24 October 2006 . Eric Rasmusen, Erasmuse@indiana.edu.
Http://www.rasmusen/org/GI/chap10_mechanisms.pdf.

10 Mechanism Design and Post-Contractual Hidden Knowledge

This is just for price discrimination.

10.5: Price Discrimination

Suppose there are two consumers, willing to pay 45 and 36 for a car with a marginal cost of 10 thousand dollars.

A single-price monopolist would charge 36, for profit of 2(36-10) = 52 instead of 45 with its profit of (45-10) = 35.

A two-price monopolist would charge 36 and 44, increasing his profits to 26+35= 61.

If the monopolist doesn't know the types, he'd like to design a mechanism to get the buyers to reveal them. He can't do much here, though. He could use an indirect mechanism like this:

p(m=36) = 36, p(m=45) = 45

By the Revelation Principle, we can find a direct mechanism which achieves the same outcome with no lying:

p(m=36) = 36, p(m=45) = 36.

In either case, the consumer willing to pay 45 can hide under the guise of being a less intense consumer and despite facing a monopolist he can end up retaining consumer surplus – an **informational rent**, a return to the consumer's private information about his own type.

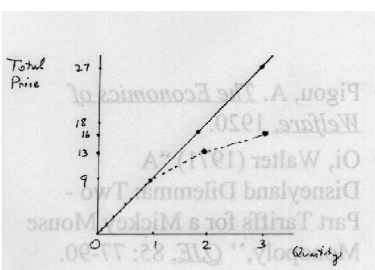
TYPES OF PRICE DISCRIMINATION

1 Interbuyer Price Discrimination. Different prices for different buyers.

Smith's price for a hamburger is $4 per burger, but Jones's is $6. (Close to Pigou's 3rd degree–coarse buyer segmentation)

2 Interquantity Price Discrimination or **Nonlinear Pricing**. Different unit prices for different quantities.

A consumer can buy a first sausage for $9, a second sausage for $4, and a third sausage for $3. Rather than paying the "linear" total price of $9 for one sausage, $18 for two, and $27 for three, he thus pays the nonlinear price of $9 for one sausage, $13 for two, and $16 for three, the concave price path shown in Figure 3. (Pigou's 2nd degree)



3 Perfect Price Discrimination. This combines interbuyer and interquantity price discrimination.

Smith might end up paying $50 for his first hot dog and $20 for his second, while next to him Jones pays $4 for his first and $3 for his second. (Pigou's first degree)

Varian's Nonlinear Pricing Game

Players

One seller and one buyer.

The Order of Play

0 Nature assigns the buyer a type, s. The buyer is "unenthusiastic" with utility function u or "valuing" with utility function v, with equal probability. The seller does not observe Nature's move, but the buyer does.

1 The seller offers mechanism $\{w_m, q_m\}$ under which the buyer can announce his type as m and buy amount q_m for lump sum w_m.

2 The buyer chooses a message m or rejects the mechanism entirely and does not buy at all.

Payoffs

The seller has a constant marginal cost of c, so his payoff is

$$w_u + w_v - c \cdot (q_u + q_v). \tag{1}$$

The buyers' payoffs are $\pi_u = u(q_u) - w_u$ and $\pi_v = v(q_v) - w_v$ if q is positive, and 0 if $q = 0$, with $u', v' > 0$ and $u'', v'' < 0$. The marginal willingness to pay is greater for the valuing buyer: for any q,

$$u'(q) < v'(q) \tag{2}$$

This second is an example of **the single-crossing property**. Combined with the assumption that $v(0) = u(0) = 0$, it also implies that

$$u(q) < v(q) \tag{3}$$

for any value of q.

Perfect Price Discrimination

The game would allow perfect price discrimination if the seller did know which buyer had which utility function. He can then just maximize profit subject to the participation constraints for the two buyers:

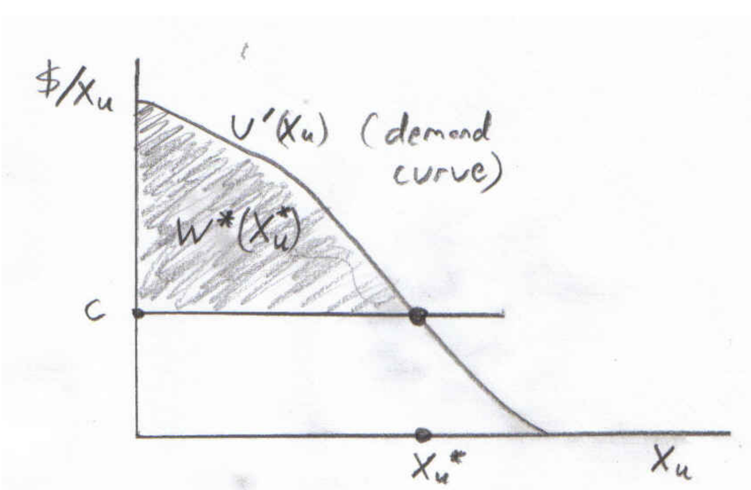
$$\underset{w_u, w_v, q_u, q_v}{\text{Maximize}} \quad w_u + w_v - c \cdot (q_u + q_v). \tag{4}$$

subject to

$$\begin{aligned} &(a) \;\; u(q_u) - w_u \geq 0 \;\; \text{and} \\ &(b) \;\; v(q_v) - w_v \geq 0. \end{aligned} \tag{5}$$

The constraints will be satisfied as equalities, since the seller will charge all that the buyers will pay. Substituting for w_u and w_v into the maximand, the first order conditions become

$$\begin{aligned} &(a) \;\; u'(q_u^*) - c = 0 \quad \text{and} \\ &(b) \;\; v'(q_v^*) - c = 0. \end{aligned} \tag{6}$$



Interbuyer Price Discrimination

Now suppose the seller can charge the two players different prices (**and prevent resale**, but they have to be linear, per-unit, prices. p, not w.

$$\underset{q_u, q_v, p_u, p_v}{\text{Maximize}} \quad p_u q_u + p_v q_v - c \cdot (q_u + q_v), \tag{7}$$

subject to the participation constraints

$$\begin{aligned} &u(q_u) - p_u q_u \geq 0 \quad \text{and} \\ &v(q_v) - p_v q_v \geq 0 \end{aligned} \tag{8}$$

and the incentive compatibility constraints

$$\begin{aligned} &q_u = argmax[u(q_u) - p_u q_u] \quad \text{and} \\ &q_v = argmax[v(q_v) - p_v q_v]. \end{aligned} \tag{9}$$

This is like moral hazard with two contracts for two agents.

The agents will solve their quantity choice problems in (9), yielding

$$\begin{aligned} u'(q_u) - p_u = 0 \quad \text{and} \\ v'(q_v) - p_v = 0. \end{aligned} \tag{10}$$

Thus, we can simplify the original problem in (7) to

$$\underset{q_u, q_v}{Maximize} \quad u'(q_u)q_u + v'(q_v)q_v - c \cdot (q_u + q_v), \tag{11}$$

subject to the participation constraints

$$\begin{aligned} u(q_u) - u'(q_u)q_u \geq 0 \quad \text{and} \\ v(q_v) - v'(q_v)q_v \geq 0. \end{aligned} \tag{12}$$

The participation constraints cannot be binding. If they were, then $u(q)/q = u'(q)$, but since $u'' < 0$ there is diminishing utility of consumption and the average utility, $U(q)/q$, will be greater than the marginal utility, $u'(q)$. Thus we can solve problem (11) as if there were no constraints.

The first-order conditions are

$$\begin{aligned} u''(q_u)q_u + u' = c \quad \text{and} \\ v''(q_v)q_v + v' = c. \end{aligned} \tag{13}$$

This is just the 'marginal revenue equals marginal cost' condition that any monopolist uses, but one for each buyer instead of one for the entire market.

Back to Nonlinear Pricing and Mechanism Design

Now, the seller does not know the buyer's type, **but he can prevent resale**. The seller's problem is

$$\underset{q_u, q_v, w_u, w_v}{\text{Maximize}} \quad w_u + w_v - c \cdot (q_u + q_v), \tag{14}$$

subject to the participation constraints,

$$\begin{array}{ll} (a) & u(q_u) - w_u \geq 0 \quad \text{and} \\ (b) & v(q_v) - w_v \geq 0, \end{array} \tag{15}$$

and the self-selection constraints,

$$\begin{array}{ll} (a) & u(q_u) - w_u \geq u(q_v) - w_v \\ & \\ (b) & v(q_v) - w_v \geq v(q_u) - w_u. \end{array} \tag{16}$$

Which constraints are binding?

Which constraints are binding?

(1) Suppose the optimal contract is in fact separating, and also that both types accept a contract. At least one type will have a binding participation constraint.

(2) Since the valuing consumer gets more consumer surplus from a given w and q than an unenthusiastic consumer, it must be the unenthusiastic consumer who is driven down to zero surplus for (w_u, q_u).

The valuing consumer would get positive surplus from accepting that same contract.

(3) To persuade the valuing consumer to accept (w_v, q_v) instead, the seller must give him that same positive surplus from it.

The seller will not be any more generous than he has to, though, so the valuing consumer's self-selection constraint will be binding.

Efficiency in the Second-Best

Rearranging our two binding constraints and setting them out as equalities we can reformulate the seller's problem from (14) as

$$\underset{q_u, q_v}{\text{Maximize}} \quad u(q_u) + u(q_u) - v(q_u) + v(q_v) - c \cdot (q_u + q_v), \tag{17}$$

which has the first-order conditions

$$\begin{aligned} &(a) \;\; u'(q_u) - c + [u'(q_u) - v'(q_u)] = 0 \\ &(b) \;\; v'(q_v) - c = 0 \end{aligned} \tag{18}$$

Equation (b) tells us that the valuing type of buyer buys a quantity such that his last unit's marginal utility exactly equals the marginal cost of production; his consumption is at the efficient level.

The unenthusiastic type, however, buys less than his first-best amount.

From the single-crossing property, $u'(q) < v'(q)$, which implies from (18a) that $u'(q_u) - c > 0$ and the unenthusiastic type has not bought enough to drive his marginal utility down to marginal cost.

The seller must sell less than first-best optimal to the unenthusiastic type so as not to make that contract too attractive to the valuing type. On the other hand, making the valuing type's contract more valuable to him actually helps separation, so q_v is chosen to maximize social surplus.

Quantities Bought

Substituting from first-order condition (b) into first-order condition (a) yields

$$[u'(q_u) - v'(q_v)] + [u'(q_u) - v'(q_u)] = 0 \tag{19}$$

The second term in square brackets is negative by the single-crossing property.

Thus, the first term must be positive.

But since the single-crossing property tells us that $[u'(q_u) - v'(q_u)] < 0$, it must be true, since $v'' < 0$, that if $q_u \geq q_v$ then $[u'(q_u) - v'(q_v)] < 0$ – that is, that the first term is negative.

We cannot have that without contradiction, so it must be that $q_u < q_v$. No pooling. Unenthusiastic buyer buys less.

Pooling vs. Separating Contracts

The separating contract in which both types buy positive amounts is not necessarily optimal.

Drop the assumption that the two types are of equal probability.

(1) Would the seller ever prefer a contract in which only the Enthustiastic buyer bought anything?

First– what would such a contract look like?

(2) Would the seller ever prefer a contract in which only the Unenthusiastic buyer bought anything?

First, what would that contract look like?

A Graphical Approach to the Same Problem

Here, just draw the diagram.

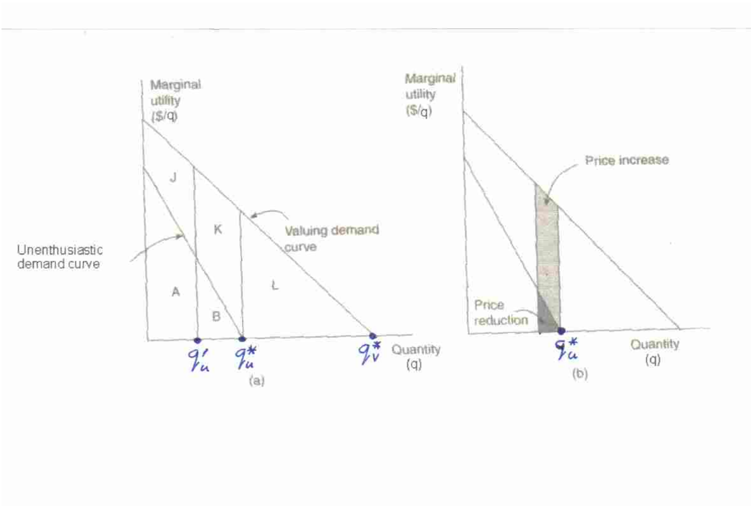


Diagram A:

PERFECT PRICE DISC, or INTERBUYER with lump sum prices:

A+B

A+B+J+K+L

SECOND BEST SEPARATING, BOTH BUY: wu A

wv = A+B+K+L

rent: J

deadweight loss: B

SECOND BEST SEP: JUST HIGH BUYS: w = A+B+J+K+L

The Single-Crossing Property

Condition (2) is an example of **the single-crossing property**, since it implies that the indifference curves of the two agents cross at most one time.

Combined with the assumption that $v(0) = u(0) = 0$, it implies that $u(q) < v(q)$ for any value of q.

When we say that Buyer V's demand is stronger than Buyer U's, however, there are two things we might mean:

1. Buyer V's *average demand* is stronger: $\frac{v(q)}{q} > \frac{u(q)}{q}$. Buyer V would pay more for quantity q than Buyer U would.

2. Buyer V's *marginal demand* is stronger: $v'(q) > u'(q)$. Buyer V would pay more for an additional unit than Buyer U would.

Definitions (1) and (2) are not equivalent. In Figure 6a, Buyer U is willing to pay 5 per unit up to $q = 4$, but only 1 per unit thereafter. Buyer V is willing to pay only 2 per unit up to $q = 10$, and 1 per unit thereafter. As a result $u(q) > v(q)$, and U has the stronger demand by definition (1). But for $q \in [4, 10]$, Buyer V is willing to pay 2 per new unit while Buyer U is only willing to pay 1, so in that interval Buyer V has the stronger demand by definition (2).

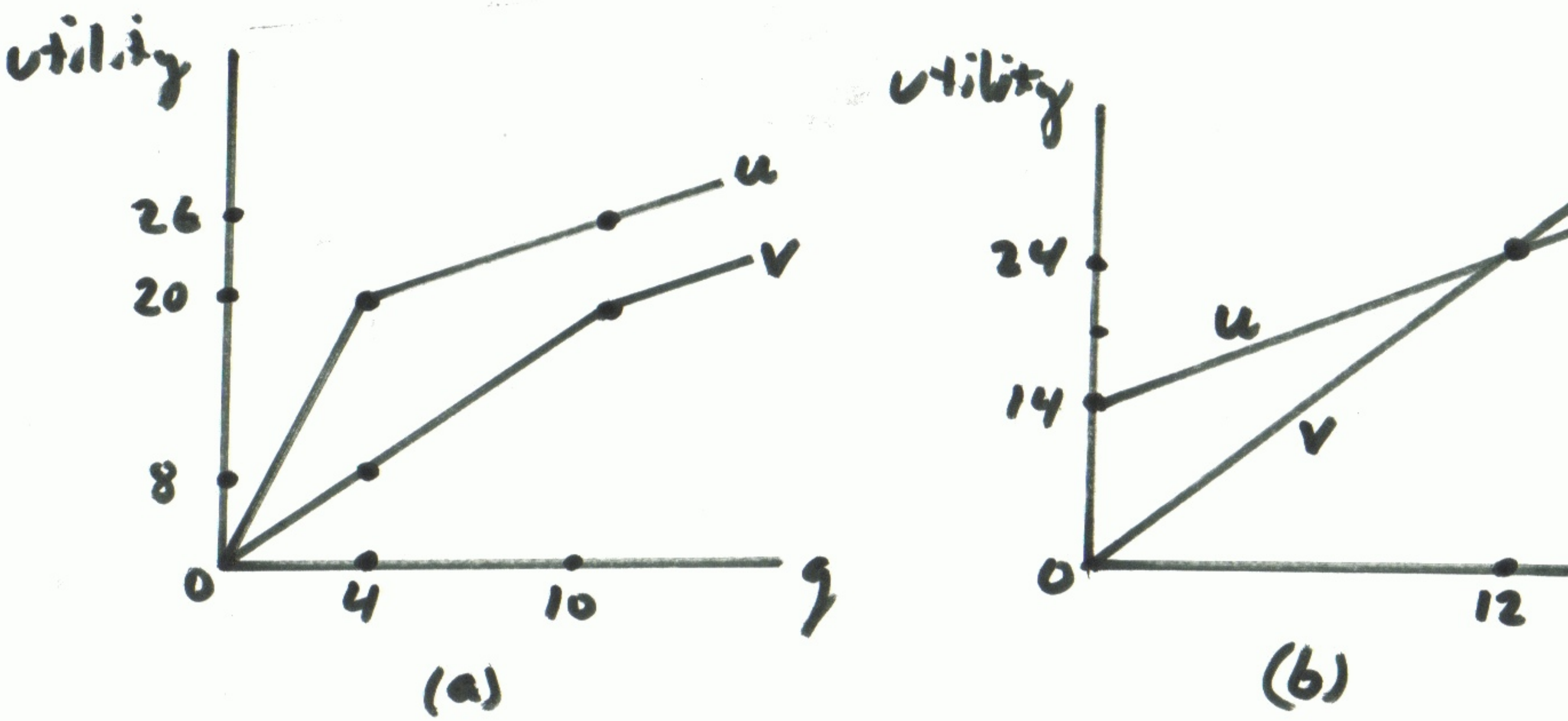


Figure 6: Marginal versus Average Demand

It is the marginal demand that is more important to economic behavior and which forms the basis for the single-crossing property. The utility functions in Figure 6b satisfy the single-crossing property, even though it is not true that $u(q) < v(q)$ (because they don't satisfy $u(0) = v(0) = 0$.)

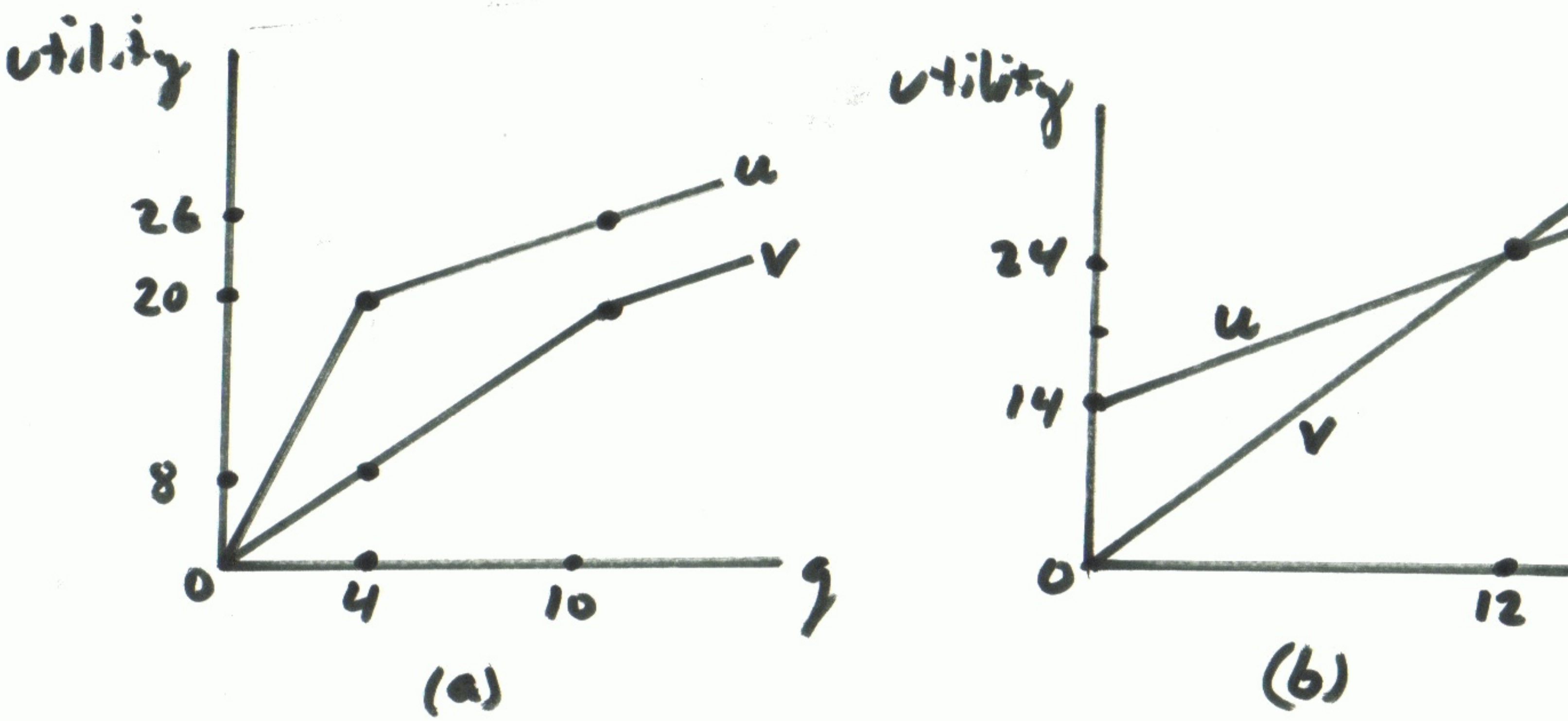
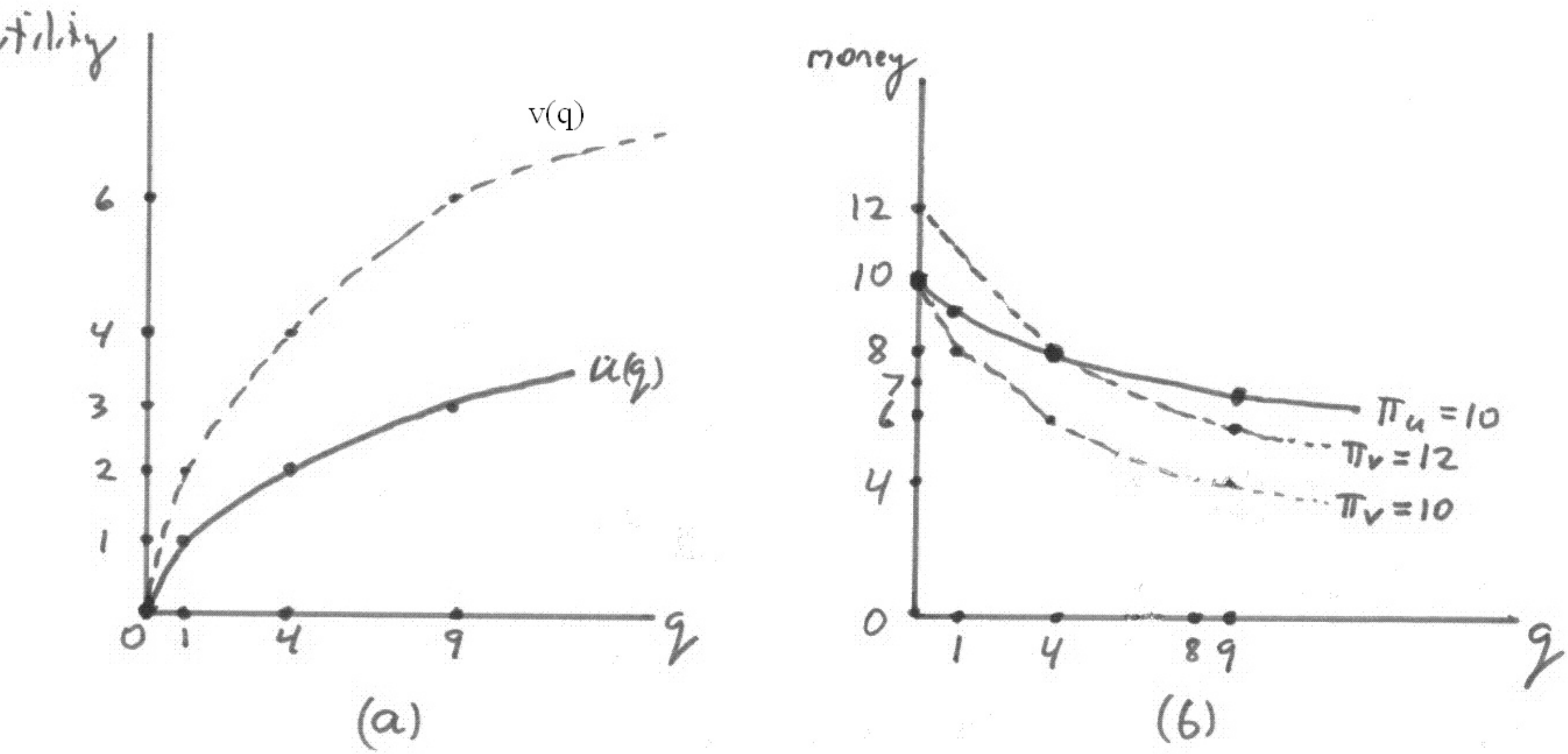


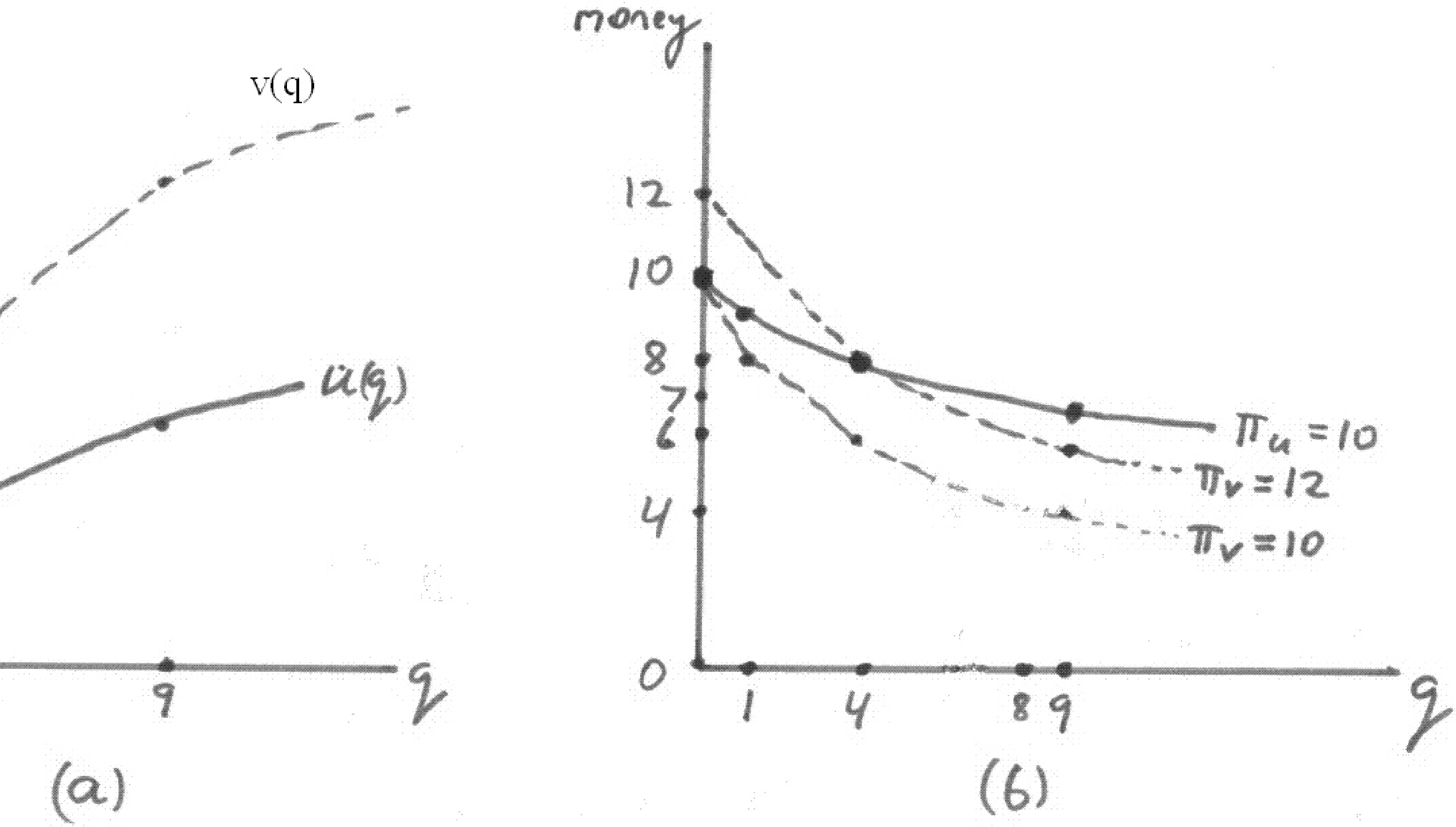
Figure 6: Marginal versus Average Demand

We wrote utility as $\pi = u(q) - w(q)$ before. This is the same as the quasilinear payoff function:

$\pi_u(q, money) = money + u(q)$, where $money = wealth - w(q)$.

Figure (a) satsifies single crossing. $u = \sqrt{q}$ and $v = 2\sqrt{q}$.





Utility is abstract, so it's nice to look at a graph without it on the axis.

Figure (b) shows how the buyers trade off money and the commodity.