A SIMPLE MODEL OF PRODUCT QUALITY WITH ELASTIC DEMAND

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In various models starting with Klein and Leffler (1981), sellers produce high quality for the sake of future profits. These models have assumed that a consumer demands one or zero units, but a simple model with elastic demand and entry costs can illustrate the same point with fewer problems of existence or determinacy of equilibrium.

1. Introduction

Klein and Leffler (1981) can be interpreted as having constructed an equilibrium for infinite repetition of the one-shot game between a seller who can choose between high and low quality and a consumer who chooses whether or not to buy without observing the quality in advance. In equilibrium, the firm is willing to produce high quality to attract repeat business, and the consumer keeps buying so long as the firm produced high quality in the past. The equilibrium price is high enough that the firm is unwilling to sacrifice its future profits for a one-time windfall from producing low quality. Although this is only one of a large number of equilibria, consumer behavior is simple, rational, and plausible.

A difficulty for this model is that the equilibrium price is above average cost and yields positive profits. Various ways that profits might dissipate have been suggested – brand name capital in Klein and Leffler (1981), initial operating losses in Shapiro (1983), entry costs in Allen (1984), and advertising in Rogerson (1988), but problems arise of non-existence or indeterminacy of equilibrium. The source of the problem may lie in the standard assumption of unit demand, and in this note I construct a model with entry costs and fully elastic demand that illustrates the Klein–Leffler point simply and with only the standard integer problem for existence.

2. The model

Denote by $n$ the number of firms that choose to sink cost $F$ and enter the market being analyzed. A firm in the market picks a price $p$ each period and decides whether to produce high or low quality at constant marginal cost $c_h$ or $c_l$, where $c_h > c_l > 0$. The quantity sold per period by firm $i$ is denoted $q_i$, the number of periods is infinite, and the discount rate is $r > 0$.

There is a continuum of identical consumers. Each period consumers pick firms from which to buy. Payoff functions are such as to yield a market demand of $q(p)$ if consumers believe the product is of high quality and zero otherwise. Consumers observe the past quality produced by every firm, but not the present quality.
The Folk Theorem of repeated games tells us that this game has a wide range of perfect outcomes, including a large number with erratic quality patterns like High, High, Low, High, Low, High... [see Rasmusen (n.d.)]. Even if we confine ourselves to pure strategy equilibria with constant quality and identical behavior by all firms, then either low or high quality can result. Low quality is an equilibrium outcome because it is the outcome in the one-shot game. If the discount rate is low enough, high quality is also an equilibrium outcome, and since this is a Paretodominant outcome it will be the focus of our attention. Consider the following strategy combination [for which the values for $p^*$ and $n^*$ are taken from eqs. (2) and (6)].

**Firms.** $n^*$ firms enter. Each sells high quality at price $p^*$. If a firm ever deviates from this, it drops out of the market, and a new firm enters and sells high quality at price $p^*$.

**Buyers.** Buyers choose randomly among firms that charge $p^*$ and have never produced low quality. If there are no such firms, they refrain from buying.

For low discount rates, this strategy combination is a perfect equilibrium. Each firm is willing to produce high quality and charge $p^*$ because of the threat of losing its customers. If a firm has deviated, it is willing to drop out because no customer will buy from it. Buyers are willing to stay away from a firm that has produced low quality because they can find high quality at another firm. For this story to work, however, the equilibrium must satisfy three constraints that pin down the price, the output per firm, and the number of firms.

The price must be high enough if a firm is to produce high quality. Given the buyers' strategy, if the firm even produces low quality it receives a one-time windfall profit, but loses its future profit stream. The tradeoff is shown in constraint (1), which is satisfied if the discount rate is low enough.

\[ q_1(p - c_1) \leq \frac{q_1(p - c_h)}{r}. \]  

Inequality (1) determines a lower bound for the price, which must satisfy

\[ p^* = \frac{c_h - rc_1}{1 - r}. \]  

We can write eq. (2) as an equality because any firm trying to charge a price higher than $p^*$ also loses its customers.

The second constraint is that sales per firm at price $p^*$ yield zero profits, so firms are indifferent between entering the market and staying out.

\[ \frac{q_1(p - c_h)}{r} = F. \]  

Substituting from eq. (2) for $p$, we obtain

\[ q^n_1 = \frac{F(1 - r)}{c_h - c_1}. \]  

Having determined $p$ and $q_1$, only $n$ remains, which is determined by the equality of market supply
and demand. Each firm would like to sell more than $q_i^*$ at price $p^*$, but the market output must equal the quantity demanded by the market, so

$$nq_i^* = q(p^*)$$

(5)

Combining eqs. (2), (4), and (5) yields

$$n^* = \frac{(c_h - c_i)}{F(1 - r)} \cdot q \left( \frac{c_h - rc_i}{1 - r} \right).$$

(6)

We have now determined the equilibrium values. The only difficulty is the standard integer problem [that the $n$ solving eq. (6) might not be an integer], which is no more severe here than in any model with a U-shaped cost curve.

3. Further discussion

The exogenous sunk cost $F$ is crucial to the determinacy of this model. In particular, it cannot be replaced by a fixed cost paid every period that a firm is in the market. If the fixed cost was equal to $q_i(p - c_h)$ or greater, firms would cut quality to obtain the immediate profit, and if the fixed cost was less, profits would be positive. What matters about the sunk cost $F$ is that it allows quasi-rents to be positive for firms that have already entered, but nonpositive for potential entrants.

The equilibrium price is determinate because $F$ is exogenous and demand is not perfectly inelastic, which pins down the size of firms. If $F$ were not exogenous, but demand was elastic, the equilibrium price would still be the unique $p^*$ that satisfies constraint (2) and the market quantity would still be $q(p^*)$, but $n$ and $q_i$ would be undetermined. A continuum of equilibria would be possible, indexed by the $F$ that consumers arbitrarily believe a firm must dissipate if it is to produce high quality. The firms' best response is for $n^*$ of them to pay the appropriate $F$ and produce high quality at price $p^*$, where $n^*$ is determined by condition (6) as a function of $F$.

References


