Picking a Sealed Bid, Two Bidders, April 28, 2002

Suppose you think each persons value is equally likely to be anywhere between 0 and 10. You know your own value is X.

Start with just 2 bidders, you and your rival. It is a Nash equilibrium for each of you to follow a strategy of bidding half your value–.5X for you and .5R for your rival. If you bid b (which we will show should be .5X), your payoff is

$$Probability(you\ win)(X-b) + Probability(you\ lose)(0)$$

$$= Probability(he\ bids\ less\ than\ b)(X-b)$$

$$= Probability(.5R < b)(X-b)$$

$$= Probability(R < 2b)(X-b)$$

$$= \tfrac{2b}{10}(X-b)$$

$$= \tfrac{bX}{5} - \tfrac{b^2}{5}$$

The payoffs derivative is

$$\frac{d\ payoff}{db} = \frac{X}{5} - \frac{2b}{5}$$

Setting this derivative equal to 0 yields $b = X/2$ as your payoff-maximizing bid.

Picking a Sealed Bid, N Bidders

Suppose you think each persons value is equally likely to be anywhere between 0 and 10. You know your own value is X. There are N bidders, including you. It is a Nash equilibrium for each of you to follow a strategy of submitting a bid of your value shaded by fraction $1/N$, so if $N = 6$ you would bid $(5/6)X$.

Let us denote by V_i the value of bidder i, for the other $N - 1$ bidders. Your payoff is

$$Probability(you\ win)(X - b) + Probability(you\ lose)(0)$$

$$= Probability(everyone\ else\ bids\ less\ than\ b)(X - b)$$

$$= Probability(V_i < b, for\ all\ i)(X - b)$$

$$= Probability(V_i < b)^(N - 1)(X - b)$$

$$= (b/10)^{N-1}(X - b)$$

$$= (1/10)^{N-1} b^{N-1}(X - b)$$

$$= (1/10)^{N-1}[b^{N-1}X - b^N]$$

The derivative of the payoff with respect to b is

$$\frac{D\ payoff}{Db} = (1/10)^{N-1}[(N-1)b^{N-2}X - Nb^{N-1}].$$

Setting the derivative equal to 0 and simplifying yields $0 = [(N - 1)X - Nb]$, so $b = (\frac{N-1}{N})X$.