A Multi-Sector Macro Model

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Introduction

I was thinking about how Keynesian stimulus would work in model where prices are sticky downwards and inputs cannot adjust across sectors. Then, it is possible to have full employment in one sector but unemployment in the other, The economy as a whole is not at full employment, but purchases in the wrong sector will have a zero Keynesian effect, not a positive one.

This model will also provide what might be the simplest possible Keynesian model. As such, it inevitably excludes subjects of interest— notably, the labor market and the asset market. The model will simply be a one-period N-good general equilibrium model with labor as the one input, no investment or saving, prices that are rigid downwards, and a fixed relation between money and nominal output. One does not need investment, saving, or interest rates for monetary and fiscal policy to be effective in such an economy.

The assumption on nominal output having a constant relationship to the money supply can be stated in various ways. It could be a cash-in-advance constraint, that trade requires money. It could be a money demand equation, that demand for money depends on prices and output. The assumption is necessary to make sense of price rigidity downwards. Inefficiency will result in this model because relative prices cannot adjust. If all price could rise, but some more than others, relative prices could always adjust in the midst of the general inflation.

The Model

The economy has N sectors. Labor is the only input, and workers in sector i only produce good i, with constant returns to scale. We will denote output, labor employment, and consumption of good i by x_i, where because labor is sector specific, $x_i \leq \overline{x_i}$.

Consumers (who are the same as the workers) have a Cobb-Douglas

utility function. Wages are sticky downwards but not upwards.

$$U = \Pi_{i=1}^{N} x_i^{\alpha_i}, 0 < \alpha_i. \tag{1}$$

The money supply and velocity are exogenous. The equation of exchange tells us that the money supply M times velocity V equals the sum of prices times quantities:

$$MV = \sum_{i=1}^{N} p_i x_i \tag{2}$$

This is a one-period model. We can add a second period, but its utility would be independent of the first and there is no saving or investment.

The Cobb-Douglas utility function has two properties that are useful here to keep things simple: good i's share of national revenue at equilibrium prices is α_i and the share of good i in a consumer's expenditure is constant at α_i.

A consumer's maximization problem Lagrangian is, if his income is Y, for each good j,

$$\underset{x_j}{Maximize} \ \Pi_{i=1}^{N} x_i^{\alpha_i} + \lambda(Y - \sum_{i=1}^{N} x_i p_i \tag{3}$$

The first order condition is, for every j

$$\frac{\alpha_j \Pi_{i=1}^{N} x_i^{\alpha_i}}{x_j} - \lambda p_j = 0 \tag{4}$$

so for any two goods j and k

$$\frac{p_j x_j}{\alpha_j} = \frac{p_k x_k}{\alpha_k} \tag{5}$$

and since the revenue shares need to add up to income ($Y = \sum_{i=1}^{N} x_i p_i$):

$$x_i = \frac{\alpha_i Y}{p_i} \tag{6}$$

Since $MV = \sum_{i=1}^{N} p_i x_i = Y$, the last equation implies that

$$x_i = \frac{\alpha_i MV}{p_i} \tag{7}$$

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Thus, if we start at full employment, where $x_i = \overline{x_i}$, the prices are

$$p_i = \frac{\alpha_i MV}{\overline{x_i}} \tag{8}$$

We will denote these initial price levels as $\overline{p_i}$, keeping in mind that the price cannot fall but it can rise.

A Supply Shock

I should do this second. Put in a parameter for the productivity of labor in the X sector. Increase it, and decrease it.

A Demand Shock

Then a demand shock occurs, as people desire less of good 1: α_1 falls to β_1. We only have one period, but with many periods, imagine that α_1 will go back to its old value next year, and everybody knows it.

The price of good 1 ought to fall, by its demand equation, to:

$$p_1 = \frac{\beta_1 MV}{\overline{x_1}}. \tag{9}$$

That cannot happen, by the assumption of price rigidity. Thus, x_1 must fall instead, to less than $\overline{x_1}$, and we get unemployment.

We are not back to an equilibrium, though, because if x_1 falls but nothing else changes, it is no longer true that

$$MV = \sum_{i=1}^{N} p_i x_i \tag{10}$$

So we must raise prices or outputs of other goods. Their outputs cannot rise, since we are at full employment, so their prices rise instead, so that good $i = 2., , , N$ has expenditure share

$$p_i x_i = \frac{\alpha_i}{\beta_1 + \sum_{i=2}^{N}} \tag{11}$$

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(1) Now suppose the government borrows amount G_1 from abroad and adds it to demand for good 1. Good 1's demand becomes

$$x_1 = \frac{\beta_i MV}{p_1} + G_1 \tag{12}$$

Good 1's price does not have to increase to make this true, because x_1 can increase, at least so long as $x_1 < \overline{x_1}$. So the stimulus reduces unemployment.

We still need $MV = PX$, though, and with x_1 rising, that is violated unless something else changes. If the prices of goods 2 to N can go back down, that can restore equilibrium.

If, however, the prices of goods 2 to N cannot go down from their new levels (a ratchet effect), then output of those goods must fall. We have pushed the unemployment out of industry 1 into all the others.

It would not help to spend in all sectors equally, because the problem is that when even one price has risen and cannot decline again, PQ still say too big for our fixed MV unless output declines somewhere in the economy.

(2) Suppose, instead, that the government buys good 2 by mistake, in its stimulus plan, so, letting p'_2 denote the current, raised, price of good 2,

$$x_2 = \frac{\alpha_2 MV}{p'_2} + G_2 \tag{13}$$

Since x_2 cannot rise, the price must rise instead. When that price rises, other prices in the economy must fall, or outputs must fall. If the newly raised prices of goods 3 to N can fall, that can restore equilibrium. If they can't, their output will have to fall.

I have not said how the government pays for the stimulus. This would be a problem, because if it does so with taxes in period 2, even lump-sum taxes, that will reduce worker income and demand in period 2, and create unemployment there.

References

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