The Transfer Paradox

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Abstract

The transfer paradox occurs when one country transfers wealth to another and the result once prices reach their new equilibrium is that the giver is better off and the recipient worse off. We revisit this classic paradox, distinguishing five different ways it can occur and offering intuition and numerical examples not available in the literature

Keywords: transfer paradox, multiple equilibria, unstable equilibrium, terms of trade, immiserizing growth

JEL classification:

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This paper: `http://www.rasmusen.org/papers/transfer-kang-rasmusen.pdf`.

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1. Introduction

In certain circumstances , if country 1 transfers wealth to country 2, it is possible that country 1's income and utility rises while country 2's falls. This is the well-known transfer paradox. The paradox can occur in a world with just two countries, but only if multiple equilibria exist. One way is if the transfer induces a large shift in expectations from one equilibrium to another. A second way is if the world begins at an unstable equilibrium, one in which aggregate demand for a good rises with its price near the equilibrium. For quite different reasons, the transfer paradox also can arise in a world with three or more countries even if the equilibrium is unique. In that case it arises because different countries have different marginal propensities to consume different goods and the transfer can change the terms of trade enough in the donor's favor to outweigh the direct loss from giving away wealth.

All this is well known. We have the modest aim in this paper of explaining the intuitions behind the transfer paradox and of presenting numerical examples for the two-country paradox and three versions of the three-country one. The intuitions are somewhat different for each of the five equilibrium shifts we will examine, and while previous papers have established that all five exist, they have not been written with an eye to explaining the economics behind the results. We offer this paper in the belief that the reader's understanding of how the transfer paradox works is important to his learning from it.

2. The Literature

Sergio Turner (2006) has an excellent history of the literature. The phrase "transfer problem" was introduced by Keynes (1929) and Ohlin (1929) in a debate over the effect of German reparation payments to the Allies after World War I. Leontief (1936) came up with the first rigorous example. Samuelson (1947 footnote at p. 29, 1952, 1954) conjectured that the effect only arises when the equilibrium is unstable under tatonnement, and Balasko (1978, 2014) has proved a version of his conjecture for the case of n goods. Gale (1974), however, showed that the paradox can also arise for completely different reasons even for stable equilibria, so long as the model has three or more countries. He says Jaskold-Gabszewicz & Dreze (1971) have a similar example. A flurry of papers in the early 1980's picked up on Gale's three-country paradox. Chichilnisky (1980, 1983), Geanakoplos and Heal (1983), Polemarchakis (1983), Leònard and Manning (1983), Postlewaite and Webb (1984).

The term "weak transfer paradox" covers the cases where both donor and recipient are helped by the transfer or both lose by the transfer. The term "strong transfer paradox covers the case where the donor is helped and the recipient is hurt. Yano (1983) shows how all the combinations of gains and losses to donor, recipient, and third party are possible for appropriate parameters. A separate classification, from Balasko (1978), is between the "local transfer paradox", where the transfer causes only a continuous change in the equilibrium played out and the "global transfer paradox" where any size of transfer causes a shift from one to another of multiple equilibria.

3. The Transfer Paradox with Two Countries

With only two countries, the transfer paradox can occur in two ways. The first way is by changing the level of the price at an unstable equilibrium, an equilibrium where aggregate demand for a good rises with its price. Such an equilibrium is on a knife edge; if even a small shock to the system occurs, tatonnement would carry the economy to a stable equilibrium with a much higher or lower price. The second way is if the transfer changes expectations and shifts the economy from one stable equilibrium to the other, shifting from the recipient's preferred equilibrium to the donor's. There, the donation merely acts as an expectations shifter, a function which could equally well be performed by cheap talk or sunspots.

Our example adapts Shapley & Shubik's 1977 example of a smooth economy with multiple equilibria, relabelled in our setting and with the addition of a transfer from one agent to the other. There are two countries. Country 1 has 50 apples and desires butter.[2] Country 2 has 40 butter and desires apples. Country 1 transfers amount t of apples to Country 2. The utility functions ($a = apples$, $b = butter$) are

$$u_1(a_1, b_1) = a_1 + 110(1 - e^{-b_1/10}) \tag{1}$$

and

$$u_2(a_2, b_2) = b_2 + 100(1 - e^{-a_2/10}) \tag{2}$$

We will normalize by setting $p_a \equiv 1$. For optimal consumption, the ratio of a country's marginal utilities from the two products must equal the ratio of the prices, so for Country 1,

$$\frac{\partial u_1/\partial a_1}{\partial u_1/\partial b_1} = \frac{1}{11e^{-b_1/10}} = \frac{1}{p_b} \tag{3}$$

[2] xxx We maybe should change this to be consistent with the 3-country examples so country 1 has apples, not butter. We'll have to exchange the names apple and butter where they appear, and change p to 1/p and change the diagrams. Or would it be easier to change the 3-county examples to have apples as Country 1's export good? then the diagrams wouldn't have to be changed.

and for country 2,

$$\frac{\partial u_2/\partial b_2}{\partial u_2/\partial a_2} = \frac{1}{10e^{-a_2/10}} = \frac{p_b}{1}. \tag{4}$$

The endowments are

$$(\overline{a}_1, \overline{b}_1) = (50 - t, 0), \tag{5}$$

and

$$(\overline{a}_2, \overline{b}_2) = (t, 40), \tag{6}$$

so the budget constraints are

$$a_1 + b_1 p_b = 50 - t, \tag{7}$$

and

$$a_2 + b_2 p_b = 40 p_b + t. \tag{8}$$

Solving a country's marginal rate of substitution equation for its consumption of each good using its budget constrain yields its demand curves:

$$a_1 = 50 - t - 10 p_b \log \frac{11}{p_b} \tag{9}$$

$$b_1 = 10 \log \frac{11}{p_b} \quad \text{for} \quad p_b \in [xx, yy] \tag{10}$$

<< From MW: I have checked the demand for b_1 and a_2. They are always positive for any $p_b > 0$. Therefore, we do not need to range "$p_b \in [xx, yy]$". "a_1" can be negative for some range of p_b only if "t" is very large. The numerical solution shows that if t>9.3, for some open set of p_b, a_1 can be negative. However, we only consider the small transfer, t=0.5. >>

and

$$a_2 = 10 \log 10 p_b \quad \text{for} \quad p_b \in [xx, yy] \tag{11}$$

$$b_2 = \frac{40 p_b + t - 10 \log (10 p_b)}{p_b} \tag{12}$$

Demand functions b_1 and a_2 can take negative values, so their range must be restricted, though that restriction is not binding at equilibrium prices. This happens because for each country the marginal utility of one good is constant, so after enough consumption of the other good the other good's marginal utility falls beneath the first good's and all further consumption is of the first good. The marginal rate of substitution condition is then invalid because there is a corner solution. This not satiation. The country would still get extra utility from consumption of the second good; it is just that it can get higher marginal utility

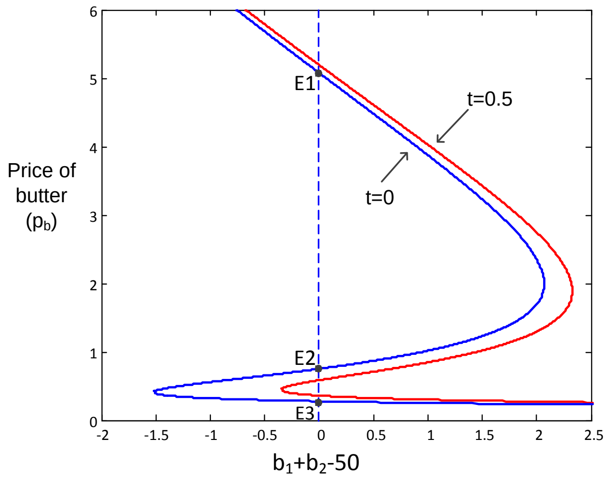


Figure 1: The World's Excess Demand for Butter

from the first good and so would prefer it if both had positive price.

Numerically solving equation of the excess demand for butter (or apples) being zero where $t = 0$ results in the three multiple equilibria shown in Shapley & Shubik (1977). We will assume that 1% of country 1's endowment is transferred to country 2, i.e., $t = 0.5$, and see what happens.

Figure 1 shows the excess demand for butter as the price of butter increases, with the blue curve being the pre-transfer excess demand function.

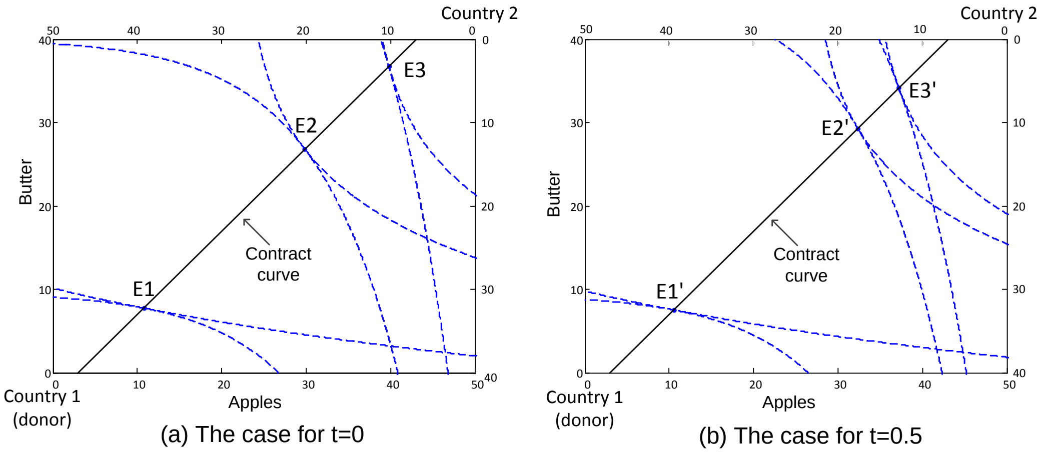
Figures 2 and 3 show the three equilibria in an Edgeworth box for the cases of $t = 0$ and $t = 0.5$. The interior contract curve can be derived from utility functions (1) and (2):

$$a_1 = b_1 + 50 - 10 \log (110) . \tag{13}$$

In Figure 2, the indifference curves are indicated by the dashed curves. The three equilibria are $E1$, $E2$ and $E3$ when $t = 0$, and $E1'$, $E2'$ and $E3'$ when $t = 0.5$. $E1, E3, E1'$ and $E3'$ are stable equilibria while $E2$ and $E2'$ are unstable.

Table 1 summarizes the equilibrium consumption and utilities before and after the transfer. Two types of the transfer problem can be observed in Table 1. Balasko (1978) distinguished the "local transfer paradox", in which the equilibrium selection map is continuous, with the "global transfer paradox", in which the selection map is permitted to be discontinuous.

The global transfer paradox occurs if and only if there exist multiple equilibria. It can be seen in Table 1. Country 1's utility in $E3'$ is higher than in $E1$ while country 2's utility in $E3'$ is lower than in $E3$, all of which are stable equilibria.



(a) The case for t=0 (b) The case for t=0.5

Figure 2: Edgeworth Boxes

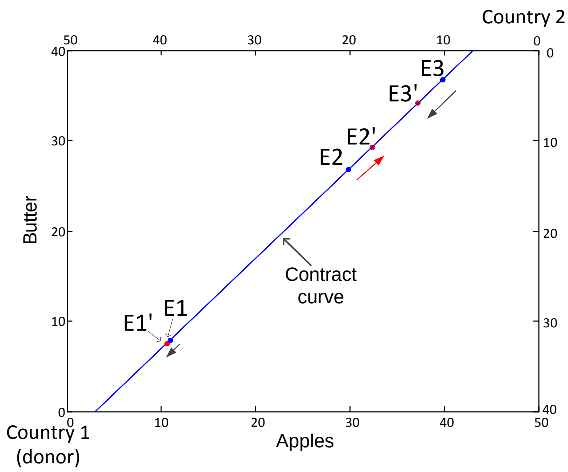


Figure 3: How the Equilibrium Consumptions Change

The local transfer paradox occurs if and only if there exists an unstable equilibrium. The utility of country 1 (the donor) in $E2'$ is higher than in $E2$ while the utility of country 2's (the recipient) in $E2'$ is lower than in $E2$.

TABLE 1:
TRANSFER PARADOX WITH TWO COUNTRIES

	Equilibrium 1		Equilibrium 2		Equilibrium 3	
	Ctry 1	Ctry 2	Ctry 1	Ctry 2	Ctry 1	Ctry 2
Original situation:						
Price of butter	$E1 : p_b = 5.07$		$E2 : p_b = 0.75$		$E3 : p_b = 0.28$	
Apple consumption (a_i)	10.74	39.26	29.82	20.18	39.77	10.22
Butter consumption (b_i)	7.74	32.26	26.83	13.17	36.78	3.22
Utility	70.01	130.29	132.30	99.87	146.99	67.26
After a transfer of $t = .5$ in apples:						
Price of butter	$E1' : p_b = 5.19$		$E2' : p_b = 0.59$		$E3' : p_b = 0.36$	
Apple consumption (a_i)	10.51	39.49	32.33	17.67	37.18	12.82
Butter consumption (b_i)	7.52	32.48	29.34	10.66	34.19	5.81
Utility	68.63	130.56	136.48	93.57	143.58	78.06

Note: By definition, $p_a \equiv 1$. Country 1 is the donor, with endowment $\bar{a}_1 = 50, \bar{b}_1 = 0$. Country 2 is the recipient, with endowment $\bar{a}_2 = 0, \bar{b}_2 = 40$.

What has happened here? Look at the excess demand function. The gift has reduced the demand for butter. Figure 1 shows that for any price, excess demand for butter is higher after the transfer.

This is because country 2's demand for butter depends on its wealth, but country 1's depends only on butter's price:

$$b_1 = 10\log(11/p_b) \tag{14}$$

$$b_2 = \frac{40p + t - 10\log(10p)}{p_b} \tag{15}$$

Country 1 wants to consume butter up to the level where its marginal utility of $10\log(11/p_b)$ equals p_b, which it can afford to do for any p_b in the relevant range (0,11) (the maximum of the cost, $10\log(11/p_b)$, is about 40.5, which is less than its income of 50). Thus, it will use any increase in income to buy apples. Country 2, on the other hand, uses its residual income to consume butter: if we increased its income to $40p_b$ from $40p_b + 2$, the extra 2 would go to buy butter, not apples. Thus, a transfer from country 1 to country 2 increases the demand

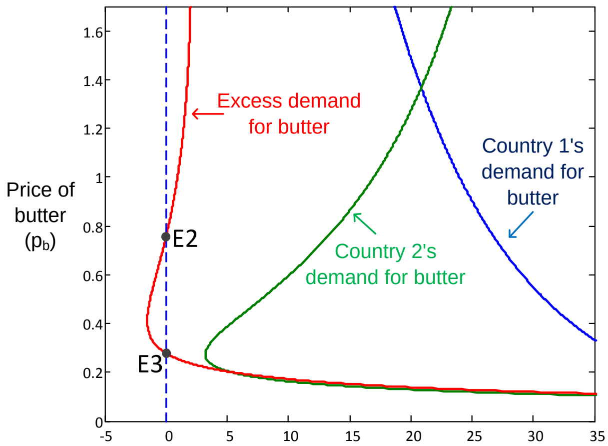


Figure 4: Demand for Butter Near the Unstable Equilibrium

for butter. As a result of the shift up in demand for butter, the prices at equilibria 1 and 3 rise.

Figure 4 is a close-up part of Figure 1, shows how the two countries' demands change with price. Initially, world demand for butter is very high because it is cheap and both countries demand a lot. Demand falls till about $p_b = .4$. It starts to rise then because of country 2, whose wealth is increasing and for whom butter is the income-elastic good. At some point, however, country 1's demand falls faster than country 2's demand increases, and excess demand eventually becomes positive again.

The unstable equilibrium lies in the region where country 2's demand for butter is increasing faster in price than country 1's is declining. Because of the transfer, country 2's influence on excess demand increases, so it rises at a faster rate with price. This means that excess demand reaches zero sooner— at a lower price for butter, which is what country 1 wants.

The transfer effect works precisely because the equilibrium is unstable, at a price where demand for butter is increasing in its price. Country 2's income effect is responsible for demand increasing with price, so if country 1 can increase country 2's income, demand for butter will rise faster and the price at which demand equals supply will be lower— which is what country 1 wants.

The increased income of country 1 is not affecting this process, unlike what we will see in the three-country case, where country 2's reduced demand for its export good and country 1's increased demand for the other good reinforce each other in pushing the relative price down. Country 1's demand is falling in price, with the income and the substitution effects both going in the same direction. Adding to country 1's income, however, does not affect its demand for butter, either in level or the rate at which it falls with price. This is special to the example, of course. More usually, adding to country 1's income would increase its demand for butter, which would run counter to country 2's desired discouragement of the demand.

4. The Transfer Paradox with Three Countries

The transfer paradox with two countries occurs only in somewhat peculiar circumstances. When there are three countries, the paradox is more plausible but it occurs for completely different reasons. The central idea is always that a transfer shifts the pattern of world consumption because countries differ in their tastes, and that the shift will result in new world prices that can benefit the donor country enough to outweigh the cost of the gift it makes. The way the terms of trade shift is somewhat different, though, in each of the three cases we will examine. In each, the donor country will benefit, but in example 1 the recipient country wins but the third party loses, in example 2 the recipient loses but the third party wins, and in example 3 both the recipient and the third party lose.

For each of three countries $i = 1, 2, 3$ let the endowment be $\bar{a}_i$ and $\bar{b}_i$ of apples and butter, let consumption be a_i and b_i, and let utility follow the Leontief function $u_i(a, b) = Min(\frac{a}{\alpha_i}, b)$. We will refer to α_i as country i's taste for apples; if it rises, the country needs more apples to reach the same utility level.[3]

The Leontief utility function is useful for illustrating the transfer paradox because changes in the quantity demanded of each good are determined entirely by the income effect, with zero substitution effect. The heart of the paradox is to use changes in income to reduce demand (and thus the price) of a good, and this is counteracted if substitution effects increase demand when the price falls. Note, however, that the Leontief utility function we use is homothetic; a country's desired ratio of apples and butter does not change with its income, only the quantity of each. These examples of the transfer paradox are not driven by one good being more income-elastic so a transfer to a country shifts its consumption more towards that good. Rather, they are driven by different countries having different desired ratios, so that the transfer changes the ratio of aggregate demand.

In equilibrium, people in country i will choose a and b so that $\frac{a_i}{\alpha_i} = b_i$. The country's

[3] This utility function violates the standard "no satiation" assumption, because a country does not value a good consumed in excess of the fixed proportions determined by α. Nothing important would change, however, if we changed the utility function to $u_i(a, b) = Min(\frac{a}{\alpha_i}, b) + a^\beta + b^\gamma$ with β and γ chosen sufficiently small, which would add a little bit of second-order substitutability to the utility function to change the sharp angle in the indifference curve to a differentiable curve.

consumption bill will equal its income, $\overline{a}_i p_a + \overline{b}_i p_b$, so

$$a_i p_a + b_i p_b = \overline{a}_i p_a + \overline{b}_i p_b.$$

Substituting using $\frac{a_i}{\alpha} = b_i$ we can work out demand:

$$a_i p_a + \frac{a_i}{\alpha_i} p_b = \overline{a}_i p_a + \overline{b}_i p_b,$$

so

$$a_i = \alpha_i \left(\frac{\overline{a}_i p_a + \overline{b}_i p_b}{\alpha_i p_a + p_b} \right),$$

and similarly

$$b_i = \frac{\overline{a}_i p_a + \overline{b}_i p_b}{\alpha_i p_a + p_b}.$$

A price increase has no substitution effect, just an income effect. Thus, if a gift is made, it does not matter in which commodity it is given, because the endowments enter demand only via the income term $(w_i^a + w_i^b p)$. So long as the gift's value at the eventual equilibrium prices are the same, it can be either apples or butter.

In equilibrium, quantity demanded must equal quantity supplied, so if there are three countries,

$$a_1 + a_2 + a_3 = \overline{a}_1 + \overline{a}_2 + \overline{a}_3, \tag{16}$$

and

$$b_1 + b_2 + b_3 = \overline{b}_1 + \overline{b}_2 + \overline{b}_3.$$

The three countries each have two demand equations. Adding the two market-clearing conditions just above makes eight equations for our six demand quantities plus the two prices, eight unknowns. Not all these equations are independent, however, because Walras's Law says that the total value of production equals the total value of consumption, so we will add one more condition, the normalization $p_a = 1$.

Below we will show how gifts change consumption under three different sets of choices for the nine parameters— the taste for apples, the endowment of apples, and the endowment of butter for the three countries.

5. Example 1: The Weak Paradox with the Recipient Benefitting but the Third Party Losing

In example 1, countries 1 and 2 start with endowments of apples and country 3 of butter. Country 1 has a preference for butter, country 2 a preference for apples, and country 3 is between them. Country 1's aim is to reduce the equilibrium price of butter, which both diminishes the price of its import and increase the price of its export. If it succeeds, this benefits country 2, which also imports butter and exports apples, even though country 2's consumption is heavily skewed towards apples. The price decline will hurt country 3, however, which exports butter and imports apples.

TABLE 2:

EXAMPLE 1: DONOR AND RECIPIENT GAIN, THIRD PARTY LOSES

	Country 1 (donor)	Country 2 (recipient)	Country 3 (third party)
Taste for apples (α_i)	.25	4	1
Apple endowment ($\overline{a}_i$)	100	100	0
Butter endowment ($\overline{b}_i$)	0	0	200
Original situation:			
Apple consumption (a_i)	20	80	100
Butter consumption (b_i)	80	20	100
Income	100	100	200
After a gift of 20 in apples:			
Apple consumption (a_i)	$26\frac{2}{3}$	$106\frac{2}{3}$	$66\frac{2}{3}$
Butter consumption (b_i)	$106\frac{2}{3}$	$26\frac{2}{3}$	$66\frac{2}{3}$
Income	80	120	100

Note: The original equilibrium prices are ($p_a = 1$, $p_b = 1$). After the gift they are ($p_a = 1$, $p_b = .5$). Country i's utility equals b_i in equilibrium. An example of Yano (1983) case (iii).

In the initial equilibrium, the price of butter is $p_b = 1$. Recall the demand curve derived earlier,

$$b_i = \frac{\overline{a}_i p_a + \overline{b}_i p_b}{\alpha_i p_a + p_b}$$

Here the demands for butter are

$$b_1 = \frac{100}{1+p_b}, b_2 = \frac{100}{2+p_b}, b_3 = \frac{200p}{4+p_b}$$

In the initial equilibrium with $p_b = 1$, these are $b_1 = 80, b_2 = 20$ and $b_3 = 100$, which add up to the world endowment of 200.

Country 1 then donates 20 in apples to country 2. We will look at how tatonnement reaches the new equilibrium price. The demands for butter change to

$$b_1 = \frac{80}{1+p_b}, b_2 = \frac{120}{2+p_b}, b_3 = \frac{200p_b}{4+p_b}$$

At the initial price of $p_b = 1$, country 1's demand for butter falls from 80 to 64 after it makes the gift to country 2. Country 2's demand rises from 20 to 24. Country 3 is so far unaffected, so its demand stays at 100. Country 1's demand has fallen more than country 2's has risen, so there is excess supply of butter. The world endowment is 200, but the quantity demanded is 188.

To increase demand, let's use tatonnement and try a lower price: $p_b = .9$. Country 1's demand for butter rises from 64 to 69.6 (rounded, as will be the case throughout theses examples) and country 2's from 24 to 24.5. Country 3's demand falls, however, from 100 to 94.7. Note that country 1's desired butter consumption is still smaller than its original level of 80 before the gift, but country 2 is better off because of both the gift itself and the decline in the price of butter, the good it imports. Total demand for butter rises from 188 to 188.8, but there is still excess supply.

Reduce the price still further, to $p_b = .7$. At this price, country 1 demands 84.2 in butter, and thus is better off than with its pre-gift consumption of 80. Country 2's demand rises to 25.5, and country 3's falls to 82.4. Total demand rises to 192.1, still less than the supply of 200.

When the price falls to $p_b = .5$ the market reaches equilibrium. Country 1 demands $106\frac{2}{3}$, country 2 demands $66\frac{2}{3}$, and country 3 demands $66\frac{2}{3}$, which sum to 200, the amount supplied.

The gift helps country 1 because when it transfer income to country 2, the direct effect is that it becomes poorer and its demand for butter falls. This fall is greater than the increase in demand for butter by apple-loving country 2, putting downward pressure on the price. When the price of butter falls, demand by countries 1 and 2 increases, but this is outweighed by the decline in demand of country 3, whose income consists of butter. This diminishes excess supply, but only when the price has fallen all the way to .5 does demand equal supply again.

Country 3 plays an essential role. Country 2's wealth, like country 1's, is an endowment of apples, so a gift from country 1 to country 2 cannot cause a price change that will increase the donor's wealth at the expense of the recipient's. Rather, the gift destabilizes the original equilibrium because country 1's demand for butter falls sharply while country 2's increases only slightly, and when this drives down the price of butter, country 3 responds by reducing its demand, which drives down the price still further.

This case of the weak paradox with the recipient being helped but the third party being hurt is the case in the numerical example in Gale (1974). He also uses with Leontief utility and has the same pattern of endowments and preferences. We have used different endowments and preferences so that the numbers for the equilibrium quantities and prices come out exact.

6. Example 2: The Strong Paradox with the Recipient Losing and the Third Party Gaining

In this example, countries 1 and 3 have endowments of apples and country 2 has butter. Country 3 has the strongest preference for apples, then country 2, and country 1 the weakest. Thus, country 1 starts with apples and wants to import butter, country 2 starts with butter and wants to import apples, while country 3 starts with apples and despite its strong preference for apples needs to import butter.

Table 3:
Example 2: Donor and Third Party Gain, Recipient Loses

	Country 1 (donor)	Country 3 (recipient)	Country 3 (third party)
Taste for apples (α_i)	1	2	4
Apple endowment ($\overline{a}_i$)	100	0	100
Butter endowment ($\overline{b}_1$)	0	100	0
Original situation:			
Apple consumption (a_i)	$33\frac{1}{3}$	100	$66\frac{2}{3}$
Butter consumption (b_i)	$33\frac{1}{3}$	50	$16\frac{2}{3}$
Income	100	200	100
After a gift of 20 apples:			
Apple consumption (a_i)	40	80	80
Butter consumption (b_i)	40	40	20
Income	80	120	100

Note: The original equilibrium prices are ($p_a = 1$, $p_b = 2$). After the gift they are ($p_a = 1$, $p_b = 1$). Country i's utility equals b_i in equilibrium.

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The demands for butter for example 2 are

$$b_1 = \frac{100}{1 + p_b}, b_2 = \frac{100p}{2 + p_b}, b_3 = \frac{100}{4 + p_b}$$

In the initial equilibrium, $p_b = 2$, so the quantities demanded are $b_1 = 33\frac{1}{3}, b_2 = 50$ and $b_3 = 16\frac{2}{3}$, which add up to the world endowment of 100.

Country 1 then donates 20 in apples to country 2. We will look at how tatonnement reaches the new equilibrium price. The demands for butter change to

$$b_1 = \frac{80}{1 + p_b}, b_2 = \frac{20 + 100p_b}{2 + p}, b_3 = \frac{100}{4 + p_b}$$

At the initial price of 2, country 1's demand falls from $33\frac{1}{3}$ to $26\frac{2}{3}$. Country 2's demand rises from 50 to 55. Country 3 is so far unaffected, so its demand stays at $16\frac{2}{3}$. Country 1's demand has fallen more than country 2's has risen, so there is excess supply of butter. The

[4] Using the quantity theory of money, total income should stay the same before and after the transfer, which would be helpful. Maybe convert to that later.

endowment is only 100 but the quantity demanded is 98.3 (rounded, as we will frequently do in these examples).

Thus, let's use tatonnement and try a lower price: $p_b = 1.8$. Country 1's demand rises to 28.6 and country 3's to 17.2. Country 2's demand falls, however, to 52.6. That is because the price reduction reduces country 2's income, and with Leontief utility functions, a reduction in income always reduces one's consumption of every good. Note that country 1's butter consumption is still smaller than originally, and country 2's is greater, so country 1 would still be worse off and country 2 better off as a result of the transfer, so long as the price did not drop further. Country 3 would also be better off, because it did not give away anything and it benefits from a lower price for the good it imports, butter. Total demand for butter rises, to 98.4, but there is still excess supply.

Reducing the price further, try $p_b = 1.5$. Country 1's butter demand rises from 28.6 to 32, still below the original level. Country 2's demand falls to 48.6— so now it starts to be hurt by the transfer. Country 1, by reducing its own demand for butter, has driven down the price of butter so much that country 2, with its income in butter, has been hurt by accepting the gift. These two countries' demand for butter has fallen, but country 3's has risen, to 18.2, and this raises total demand to 98.8, closer to the equilibrium level of 100.

Finally let us go to the new equilibrium price of butter, $p_b = 1$. Country 1's butter demand— which will now, in equilibrium, equal its consumption— has risen to 40, higher than its original consumption because consumption is so much lower. Country 3's consumption is 20, also higher. Country 2's consumption, however, has fallen from 50 to 40.

What is going on is that country 1's demand for butter falls because it has spent part of its income on the gift to country 2, and that reduces the price of butter. When the price of butter falls, country 2, a butter exporter, is worse off. Country 3 is better off, however, and while its demand for butter rises, its demand for apples rises even more given its very strong taste for them. That pushes down the price of butter still further. Eventually country 2 is so much worse off that declines in the price of butter have less effect on its demand than they do on country 3's.

Do not be misled by the last line of Table 3, which says that country 1's income has fallen from 100 to 80. Country 1's nominal income has fallen, but the average price in this little economy has fallen too. If we dropped the normalization of $p_a \equiv 1$ and added the quantity-theory-of-money equation $(a_1 + a_2 + a_3)p_a + (b_1 + b_2 + b_3)p_b = M$, nominal GDP would remain unchanged and country 1's nominal income would show an increase.

Why is it that country 3 is necessary to this process? As apple-loving country 3 becomes richer, it demands more apples even though the relative price of apples has risen, and that helps country 1, which exports apples. Since country 3 always wants 4 times as much apples as butter, in the tatonnement process above its apple demand is going from 66.7 to 68.8 to 72.8 to 80. Since it starts with an endowment of 100 apples, it is still exporting 20 apples, but its exports have fallen enough to put upwards pressure on the relative price of apples, to the benefit of country 1. Thus, while country 1's fallen demand starts off the process by pushing down the price of butter, as the price falls, the process is reinforced by country 3 pushing up the price of apples. Country 2 is caught in the middle.

Part of the mystery of this version of the transfer paradox is that it seems that country 2's initial rise in income is what triggers the change in relative prices, yet in the end country 2's income has fallen. As the price adjusts and country 2's income falls from its initial transfer-induced level, shouldn't there be a price at which country 2's income has reached its original level and the process stops? Country 3 is why this logic fails. There is a price at which country 2's real income has reached its original level, but country 3's real income does not reach it at the same point. Country 3's demand for butter is still too low, so the price has to fall further.

7. Example 3: The Strong Paradox with Recipient and Third Party Losing

In example 3, country 1 starts with butter while countries 2 and 3 start with apples. Country 3 has the strongest preference for apples and country 2 the weakest. Thus, when the price of butter rises, country 1 is better off via both consumption and income, country 2 gains in consumption but loses in income, and country 3 loses in both consumption and income.

Table 4:
Example 3: Donor Gains, Recipient and Third Party Lose

	Country 1 (donor)	Country 3 (recipient)	Country 3 (third party)
Taste for apples (α_i)	1/2	1	1/3
Apple endowment ($\overline{a}_i$)	100	20	0
Butter endowment ($\overline{b}_1$)	20	100	80
Original situation:			
Apple consumption (a_i)	40	60	20
Butter consumption (b_i)	80	60	60
Income	120	120	80
After a gift of 11.75 apples:			
Apple consumption (a_i)	45.97	57.02	17.02
Butter consumption (b_i)	91.94	57.02	51.05
Income	100	90.52	47.02
After a gift of 20 butter:			
Apple consumption (a_i)	45.97	57.02	17.02
Butter consumption (b_i)	91.94	57.02	51.05
Income	100	90.52	47.02
After a gift of 20 apples and 20 butter:			
Apple consumption (a_i)	80	40	0
Butter consumption (b_i)	160	40	0
Income	80	40	0

Note: The original equilibrium prices are ($p_a = 1$, $p_b = 1$). After the gift of 11.75 apples or 20 butter they are ($p_a = 1$, $p_b = .59$). After the gift or 20 apples and 20 butter they are ($p_a = 1$, $p_b = 0$).

The demands for butter in example 3 are

$$b_1 = \frac{120 - (t_a \times 1 + t_b * p_b)}{\frac{1}{2} + p_b}, b_2 = \frac{120p_b + t_a \times 1 + t_b \times p_b}{2 + p_b}, b_3 = \frac{80p_b}{\frac{1}{3} + p_b} \tag{17}$$

In the initial equilibrium, the price of butter is $p_b = 1$, so the quantities demanded are $b_1 = 80, b_2 = 60$ and $b_3 = 60$, which add up to the world endowment of 200. We will again go through a few steps of tatonnement. The choice of the quantity 11.75 for the transfer of apples may seem arbitrary, but we will see shortly that it is equivalent to a transfer of 20 in butter.

The first step is to see what happens when the transfer of 11.75 in apples changes income but price has not adjusted. Country 1's demand falls from 80 to 72.2, Country 2's rises from 60 to 65.9, and country 3's demand is unchanged at 60. Aggregate demand thus falls from 200 to 198, so there is excess supply.

Consider a reduction in price to $p_b = 0.9$. Country 1's endowment is mostly in apples, so its income has risen and its demand rises from 72.2 to 75.9. Countries 2 and 3 have their income mostly in butter, so b_2 falls from 65.9 to 64.1 and b_3 falls from 60 to 58.4. Since the price of butter has fallen 10%, the real value of their butter income has fallen 10%, though country 2 also has its transfer of 11.75 from country 1. Aggregate demand is 198.4, still resulting in excess supply.

Going further, let the price drop to $p_b = 0.7$. The real value of country 1's income is very high in terms of apples, so it demands $b_1 = 85.2$. Country 3's income has correspondingly fallen, so it demands only $b_3 = 54.2$. Country 2 still has the transfer of 11.75 and its original endowment of butter, so its demand falls just to $b_2 = 59.9$. There is still excess supply however, since aggregate demand is 199.3.

When the price falls to $p_b = 0.59$, the market reaches equilibrium. Country 1 demand 91.8, country 2 demands 57.1, and country 3 demands 51.1, which sum to 200.

Now consider a transfer of 20 in butter instead of donating apples. This reaches the exact same outcome, because it has the exact same value at the new equilibrium price of $p_b = .59$. You can see this must happen from the demand equations (17), because the transfers only enter it via the expression $(transfer_a \times 1 + transfer_b \times p_b)$, the nominal value of the transfers. If the way the transfers are split between apples and butter doesn't affect the demand for butter, it can't affect the demand for apples (which just uses the money that's left after buying butter) or the equilibrium price (which depends on supply and demand).

Let's go through the tatonnement in this case too. The first step is to see what happens when the transfer of 20 in butter changes income but price has not adjusted. Country 1's demand falls from 80 to 66.7, Country 2's rises from 60 to 70, and country 3's demand is unchanged at 60. Aggregate demand thus falls from 200 to 196.7, so there is excess supply.

Consider a reduction in price to $p_b = 0.9$. Country 1's endowment is mostly in apples, so its income has risen and its demand rises from 66.7 to 71.4. Countries 2 and 3 have their income mostly in butter, so b_2 falls from 70 to 67.4 and b_3 falls from 60 to 58.4. Aggregate demand is 197.2, still resulting in excess supply.

Going further, let the price drop to $p_b = 0.7$. Country 1 demands $b_1 = 83.3$. Country 3 demands only $b_3 = 54.2$. Country 2's demand falls just to $b_2 = 61.2$. There is still excess supply however, since aggregate demand is 198.1.

When the price falls to $p_b = 0.59$, the market reaches equilibrium. Country 1 demand 91.8, country 2 demands 57.1, and country 3 demands 51.1, which sum to 200

In the previous examples, we have neglected the question of what size transfer is best for country 1. In example 3, country 1 should transfer a big enough gift that the price of butter is driven to zero. A transfer of 20 in apples and 20 in butter to country 2 will do this, as illustrated in the last part of Table 3. In the new equilibrium, country 1 consumes 160 butter, country 2, 40, and country 3, 0. This is an equilibrium because if the price of butter is 0, country 3 is unable to buy any apples and so has no use for its butter, which it will give away. For it to be an equilibrium, however, country 3 must send all 80 of its butter to country 1, none to country 2. Otherwise, country 1 would be demanding more butter than it would be getting.

8. Monopoly Pricing, Immiserizing Growth, and Prosperifying Decline

We have gone through several situations in which country 1 can improve its welfare by giving away wealth to country 2. We have explained this using the ideas of shifting equilibria, unstable equilibria with strong income effects, and various three-country income effects. It is natural, though, to wonder if the best way to understand the transfer paradox might be by relating it to an even better-known paradox of international trade, the paradox of immiserizing growth.

In the paradox of immiserizing growth, a country increases its productivity but ends up poorer. Reversing the process would give us what we will call "prosperifying decline": the country reduces its productivity but ends up richer. Here, country 1 can help itself by destroying some of its export good, apples. In fact, this is cheaper than getting the same equilibrium price change by transferring the apples to country 2.

Consider example 3, in which country 1 donates 11.75 in apples or 20 in butter to country 2 to drive the price down to $p_b = .59$. To get the same price decline, country 1 only needs to destroy 3.7 in apples, which is cheaper than 11.75.

Bhagwati, Brecher & Hatta (1983) show that the transfer paradox will not occur if countries use their optimal tariffs against each other. Tariffs, like destruction, are a way to change the terms of trade that is cheaper than donations. In fact, since tariffs do not destroy goods, I bet they are a better way than destruction. [IS THAT TRUE?] Note, too, that neither destruction nor tariffs require the cooperation of another country— though it is true that the other country can react (if it is a "large" country able to affect the terms of trade) by balancing with its own destruction and tariffs.

One might think that the transfer paradox operates through the same basic mechanism as immiserizing growth, just less effectively. After all, in both a country is getting rid of some of its production and ending up better off because of the resulting change in equilibrium prices. The similarity is deceptive, however. The intuition behind immiserizing growth is different from that behind the transfer paradox, and simpler. One might think that what is happening with the transfer paradox is that country 1 is inducing immiserizing growth in country 2 by donating to it, or is inducing prosperifying decline in itself, but neither is true. In immiserizing growth, a country raises its total factor productivity and becomes poorer as a result. The reason is that if a country increases the output of its export good, it reduces the price, so if it is big enough to influence the world price— that is, if it has market power— it may want to reduce its sales, even if the marginal cost is zero (or negative— a destruction cost!). In an exchange economy, this amounts to an increase in the endowment of the export good. In a production economy, it takes the form of a bigger increase in exports than other goods, given the form of the productivity increase and the tastes of the home country.

In the transfer paradox, no goods are created or destroyed, and it does not matter whether country 1 donates its export good or its import good to country 2, so long as the value at the new equilibrium price of the goods transferred is the same. Income effects and different preferences across countries are crucial, whereas they are not necessary for immiserizing growth. Immiserizing growth is a supply-side effect, depending on quantities available to be treated; the transfer paradox is a demand-side effect, depending on how changing income affects countries' demands.

To be sure, Jones (1984) shows that immiserizing growth can be viewed using the idea of the transfer paradox. Suppose that instead of Country 1 existing we have Nature making the transfer to Country 2. Moreover, view the expansion of supply by Country 2 as being net of a reduction in demand by Nature, the same reduction as we would have had with Country 1 being the donor. This gives rise to conditions for the transfer paradox identical to those in the initial paradox. This seems to us, however, to make introduce a separate hard idea into the process of understanding immiserizing growth.

Polemarchakis (1983) also relates immiserizing growth to the transfer paradox. He says that the intuition is simple, but he explains it in equations rather than words (in his section 4) and his explanation is misleading. It is based on the idea that if country 1 loses some of its endowment of a good that it exports, the equilibrium price of that good can rise enough to make up for the loss. If the good is transferred to an imaginary country 3 that consumes only that one good, the effect on trade between countries 1 and 2 is the same as if it that amount of good was destroyed, but country 1 is now better off as the result of a transfer rather than pure destruction. That idea is true, but as we will explain, it is best not classified as an example of the transfer paradox because it works only if the gift is of that particular good, whereas the transfer paradox proper works whichever good is transferred: it works through wealth transfer, not manipulation of the quantity of a particular good. We will return to this in a later section where we discuss the paradox of immiserizing growth.

In the limit, when the recipient country's marginal consumption of the donated good is zero, Polemarchakis is correct that destruction and gifts is true, and the gift does not have to be of the recipient's desired good. Otherwise, destruction is better.

9. Concluding Thoughts

We may also ask what the recipient country can do with the gift to prevent the transfer effect. It couldn't subsidize its citizens' purchases of butter. That wouldn't work, though, in this Leontief utility world. I think the government has to restrain the demand of its citizens or it won't work. Knowing they are richer, they will demand more apples. They *must* keep some of their own apples off of the market.

The transfer problem in a static economy with two agents was shown to be theoretically possible. However, it has been difficult to show its applicability to a broader set of economic contexts due to the lack of concrete examples of the concept. In this regard, this paper has a clear contribution to the area of the transfer problem.

It is interesting that a similar type of transfer problem can also occur in a monetary economy with nominal taxes and transfers. Kang (2014) showed the existence of paradoxical outcomes in an economy with three consumers, in which inflation makes the tax payer worse off and the recipient of the subsidy better off. However, this "nominal" transfer problem has not been addressed in a two-agent economy, which should be an interesting topic for future research.

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