same density $f(p)$. For example, both might mix over [.4, .6], with an atom at .4 and a density rising from .4 to .6, as in the figure.



The Symmetric-Equilibrium Mixing Density $f_1(p)$ and Atom K_1 for $a = .10, .20, .30, .40$

Proposition 1: Any equilibrium in which each player mixes over an interval
(a) consists of probability atom K_1 at a and density $f_1(p)$ on $[a, b]$ for Player 1
(b) and of probability atom K_2 at $1-b$ and density $f_2(q)$ on $[1-b, 1-a]$ for Player 2.
(c) The densities are positive throughout, and if their derivatives exist, the densities start strictly positive with $f_1(a) = \left(\frac{a}{1-a}\right) K_1$ and $f_2(1-b) = \left(\frac{1-b}{b}\right) K_2$
(d) and increase convexly
(e) with positive derivatives of every order.

The fundamental equation that characterizes the equilibrium then says that Player 1's f_1 and K_1 must be such that Player 2's expected payoff from any q in his support equals that from playing $q = 1-a$:

$$\pi_2(q) = K_1 \left(\frac{q}{q+a}\right) + \int_a^{1-q} \left(\frac{q}{q+p}\right) f_1(p)dp = K_1(1-a) \quad (\textit{the crucial equation}) \tag{1}$$

Then:

$$\frac{d\pi_2(q)}{dq} = \frac{aK_1}{(q+a)^2} + \int_a^{1-q} \left(\frac{p}{(p+q)^2}\right) f_1(p)dp - qf_1(1-q) = 0. \tag{2}$$