Chapter 11
signalling

11.1 The Informed Player Moves First: Signalling

Signalling is a way for an agent to communicate his type under adverse selection. The signalling contract specifies a wage that depends on an observable characteristic – the signal – which the agent chooses for himself after Nature chooses his type. Figures 7.1d and 7.1e showed the extensive forms of two kinds of models with signals. If the agent chooses his signal before the contract is offered, he is signalling to the principal. If he chooses the signal afterwards, the principal is screening him. Not only will it become apparent that this difference in the order of moves is important, it will also be seen that signalling costs must differ between agent types for signalling to be useful, and the outcome is often inefficient.

We begin with signalling models in which workers choose education levels to signal their abilities. Section 11.1 lays out the fundamental properties of a signalling model, and section 11.2 shows how the details of the model affect the equilibrium. Section 11.3 steps back from the technical detail to more practical considerations in applying the model to education. Section 11.4 turns the game into a screening model. Section 11.5 switches to diagrams and applies signalling to new stock issues to show how two signals need to be used when the agent has two unobservable characteristics. Section 11.6 addresses the rather different idea of signal jamming: strategic behavior a player uses to cover up information rather than to disclose it.

Spence (1973) introduced the idea of signalling in the context of education. We will construct a series of models which formalize the notion that education has no direct effect on a person's ability to be productive in the real world but useful for demonstrating his ability to employers. Let half of the workers have the type "high-ability" and half "low-ability," where ability is a number denoting the dollar value of his output. Output is assumed to be a noncontractible variable and there is no uncertainty. If output is contractible, it should be in the contract, as we have seen in chapter 7. Lack of uncertainty is a simplifying assumption, imposed so that the contracts are functions only of the signals rather than a combination of the signal and the output.

Employers do not observe the worker's ability, but they do know the distribution of abilities, and they observe the worker's education. To simplify, we will specify that the players are one worker and two employers. The employers compete profits down to zero and the worker receives the gains from trade. The worker's strategy is his education level

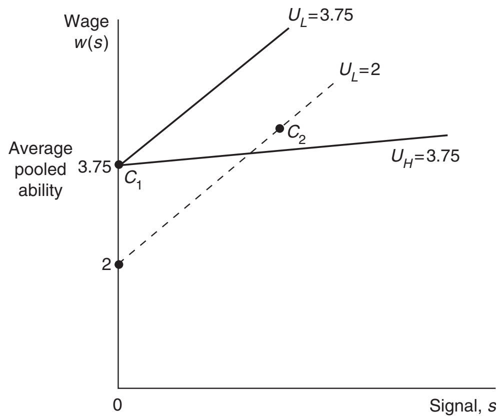

Figure 11.1 Education VI: no pooling equilibrium in a screening game.

and his choice of employer. The employers' strategies are the contracts they offer giving wages as functions of education level. The key to the model is that the signal, education, is less costly for workers with higher ability.

In the first four variants of the game, workers choose their education levels before employers decide how pay should vary with education.

Education I

PLAYERS
A worker and two employers.

THE ORDER OF PLAY

0 Nature chooses the worker's ability $a \in \{2, 5.5\}$, the *Low* and *High* ability each having probability 0.5. The variable a is observed by the worker, but not by the employers.
1 The worker chooses education level $s \in \{0, 1\}$.
2 The employers each offer a wage contract $w(s)$.
3 The worker accepts a contract, or rejects both of them.
4 Output equals a.

PAYOFFS
The worker's payoff is his wage minus his cost of education, and the employer's is his profit.

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w, \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the employer whose contract is accepted,} \\ 0 & \text{for the other employer.} \end{cases}$$

The payoffs assume that education is more costly for a worker if his ability takes a lower value, which is what permits separation to occur. As in any hidden knowledge game, we must think about both pooling and separating equilibria. Education I has both. In the pooling equilibrium, which we will call Pooling Equilibrium 1.1, both types of workers pick zero education and the employers pay the zero-profit wage of 3.75 regardless of the education level ($3.75 = [2 + 5.5]/2$).

$$\text{Pooling Equilibrium 1.1} \begin{cases} s(Low) = s(High) = 0 \\ w(s=0) = w(s=1) = 3.75 \\ Prob(a = Low|s=1) = 0.5 \end{cases}$$

Pooling Equilibrium 1.1 needs to be specified as a perfect Bayesian equilibrium rather than simply a Nash equilibrium because of the importance of the interpretation that the uninformed player puts on out-of-equilibrium behavior. The equilibrium needs to specify the employer's beliefs when he observes $s = 1$, since that is never observed in equilibrium. In Pooling Equilibrium 1.1, the beliefs are passive conjectures (see section 6.2): employers believe that a worker who chooses $s = 1$ is *Low* with the prior probability, which is 0.5. Given this belief, both types of workers realize that education is useless, and the model reaches the unsurprising outcome that workers do not bother to acquire unproductive education.

Under other beliefs, the pooling equilibrium breaks down. Under the belief $Prob(a = Low|s = 1) = 0$, for example, employers believe that any worker who acquired education is a *High*, so pooling is not Nash because the *High* workers are tempted to deviate and acquire education. This leads to the separating equilibrium for which signalling is best known, in which the high-ability worker acquires education to prove to employers that he really has high ability.

$$\text{Separating Equilibrium 1.2} \begin{cases} s(Low) = 0, s(High) = 1, \\ w(s=0) = 2, w(s=1) = 5.5. \end{cases}$$

Following the method used in chapters 7–10, we will show that Separating Equilibrium 1.2 is a perfect Bayesian equilibrium by using the standard constraints which an equilibrium must satisfy. A pair of separating contracts must maximize the utility of the *High*s and the *Low*s subject to two constraints: (1) the participation constraints that the firms can offer the contracts without making losses; and (2) the self-selection constraints that the *Low*s are not attracted to the *High* contract, and the *High*s are not attracted by the *Low* contract. The participation constraints for the employers require that

$$w(0) \le a_L = 2 \quad \text{and} \quad w(1) \le a_H = 5.5. \tag{11.1}$$

Competition between the employers makes the expressions in (11.1) hold as equalities. The self-selection constraint of the *Lows* is

$$U_L(s=0) \ge U_L(s=1), \tag{11.2}$$

which in Education I is

$$w(0) - 0 \ge w(1) - \frac{8(1)}{2}. \tag{11.3}$$

Since in Separating Equilibrium 1.2 the separating wage of the *Low*s is 2 and the separating wage of the *High*s is 5.5 from (11.1), the self-selection constraint (11.3) is satisfied.

The self-selection constraint of the *High*s is

$$U_H(s=1) \geq U_H(s=0), \tag{11.4}$$

which in Education I is

$$w(1) - \frac{8(1)}{5.5} \geq w(0) - 0. \tag{11.5}$$

Constraint (11.5) is satisfied by Separating Equilibrium 1.2.

There is another conceivable pooling equilibrium for Education I, in which $s(Low) = s(High) = 1$, but this turns out not to be an equilibrium, because the *Low*s would deviate to zero education. Even if such a deviation caused the employer to believe they were low-ability with probability 1 and reduce their wage to 2, the low-ability workers would still prefer to deviate, because

$$U_L(s=0) = 2 \geq U_L(s=1) = 3.75 - \frac{8(1)}{2}. \tag{11.6}$$

Thus, a pooling equilibrium with $s = 1$ would violate incentive compatibility for the *Low* workers.

Notice that we do not need to worry about a nonpooling constraint for this game, unlike in the case of the games of chapter 9. One might think that because employers compete for workers, competition between them might result in their offering a pooling contract that the high-ability workers would prefer to the separating contract. The reason this does not matter is that the employers do not compete by offering contracts, but by reacting to workers who have acquired education. That is why this is signalling and not screening: the employers cannot offer contracts in advance that change the workers' incentives to acquire education.

We can test the equilibrium by looking at the best responses. Given the worker's strategy and the other employer's strategy, an employer must pay the worker his full output or lose him to the other employer. Given the employers' contracts, the *Low* has a choice between the payoff 2 for ignorance (=2 − 0) and 1.5 for education (=5.5 − 8/2), so he picks ignorance. The *High* has a choice between the payoff 2 for ignorance (=2 − 0) and 4.05 for education (=5.5 − 8/5.5, rounded), so he picks education.

Unlike the pooling equilibrium, the separating equilibrium does not need to specify beliefs. Either of the two education levels might be observed in equilibrium, so Bayes' Rule always tells the employers how to interpret what they see. If they see that an agent has acquired education, they deduce that his ability is *High* and if they see that he has not, they deduce that it is *Low*. A worker is free to deviate from the education level appropriate to his type, but the employers' beliefs will continue to be based on equilibrium behavior. If a *High* worker deviates by choosing $s = 0$ and tells the employers he is a *High* who would rather pool than separate, the employers disbelieve him and offer him the *Low* wage of 2 that is appropriate to $s = 0$, not the pooling wage of 3.75 or the *High* wage of 5.5.

Separation is possible because education is more costly for workers if their ability is lower. If education were to cost the same for both types of worker, education would not work as a signal, because the low-ability workers would imitate the high-ability workers.

This requirement of different signalling costs is the **single-crossing property** that we have seen in chapter 10. When the costs are depicted graphically, as they will be in figure 11.1, the indifference curves of the two types intersect a single time.

A strong case can be made that the beliefs required for the pooling equilibria are not sensible. Harking back to the equilibrium refinements of section 6.2, recall that one suggestion (from Cho & Kreps [1987]) is to inquire into whether one type of player could not possibly benefit from deviating, no matter how the uninformed player changed his beliefs as a result. Here, the *Low* worker could never benefit from deviating from Pooling Equilibrium 1.1. Under the passive conjectures specified, the *Low* has a payoff of 3.75 in equilibrium versus -0.25 ($=3.75 - 8/2$) if he deviates and becomes educated. Under the belief that most encourages deviation – that a worker who deviates is *High* with probability one – the *Low* would get a wage of 5.5 if he deviated, but his payoff from deviating would only be 1.5 ($=5.5 - 8/2$), which is less than 2. The more reasonable belief seems to be that a worker who acquires education is a *High*, which does not support the pooling equilibrium.

The nature of the separating equilibrium lends support to the claim that education *per se* is useless or even pernicious, because it imposes social costs but does not increase total output. While we may be reassured by the fact that Professor Spence himself thought it worthwhile to become Dean of Harvard College, the implications are disturbing and suggest that we should think seriously about how well the model applies to the real world. We will do that later. For now, note that in the model, unlike most real-world situations, information about the agent's talent has no social value, because all agents would be hired and employed at the same task even under full information. Also, if side payments are not possible, Separating Equilibrium 1.2 is second-best efficient in the sense that a social planner could not make both types of workers better off. Separation helps the high-ability workers even though it hurts the low-ability workers.

Separation can also occur even if the signal of education has zero or negative cost for the high-ability workers, so long as it has positive cost for the low-ability workers. In such a case, however, the signalling is nonstrategic; it merely happens that the high-ability worker's efficient level of education is too high for low-ability workers to want to imitate them, and employers therefore deduce both natural and acquired ability from education.

11.2 Variants on the Signalling Model of Education

Although Education I is a curious and important model, it does not exhaust the implications of signalling. This section will start with Education II, which will show an alternative to the arbitrary assumption of beliefs in the perfect Bayesian equilibrium concept. Education III will be the same as Education I except for its different parameter values, and will have two pooling equilibrium rather than one separating and one pooling equilibrium. Education IV will allow a continuum of education levels, and will unify Education I and Education III by showing how all of their equilibria and more can be obtained in a model with a less restricted strategy space.

Education II: Modelling Trembles So Nothing Is Out of Equilibrium

The pooling equilibrium of Education I required the modeller to specify the employers' out-of-equilibrium beliefs. An equivalent model constructs the game tree to support the beliefs instead of introducing them via the equilibrium concept. This approach was briefly mentioned in connection with section 6.2's game of PhD Admissions. The advantage is that the assumptions on beliefs are put in the rules of the game along with the other assumptions. So let us replace Nature's move in Education I and modify the payoffs as follows.

Education II

THE ORDER OF PLAY

0 Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5. (a is observed by the worker, but not by the employer.) With probability 0.001, Nature endows a worker with free education of $s = 1$.

1 The game continues in the same way as Education I.

PAYOFFS

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w \text{ (ordinarily),} \\ w & \text{if the worker accepts contract } w \text{ (with free education),} \\ 0 & \text{if the worker does not accept a contract.} \end{cases}$$

With probability 0.001 the worker receives free education regardless of his ability (this might model idiosyncratic reasons someone might be educated that are unrelated to job prospects). If the employer sees a worker with education, he knows that the worker might be one of this rare type, in which case the probability that the worker is *Low* is 0.5. Both $s = 0$ and $s = 1$ can be observed in any equilibrium and Education II has almost the same two equilibria as Education I, without the need to specify beliefs. The separating equilibrium did not depend on beliefs, and remains an equilibrium. What was Pooling Equilibrium 1.1 becomes "almost" a pooling equilibrium – almost all workers behave the same, but the small number with free education behave differently. The two types of greatest interest, the *High* and the *Low*, are not separated, but the ordinary workers are separated from the workers whose education is free. Even that small amount of separation allows the employers to use Bayes' Rule and eliminates the need for exogenous beliefs.

More formally, the pooling equilibrium would have both abilities of workers choosing $s = 0$ unless a worker was one of the few endowed with automatic $s = 1$. Out-of-equilibrium, if the employer observed $s = 1$, he could use Bayes' Rule:

$$\begin{aligned} Prob(a = Low|s = 1) &= \frac{Prob(s = 1|a = L)Prob(L)}{Prob(s = 1|a = L)Prob(L) + Prob(s = 1|a = H)Prob(H)}, \\ &= \frac{(0.001)(0.5)}{(0.001)(0.5) + (0.001)(0.5)}, \\ &= 0.5. \end{aligned} \tag{11.7}$$

Normal workers would not deviate from $s = 0$ because it would not increase the employer's estimate of their ability.

Education III: No Separating Equilibrium, Two Pooling Equilibria

Let us next modify Education I by changing the possible worker abilities from $\{2, 5.5\}$ to $\{2, 12\}$. The separating equilibrium vanishes, but a new pooling equilibrium emerges. In Pooling Equilibria 3.1 and 3.2, both pooling contracts pay the same zero-profit wage of 7 (=[2 + 12]/2), and both types of agents acquire the same amount of education, but the amount depends on the equilibrium.

$$\text{Pooling Equilibrium 3.1} \begin{cases} s(Low) = s(High) = 0, \\ w(0) = w(1) = 7, \\ Prob(a = Low|s = 1) = 0.5 \text{ (passive conjectures).} \end{cases}$$

$$\text{Pooling Equilibrium 3.2} \begin{cases} s(Low) = s(High) = 1, \\ w(0) = 2, w(1) = 7, \\ Prob(a = Low|s = 0) = 1. \end{cases}$$

Pooling Equilibrium 3.1 is similar to the pooling equilibrium in Education I and II, but Pooling Equilibrium 3.2 is inefficient. Both types of workers receive the same wage, but they incur the education costs anyway. Each type is frightened to do without education because the employer would pay him not as if his ability were average, but as if he were known to be *Low*.

Examination of Pooling Equilibrium 3.2 shows why a separating equilibrium no longer exists. Any separating equilibrium would require $w(0) = 2$ and $w(1) = 7$, but this is the contract that leads to Pooling Equilibrium 3.2. The self-selection and zero-profit constraints cannot be satisfied simultaneously, because the *Low* type is willing to acquire $s = 1$ to obtain the high wage.

It is not surprising that information problems create inefficiencies in the sense that first-best efficiency is lost. Indeed, the surprise is that in some games with asymmetric information, such as Broadway Game I in section 7.4, the first-best can still be achieved by tricks such as boiling-in-oil contracts. More often, we discover that the outcome is second-best efficient: given the informational constraints, a social planner could not alter the equilibrium without hurting some type of player. Pooling Equilibrium 3.2 is not even second-best efficient, because Pooling Equilibrium 3.1 and Pooling Equilibrium 3.2 result in the exact same wages and allocation of workers to tasks. The inefficiency is purely a problem of unfortunate expectations, like the inefficiency from choosing the dominated equilibrium in Ranked Coordination.

Pooling Equilibrium 3.2 also illustrates a fine point of the definition of pooling, because although the two types of workers adopt the same strategies, the equilibrium contract offers different wages for different education. The implied threat to pay a low wage to an uneducated worker never needs to be carried out, so the equilibrium is still called a pooling equilibrium. Notice that perfectness does not rule out threats based on beliefs. The model imposes these beliefs on the employer, and he would carry out his threats, because he believes they are best responses. The employer receives a higher payoff under some beliefs than under others, but he is not free to choose his beliefs.

Following the approach of Education II, we could eliminate Pooling Equilibrium 3.2 by adding an exogenous probability 0.001 that either type is completely unable to buy education. Then every possible actions might be observed in equilibrium and we end up with Pooling Equilibrium 3.1 because the only rational belief is that if $s = 0$ is observed, the worker has equal probability of being *High* or being *Low*. To eliminate Pooling Equilibrium 3.1 requires less reasonable beliefs; for example, a probability of 0.001 that a *Low* gets free education together with a probability of 0 that a *High* does.

These first three games illustrate the basics of signalling: (1) separating and pooling equilibria both may exist, (2) out-of-equilibrium beliefs matter, and (3) sometimes one perfect Bayesian equilibrium can Pareto-dominate others. These results are robust, but Education IV will illustrate some dangers of using simplified games with binary strategy spaces instead of continuous and unbounded strategies. So far education has been limited to $s = 0$ or $s = 1$; Education IV allows it to take greater or intermediate values.

Education IV: Continuous Signals and Continua of Equilibria

Let us now return to Education I, with one change: that education s can take any level on the continuum between 0 and infinity.

Education IV

PLAYERS
A worker and two employers.

THE ORDER OF PLAY

0 Nature chooses the worker's ability $a \in \{2, 5.5\}$, the *Low* and *High* ability each having probability 0.5. The variable a is observed by the worker, but not by the employers.
1 The worker chooses education level $s \in [0, \infty)$.
2 The employers each offer a wage contract $w(s)$.
3 The worker accepts a contract, or rejects both of them.
4 Output equals a.

PAYOFFS
The worker's payoff is his wage minus his cost of education, and the employer's is his profit.

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w, \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the employer whose contract is accepted,} \\ 0 & \text{for the other employer.} \end{cases}$$

The game now has continua of pooling and separating equilibria which differ according to the value of education chosen. In the pooling equilibria, the equilibrium education level is s^*, where each s^* in the interval $[0, \bar{s}]$ supports a different equilibrium. The out-of-equilibrium belief most likely to support a pooling equilibrium is $Prob(a = Low|s \neq s^*) = 1$, so let us use this to find the value of $\bar{s}$, the greatest amount of education that can be generated by a pooling equilibrium. The equilibrium is Pooling Equilibrium 4.1, where $s^* \in [0, \bar{s}]$.

$$\text{Pooling Equilibrium 4.1} \begin{cases} s(Low) = s(High) = s^*, \\ w(s^*) = 3.75, \\ w(s \neq s^*) = 2, \\ Prob(a = Low|s \neq s^*) = 1. \end{cases}$$

The critical value $\bar{s}$ can be discovered from the incentive compatibility constraint of the *Low* type, which is binding if $s^* = \bar{s}$. The most tempting deviation is to zero education, so that is the deviation that appears in the constraint.

$$U_L(s = 0) = 2 \leq U_L(s = \bar{s}) = 3.75 - \frac{8\bar{s}}{2}. \tag{11.8}$$

Equation (11.8) yields $\bar{s} = 7/16$. Any value of s^* less than 7/16 will also support a pooling equilibrium. Note that the incentive-compatibility constraint of the *High* type is not binding. If a *High* deviates to $s = 0$, he, too, will be thought to be a *Low*, so

$$U_H(s = 0) = 2 \leq U_H\left(s = \frac{7}{16}\right) = 3.75 - \frac{8\bar{s}}{5.5} \approx 3.1. \tag{11.9}$$

In the separating equilibria, the education levels chosen in equilibrium are 0 for the *Low*'s and s^* for the *High*'s, where each s^* in the interval $[\bar{s}, \bar{\bar{s}}]$ supports a different equilibrium. A difference from the case of separating equilibria in games with binary strategy spaces is that now there are possible out-of-equilibrium actions even in a separating equilibrium. The two types of workers will separate to two education levels, but that leaves an infinite number of out-of-equilibrium education levels. As before, let us use the most extreme belief for the employers' beliefs after observing an out-of-equilibrium education level: that $Prob(a = Low|s \neq s^*) = 1$. The equilibrium is Separating Equilibrium 4.2, where $s^* \in [\bar{s}, s]$.

$$\text{Separating Equilibrium 4.2} \begin{cases} s(Low) = 0, \quad s(High) = s^*, \\ w(s^*) = 5.5, \\ w(s \neq s^*) = 2, \\ Prob(a = Low|s \notin \{0, s^*\}) = 1. \end{cases}$$

The critical value $\bar{s}$ can be discovered from the incentive-compatibility constraint of the *Low*, which is binding if $s^* = \bar{s}$.

$$U_L(s = 0) = 2 \geq U_L(s = \bar{s}) = 5.5 - \frac{8\bar{s}}{2}. \tag{11.10}$$

Equation (11.10) yields $\bar{s} = 7/8$. Any value of s^* greater than 7/8 will also deter the *Low* workers from acquiring education. If the education needed for the wage of 5.5 is too great, the *High* workers will give up on education too. Their incentive compatibility constraint requires that

$$U_H(s = 0) = 2 \leq U_H(s = \bar{\bar{s}}) = 5.5 - \frac{8\bar{\bar{s}}}{5.5}. \tag{11.11}$$

Equation (11.11) yields $\bar{\bar{s}} = 77/32$. s^* can take any lower value than $77/32$ and the *High*'s will be willing to acquire education.

The big difference from Education I is that Education IV has Pareto-ranked equilibria. Pooling can occur not just at zero education but at positive levels, as in Education III, and the pooling equilibria with positive education levels are all Pareto inferior. Also, the separating equilibria can be Pareto ranked, since separation with $s^* = \bar{s}$ dominates separation with $s^* = \bar{\bar{s}}$. Using a binary strategy space instead of a continuum conceals this problem.

Education IV also shows how restricting the strategy space can alter the kinds of equilibria that are possible. Education III had no separating equilibrium because at the maximum possible signal, $s = 1$, the *Low*'s were still willing to imitate the *High*'s. Education IV would not have any separating equilibria either if the strategy space were restricted to allow only education levels less than $\frac{7}{8}$. Using a bounded strategy space eliminates possibly realistic equilibria.

This is not to say that models with binary strategy sets are always misleading. Education I is a fine model for showing how signalling can be used to separate agents of different types; it becomes misleading only when used to reach a conclusion such as "[i]f a separating equilibrium exists, it is unique." As with any assumption, one must be careful not to narrow the model so much as to render vacuous the question it is designed to answer.

11.3 General Comments on Signalling in Education

Signalling and Similar Phenomena

The distinguishing feature of signalling is that the agent's action, although not directly related to output, is useful because it is related to ability. For the signal to work, it must be less costly for an agent with higher ability. Separation can occur in Education I because when the principal pays a greater wage to educated workers, only the *High*s, whose utility costs of education are lower, are willing to acquire it. That is why a signal works where a simple message would not: actions speak louder than words.

Signalling is outwardly similar to other solutions to adverse selection. The high-ability agent finds it cheaper than the low-ability one to build a reputation, but the reputation-building actions are based directly on his high ability. In a typical reputation model he shows ability by producing high output period after period. Also, the nature of reputation is to require several periods of play, which signalling does not.

Another form of communication is possible when some observable variable not under the control of the worker is correlated with ability. Age, for example, is correlated with reliability, so an employer pays older workers more, but the correlation does not arise because it is easier for reliable workers to acquire the attribute of age. Because age is not an action chosen by the worker, we would not need game theory to model it.

Problems in Applying Signalling to Education

On the empirical level, the first question to ask of a signalling model of education is, "What is education?" For operational purposes this means, "In what units is education measured?"

Two possible answers are "years of education" and "grade point average." If the sacrifice of a year of earnings is greater for a low-ability worker, years of education can serve as a signal. If less intelligent students must work harder to get straight As, then grade-point-average can also be a signal.

Layard & Psacharopoulos (1974) give three rationales for rejecting signalling as an important motive for education. First, dropouts get as high a rate of return on education as those who complete degrees, so the signal is not the diploma, although it might be the years of education. Second, wage differentials between different education levels rise with age, although one would expect the signal to be less important after the employer has acquired more observations on the worker's output. Third, testing is not widely used for hiring, despite its low cost relative to education. Tests are available, but unused: students commonly take tests like the American SAT whose results they could credibly communicate to employers, and their scores correlate highly with subsequent grade point average. One would also expect an employer to prefer to pay an 18-year-old low wages for four years to determine his ability, rather than waiting to see what grades he gets as a history major.

Productive Signalling

Even if education is largely signalling, we might not want to close the schools. Signalling might be wasteful in a pooling equilibrium like Pooling Equilibrium 3.2, but in a separating equilibrium it can be second-best efficient for at least three reasons. First, it allows the employer to match workers with jobs suited to their talents. If the only jobs available were "professor" and "typist," then in a pooling equilibrium, both *High* and *Low* workers would be employed, but they would be randomly allocated to the two jobs. Given the principle of comparative advantage, typing might improve, but I think, pridefully, that research would suffer.

Second, signalling keeps talented workers from moving to jobs where their productivity is lower but their talent is known. Without signalling, a talented worker might leave a corporation and start his own company, where he would be less productive but better paid. The naive observer would see that corporations hire only one type of worker (*Low*), and imagine there was no welfare loss.

Third, if ability is endogenous – moral hazard rather than adverse selection – signalling encourages workers to acquire ability. One of my teachers said that you always understand your next-to-last econometrics class. Suppose that solidly learning econometrics increases the student's ability, but a grade of A is not enough to show that he solidly learned the material. To signal his newly acquired ability, the student must also take "Time Series," which he cannot pass without a solid understanding of econometrics. "Time Series" might be useless in itself, but if it did not exist, the students would not be able to show he had learned basic econometrics.

11.4 The Informed Player Moves Second: Screening

In screening games, the informed player moves second, which means that he moves in response to contracts offered by the uninformed player. Having the uninformed player make the offers is important because his offer conveys no information about himself, unlike in a signalling model.

Education V: Screening with a Discrete Signal

PLAYERS
A worker and two employers.

THE ORDER OF PLAY
0 Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5. Employers do not observe ability, but the worker does.
1 Each employer offers a wage contract $w(s)$.
2 The worker chooses education level $s \in \{0, 1\}$.
3 The worker accepts a contract, or rejects both of them.
4 Output equals a.

PAYOFFS

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w, \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the employer whose contract is accepted,} \\ 0 & \text{for the other employer.} \end{cases}$$

Education V has no pooling equilibrium, because if one employer tried to offer the zero profit pooling contract, $w(0) = 3.75$, the other employer would offer $w(1) = 5.5$ and draw away all the *High*s. The unique equilibrium is

$$\text{Separating Equilibrium 5.1} \begin{cases} s(Low) = 0, s(High) = 1, \\ w(0) = 2, w(1) = 5.5. \end{cases}$$

Beliefs do not need to be specified in a screening model. The uninformed player moves first, so his beliefs after seeing the move of the informed player are irrelevant. The informed player is fully informed, so his beliefs are not affected by what he observes. This is much like simple adverse selection, in which the uninformed player moves first, offering a set of contracts, after which the informed player chooses one of them. The modeller does not need to refine perfectness in a screening model. The similarity between adverse selection and screening is strong enough that Education V would not have been out of place in chapter 9, but it is presented here because the context is so similar to the signalling models of education.

Education VI allows a continuum of education levels, in a game otherwise the same as Education V.

Education VI: Screening with a Continuous Signal

PLAYERS
A worker and two employers.

THE ORDER OF PLAY
0 Nature chooses worker ability $a \in \{2, 5.5\}$, each ability having probability 0.5. Employers do not observe ability, but the worker does.
1 Each employer offers a wage contract $w(s)$.
2 The worker choose education level $s \in [0, 1]$.
3 The worker chooses a contract, or rejects both of them.
4 Output equals a.

PAYOFFS

$$\pi_{worker} = \begin{cases} w - 8s/a & \text{if the worker accepts contract } w, \\ 0 & \text{if the worker rejects both contracts.} \end{cases}$$

$$\pi_{employer} = \begin{cases} a - w & \text{for the employer whose contract is accepted,} \\ 0 & \text{for the other employer.} \end{cases}$$

Pooling equilibria generally do not exist in screening games with continuous signals, and sometimes separating equilibria in pure strategies do not exist either – recall Insurance Game III from section 9.4. Education VI, however, does have a separating Nash equilibrium, with a unique equilibrium path.

$$\text{Separating Equilibrium 6.1} \begin{cases} s(Low) = 0, s(High) = 0.875, \\ w = \begin{cases} 2 & \text{if } s < 0.875, \\ 5.5 & \text{if } s \geq 0.875. \end{cases} \end{cases}$$

In any separating contract, the *Low*s must be paid a wage of 2 for an education of 0, because this is the most attractive contract that breaks even. The separating contract for the *High*s must maximize their utility subject to the constraints discussed in Education I. When the signal is continuous, the constraints are especially useful to the modeller for calculating the equilibrium. The participation constraints for the employers require that

$$w(0) \leq a_L = 2 \quad \text{and} \quad w(s^*) \leq a_H = 5.5, \tag{11.12}$$

where s^* is the separating value of education that we are trying to find. Competition turns the inequalities in (11.12) into equalities. The self-selection constraint for the low-ability workers is

$$U_L(s = 0) \geq U_L(s = s^*), \tag{11.13}$$

which in Education VI is

$$w(0) - 0 \geq w(s^*) - \frac{8s^*}{2}. \tag{11.14}$$

Since the separating wage is 2 for the *Low*s and 5.5 for the *High*s, constraint (11.14) is satisfied as an equality if $s^* = 0.875$, which is the crucial education level in Separating Equilibrium 6.1.

$$U_H(s = 0) = w(0) \leq U_H(s = s^*) = w(s^*) - \frac{8s^*}{5.5}. \tag{11.15}$$

If $s^* = 0.875$, inequality (11.15) is true, and it would also be true for higher values of s^*. Unlike the case of the continuous-strategy signalling game, Education IV, however, the equilibrium contract in Education VI is unique, because the employers compete to offer the most attractive contract that satisfies the participation and incentive compatibility constraints. The most attractive is the separating contract that Pareto dominates the other separating contracts by requiring the relatively low separating signal of $s^* = 0.875$.

Similarly, competition in offering attractive contracts rules out pooling contracts. The nonpooling constraint, required by competition between employers, is

$$U_H(s = s^*) \geq U_H(pooling), \tag{11.16}$$

which, for Education VI, is, using the most attractive possible pooling contract,

$$w(s^*) - \frac{8s^*}{5.5} \geq 3.75. \tag{11.17}$$

Since the payoff of *High*s in the separating contract is 4.23 ($=5.5 - 8 \times 0.875/5.5$, rounded), the nonpooling constraint is satisfied.

No Pooling Equilibrium in Screening: Education VI

The screening game Education VI lacks a pooling equilibrium, which would require the outcome $\{s = 0, w(0) = 3.75\}$, shown as C_1 in figure 11.1. If one employer offered a pooling contract requiring more than zero education (such as the inefficient Pooling Equilibrium 3.2), the other employer could make the more attractive offer of the same wage for zero education. The wage is 3.75 to ensure zero profits. The rest of the wage function – the wages for positive education levels – can take a variety of shapes, so long as the wage does not rise so fast with education that the *High*s are tempted to become educated.

But no equilibrium has these characteristics. In a Nash equilibrium, no employer can offer a pooling contract, because the other employer could always profit by offering a separating contract paying more to the educated. One such separating contract is C_2 in figure 11.1, which pays 5 to workers with an education of $s = 0.5$ and yields a payoff of 4.89 to the *High*s ($=5 - [8 \times 0.5]/5.5$, rounded) and 3 to the *Low*s ($=5 - 8 \times 0.5/2$). Only *High*s prefer C_2 to the pooling contract C_1, which yields payoffs of 3.75 to both *High* and *Low*, and if only *High*s accept C_2, it yields positive profits to the employer.

Nonexistence of a pooling equilibrium in screening models without continuous strategy spaces is a general result. The linearity of the curves in Education VI is special, but in any screening model the *Low*s would have greater costs of education, which is equivalent to steeper indifference curves. This is the **single-crossing property** alluded to in Education I. Any pooling equilibrium must, like C_1, lie on the vertical axis where education is zero and the wage equals the average ability. A separating contract like C_2 can always be found to the northeast of the pooling contract, between the indifference curves of the two types, and it will yield positive profits by attracting only the *High*s.

Education VII: No Pure-strategy Equilibrium in a Screening Game

In Education VI we showed that screening models have no pooling equilibria. In Education VII the parameters are changed a little to eliminate even the separating equilibrium in pure strategies. Let the proportion of *High*s be 0.9 instead of 0.5, so the zero-profit pooling wage is 5.15 (=0.9[5.5] + 0.1[2]) instead of 3.75. Consider the separating contracts C_3 and C_4, shown in figure 11.2, calculated in the same way as Separating Equilibrium 5.1. The pair (C_3, C_4) is the most attractive pair of contracts that separates *High*s from *Low*s. *Low* workers accept contract C_3, obtain $s = 0$, and receive a wage of 2, their ability. *High*s accept contract C_4, obtain $s = 0.875$, and receive a wage of 5.5, their ability. Education is not attractive to *Low*s because the *Low* payoff from pretending to be *High* is 2 ($=5.5 - 8 \times 0.875/2$), no better than the *Low* payoff of 2 from C_3 ($=2 - 8 \times 0/2$).

The wage of the pooling contract C_5 is 5.15, so that even the *High*s strictly prefer C_5 to (C_3, C_4). But our reasoning that no pooling equilibrium exists is still valid; some contract C_6 would attract all the *High*s from C_5. No Nash equilibrium in pure strategies exists, either separating or pooling.

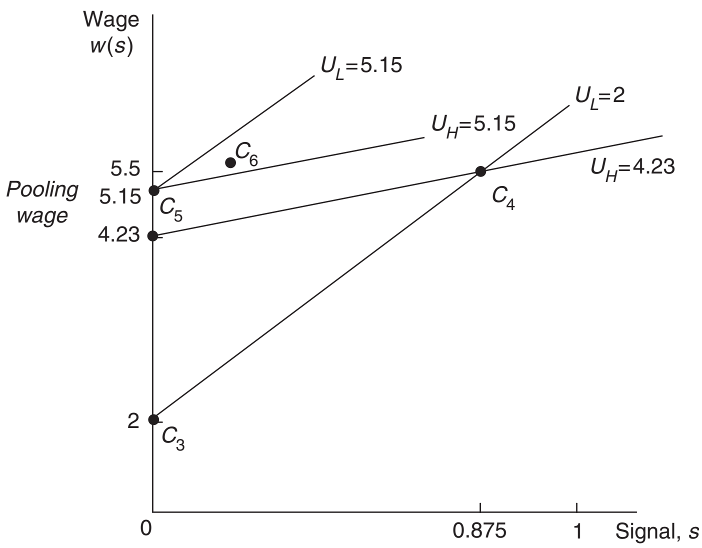

Figure 11.2 Education VII: neither separating nor pooling pure-strategy equilibria in a screening game.

Should the Modeller Worry about Multiple Equilibria, or Nonexistence of a Pure-strategy Equilibrium?

There is no reason why the modeller should be more worried about nonexistence of an equilibrium in pure strategies, per se, in a signalling model. As far back as chapter 3 we looked at games in which the only realistic equilibrium is in mixed strategies. In that chapter, and also in chapter 1 and in chapter 6 (on dynamic games with incomplete information) we discussed multiple equilibria, and why we should not be unduly worried about them either – the main reason being that under the logic of Nash equilibrium, which is self-fulfilling expectations, we should indeed expect different expectations to yield different outcomes.

It is in coordination models and signalling models that multiple equilibria have created the most anxiety among economists. For coordination models, there has been hope that cheap talk and focal points can often narrow down our predictions to a single outcome. For signalling models, the various refinements discussed in chapter 6 have been proposed, but with unenthusiastic reception. For both coordination games and signalling games, however, there is another way out: changing the order of moves. If moves are not simultaneous, and if, in the signalling game, it is not the informed player but the uninformed player who moves first, the outcome is much more likely to be both uniquely determined and efficient. Because the outcome is more likely to be efficient, it will also often be the case that the players would like the moves to be sequential, or that evolution will cause such a game to more often observed.

Consider what happens if the principal makes the offer of a particular reward for a particular signal level – a screening model instead of a signalling model. That simple change will eliminate Pareto-dominated contracts, because the uninformed player will not offer them.

As a specific example, suppose job candidates can heavily research the employer before an interview, and this works as a signal because less serious applicants are less willing to expend that effort.

One possibility is a pooling equilibrium in which both good and bad applicants signal by researching. The equilibrium is maintained by the fear that if an applicant does *not* he will be thought to be the bad type. This is inefficient, because the employer, in the end, gets no information.

An employer, therefore, would like to switch to a screening game. He would say to applicants that they should not research the company, because he knows that researching is not a good signal. In response, neither type of applicant would research. And the company would profit by reducing the salary it offers, acquiring some of the social savings from less researching. (A "no presents" rule at a birthday party is another example.)

Note, however, that just because you observe all applicants researching, an apparent pooling equilibrium, it might not be the case that the equilibrium really is pooling. It could be that the analyst's data has a self-selection problem: given that researching is necessary to have a chance at the job, only the good types apply – and they all research it. Be careful in concluding that a costly signal really is not separating out the types.

A second, related, inefficient equilibrium is when the equilibrium is separating but the signal is inefficiently high. In equilibrium, job applicants must be able to recite a list of the past four presidents of the company, but even requiring a list of two presidents would separate the good applicants from the bad. In response, the company could say before the interview that applicants need only know the history of the company for the past five years, and that it considers excess knowledge a sign of desperation on the part of the applicant.

A third, different kind of equilibrium is pooling where no signalling occurs: applicants believe that employers are unimpressed by their knowledge of company history and employers are unimpressed because only a blundering applicant would bother to research it. Such an equilibrium need not be inefficient. It might be that the cost of the signal is not worth the efficiency gains from separation. But it might be inefficient: the gains from hiring the most enthusiastic applicant, one who could stand to spend ten hours researching the history of an accounting firm merely to slightly increase his probability of getting a job there, might be worth the cost. If so, the employer could break out of the inefficient pooling equilibrium by announcing that applicants will be rated partly on their knowledge of company history. The pooling equilibrium would then be too delicate to survive.

A fourth kind of equilibrium is inefficient separation – an equilibrium where signalling occurs and separate good types of applicants from bad, but the cost to the applicants is not worth the social benefit. The good type of applicant may benefit, but the bad type loses even more, since the benefit is a pure transfer of who gets the job and this is achieved at a cost, the cost of the research.

Inefficient separation of this kind is more robust than the first three kinds of inefficient equilibria. The reason is that if one employer deviates and announces that it will no longer give applicants credit for their knowledge of company history, the result would simply be that they would attract even more low-quality applicants, while their high-quality applicants would desert them for employers who would respond to the research signal. If, however, there were only one employer, that employer could successfully pursue such a policy. It would change the signalling game to a screening game, and announce a reduction in the wage to the applicants' reservation level, but also ignore knowledge of company history in interviews. This is an interesting efficiency advantage of a monopsony.

Thus, very often, if not always, the timing of moves in a signalling game is flexible enough that in the metagame in which the players can choose the order of the moves, they will choose to convert it to a screening game. In the end, this will often eliminate all but one equilibrium outcome, and often the remaining outcome will be efficient, or at least Pareto-optimal.

A Summary of the Education Models

Because of signalling's complexity, most of this chapter has been devoted to elaboration of the education model. We began with Education I, which showed how with two types and two signal levels the perfect Bayesian equilibrium could be either separating or pooling. Education II took the same model and replaced the specification of out-of-equilibrium beliefs with an additional move by Nature, while Education III changed the parameters in Education I to increase the difference between types and to show how signalling could continue with pooling. Education IV changed Education I by allowing a continuum of education levels, which resulted in a continuum of inefficient equilibria, each with a different signal level. After a purely verbal discussion of how to apply signalling models, we looked at screening, in which the employer moves first. Education V was a screening reprise of Education I, while Education VI broadened the model to allow a continuous signal, which eliminates pooling equilibria. Education VII modified the parameters of Education VI to show that sometimes no pure-strategy Nash equilibrium exists at all.

Throughout it was implicitly assumed that all the players were risk-neutral. Risk-neutrality is unimportant, because there is no uncertainty in the model and the agents bear no risk. If the workers were risk-averse and they differed in their degrees of risk aversion, the contracts could try to use the difference to support a separating equilibrium because willingness to accept risk might act as a signal. If the principal were risk-averse he might offer a wage less than the average productivity in the pooling equilibrium, but he is under no risk at all in the separating equilibrium, because it is fully revealing. The models are also games of certainty, and this too is unimportant. If output were uncertain, agents would just make use of the expected payoffs rather than the raw payoffs and very little would change.

Ways to Communicate

We have by now seen a variety of ways one player can convey information to another. Following the Crawford–Sobel model of chapter 10, let us call these players the Sender and the Receiver. Why might the Receiver believe the Sender's message? The reason depends on the setting.

1 **Cheap talk games:** The Sender's message is costless and there is no penalty for lying.

In a cheap talk game, messages have no direct impact on payoff functions. If the Receiver ignores the message, the Sender's payoff is unaffected by the message. If the Receiver changes his action in response, though, that might affect the Sender. Usually, these are coordination games, where the Sender's preference for the action the Receiver will choose in response to the message is the same as the Receiver's, or at least correlated with it. Chapter 10's Crawford–Sobel model is an example of imperfect correlation, in which cheap talk results in some information being exchanged, even though the message is coarse and the Sender cannot convey precisely what he knows.

2 **Truthful announcement games:** The Sender may be silent instead of sending a message, but if he sends a message it must be truthful. The message might or might not be costly.

The Sender's type usually varies from bad to good in these models, and they are subject to the "unravelling" illustrated in chapter 10's example, since silence indicates bad news. As discussed there, unravelling might only be partial, for a variety of reasons such as some exogenous probability that the Sender actually is unable to send even a truthful message. One variant is when the Sender has committed to the truth, perhaps via a mechanism negotiated with the Receiver, converting a cheap talk game into a truthful announcement game.

3 **Auditing games:** The Receiver may audit the message at some cost and discover if the Sender was lying. The Sender's message might or might not be costly.

We saw examples of this in chapter 3, with commitment to auditing. A variant is when the Receiver cannot commit to auditing, in which case there will be an equilibrium in mixed strategies, with the Sender sometimes truthful, sometimes not, and the Receiver sometimes auditing, sometimes not. An example is Rasmusen (1993), in which the Sender is a lobbyist or protester who sends a costly message to the Receiver, a government official. That cost is wasted if the Receiver audits and finds the message is a lie, even if there is no additional punishment of the kind found in truthful announcement games.

4 **Mechanism games:** The Sender's message might or might not be costly. Before he sends it, he commits to a contract with the Receiver, with their decisions based on what they can observe and enforcement based on what can be verified by the courts.

Mechanisms were the topic of chapter 10. Commitment is the key. Mechanisms work out most simply if they are chosen before the Sender receives his private information, since otherwise the choice of mechanism may itself convey information.

5 **Signalling games:** The Sender's message is costly when he lies, and more costly when he lies than when he tells the truth. He sends it before the Receiver takes any action.

Signalling is the main topic of this chapter, chapter 11. The Sender's type usually varies from bad to good in these models. The single-crossing property is crucial – that if the Sender's type is better, it is cheaper for him to send a message that his type is good. It might well be costless for the Sender to send a truthful message (or even have negative costs), because what matters is the difference in costs between a truthful message and a false one. Usually in these models the message is an indirect one, conveyed by the choice of some action such as years of education seemingly unrelated to communication.

6 **Expensive-talk games:** The Sender's message is costly, but the cost is the same regardless of his type. There is no penalty for lying.

The difference between expensive talk and signalling is that in expensive talk the single-crossing property is not satisfied. Truthful communication might still work, for reasons akin to those in the Cheap-Talk Game, if the high-type Sender has a greater desire than the low type for the Receiver to adopt a high response. Chapter 6's PhD Admissions Game is an example. The student who hates economics has no incentive to pay the cost of applying for the PhD program. As with cheap talk and signalling, expectations are crucial because of multiple equilibria.

7 **Screening games:** The Sender's message is costly when he lies, and more costly when he lies than when he tells the truth. He sends it in response to an offer by the Receiver.

We have discussed screening games here in chapter 11. If the Receiver can commit to his response to a signal, this is a mechanism game. If he cannot, it is an expensive-talk game, where the Receiver's offer is made as part of an equilibrium in which the Sender's making the offer influences the Receiver's expectations of what the Sender will do.

In the rest of this chapter, we will look at more specialized models related to signalling.

*11.5 Two Signals: The Game of Underpricing New Stock Issues

One signal might not be enough when there is not one but two characteristics of an agent that he wishes to communicate to the principal. This has been generally analyzed in Engers (1987), and multiple signal models have been especially popular in financial economics, for example, the multiple signal model used to explain the role of investment bankers in new stock issues by Hughes (1986). We will use a model of initial public offerings of stock as the example in this section.

Empirically, it has been found that companies consistently issue stock at a price so low that it rises sharply in the days after the issue, an abnormal return estimated to average 11.4 percent (Copeland & Weston [1988], p. 377). The game of Underpricing New Stock Issues tries to explain this using the percentage of the stock retained by the original owner and the amount of underpricing as two signals. The two characteristics being signalled are the mean of the value of the new stock, which is of obvious concern to the potential buyers, and the variance, the importance of which will be explained later.

Underpricing New Stock Issues (Grinblatt & Hwang [1989])

PLAYERS
The entrepreneur and many investors.

THE ORDER OF PLAY
(See figure 2.3a for a time line.)

0 Nature chooses the expected value (μ) and variance (σ^2) of a share of the firm using some distribution F.
1 The entrepreneur retains fraction α of the stock and offers to sell the rest at a price per share of P_0.
2 The investors decide whether to accept or reject the offer.
3 The market price becomes P_1, the investors' estimate of μ.
4 Nature chooses the value V of a share using some distribution G such that μ is the mean of V and σ^2 is the variance. With probability θ, V is revealed to the investors and becomes the market price.
5 The entrepreneur sells his remaining shares at the market price.

PAYOFFS

$$\pi_{entrepreneur} = U([1-\alpha]P_0 + \alpha[\theta V + (1-\theta)P_1]), \quad \text{where } U' > 0 \text{ and } U'' < 0,$$
$$\pi_{investors} = (1-\alpha)(V - P_0) + \alpha(1-\theta)(V - P_1).$$

The entrepreneur's payoff is the utility of the value of the shares he issues at P_0 plus the value of those he sells later at the price P_1 or V. The investors' payoff is the true value of the shares they buy minus the price they pay.

Underpricing New Stock Issues subsumes the simpler model of Leland & Pyle (1977), in which σ^2 is common knowledge and if the entrepreneur chooses to retain a large fraction of the shares, the investors deduce that the stock value is high. The one signal in that model is fully revealing because holding a larger fraction exposes the undiversified entrepreneur to a larger amount of risk, which he is unwilling to accept unless the stock value is greater than investors would guess without the signal.

If the variance of the project is high, that also increases the risk to the undiversified entrepreneur, which is important even though the investors are risk-neutral and do not care directly about the value of σ^2. Since the risk is greater when variance is high, the signal α is more effective and retaining a smaller amount allows the entrepreneur to sell the remainder at the same price as a larger amount for a lower-variance firm. Even though the investors are diversified and do not care directly about firm-specific risk, they are interested in the variance because it tells them something about the effectiveness of entrepreneur-retained

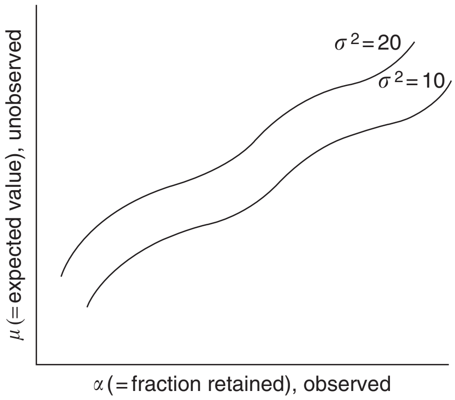

Figure 11.3 How the signal changes with the variance.

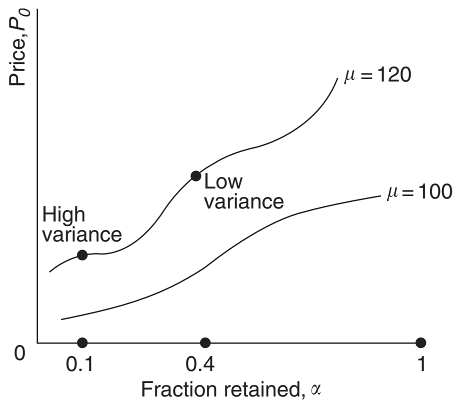

Figure 11.4 Different ways to signal a given company value.

shares as a signal of share value. Figure 11.3 shows the signalling schedules for two variance levels.

In the game of Underpricing New Stock Issues, σ^2 is not known to the investors, so the signal is no longer fully revealing. An α equal to 0.1 could mean either that the firm has a low value with low variance, or a high value with high variance. But the entrepreneur can use a second signal, the price at which the stock is issued, and by observing α and P_0, the investors can deduce μ and σ^2.

I will use specific numbers for concreteness. The entrepreneur could signal that the stock has the high mean value, $\mu = 120$, in two ways: (1) retaining a high percentage, $\alpha = 0.4$, and making the initial offering at a high price of $P_0 = 90$, or (2) retaining a low percentage, $\alpha = 0.1$, and making the initial offering at a low price, $P_0 = 80$. Figure 11.4 shows the different combinations of initial price and fraction retained that might be used. If the stock has a high variance, he will want to choose behavior (2), which reduces his risk. Investors deduce that the stock of anyone who retains a low percentage and offers a low price actually has $\mu = 120$ and a high variance, so stock offered at the price of 80 rises in price. If, on the other hand, the entrepreneur retained $\alpha = 0.1$ and offered the high price $P_0 = 90$, investors would conclude that μ was lower than 120 but that variance was low also, so the stock would not rise in price. The low price conveys the information that this stock has a high mean and high variance rather than a low mean and low variance.

This model explains why new stock is issued at a low price. The entrepreneur knows that the price will rise, but only if he issues it at a low initial price to show that the variance is high. The price discount shows that signalling by holding a large fraction of stock is unusually costly, but he is nonetheless willing to signal. The discount is costly because he is selling stock at less than its true value, and retaining stock is costly because he bears extra risk, but both are necessary to signal that the stock is valuable.

*11.6 Signal Jamming and Limit Pricing

This chapter has examined a number of models in which an informed player tries to convey information to an uninformed player by some means or other – by entering into an incentive contract, or by signalling. Sometimes, however, the informed party has the opposite problem: his natural behavior would convey his private information but he wants to keep it secret. This happens, for example, if one firm is informed about its poor ability to compete successfully, and it wants to conceal this information from a rival. The informed player may then engage in costly actions, just as in signalling, but now the costly action will be **signal jamming** (a term coined in Fudenberg & Tirole [1986c]): preventing information from appearing rather than generating information.

Limit pricing refers to the practice of keeping prices low to deter entry. Limit pricing can be explained in a variety of ways; notably, as a way for the incumbent to signal that he has low enough costs that rivals would regret entering, as in problem 6.2 and Milgrom & Roberts (1982a). Here, the explanation for limit pricing will be signal jamming: by keeping profits low, the incumbent keeps it unclear to the rival whether the market is big enough to accommodate two firms profitably. In previous editions I used a model from Rasmusen (1997) as an illustration, but in this edition I will use a simpler model that is still able to convey the idea of signal jamming. The market will be one with inelastic demand up to a reservation price. Entry would split that inelastic quantity evenly between the firms, and the price would fall. What is crucial is that before entry the entrant does not know the reservation price, and hence does not know how far the price would fall after entry. The incumbent may or may not find it worthwhile to keep his price low to prevent the entrant from learning something about the reservation price.

The incumbent needs to trade off a high price in the first period against the possibility of inducing entry. As always, work back from the end.

The incumbent's choice of p_{2m} is easy: choose $p_{2m} = v$, since there is no threat of future entry.

The rival's expected payoff from entry depends on his beliefs about v, which in turn depend upon which of the multiple equilibria of this game is being played out.

If the incumbent were nonstrategic, he would charge $p_1 = v$, maximizing his first-period profit. The rival would deduce the value v and would enter in the second-period

Limit Pricing as Signal Jamming

PLAYERS
The incumbent and the rival.

THE ORDER OF PLAY

0 Nature chooses the reservation price v using the continuous density $h(v)$ on the support $[c, d]$, observed only by the incumbent. (The parameter c is marginal cost.)
1 The incumbent chooses the first-period price p_1, generating sales of q if $p_1 \leq v$ and 0 otherwise.The variables p_1 and q are observed by both players.
2 The rival decides whether to enter at cost F or to stay out.
3 If the rival did not enter, the incumbent chooses the second period price, $p_{2,m}$, generating sales of q if $p_{2m} \leq v$ and 0 otherwise.
4 If the rival did enter, the duopoly price is $p_2(v)$, with $p_2 \geq c$ and $dp_2/dv > 0$.

Let us make more specific assumptions to avoid caveats later. Assume that the second-period duopoly price is halfway between marginal cost and the monopoly price, so

$$p_2(v) = \frac{c + v}{2}, \tag{11.18}$$

the distribution of reservation prices is uniform, so

$$h(v) = \frac{1}{d - c}, \tag{11.19}$$

that $c = 0$, and that $d > 6.4F/q$. We will defer using these special assumptions until midway through our exposition.

PAYOFFS
If the rival does not enter and p_1 and p_{2m} are no greater than v, the payoffs are

$$\begin{aligned} \pi_{incumbent} &= (p_1 - c)q + (p_2 - c)q, \\ \pi_{rival} &= 0 \end{aligned} \tag{11.20}$$

If the rival does enter and p_1 are no greater than v, the payoffs are

$$\begin{aligned} \pi_{incumbent} &= (p_1 - c)q + (p_2(v) - c)\left(\frac{q}{2}\right) \\ \pi_{rival} &= -F + (p_2(v) - c)\left(\frac{q}{2}\right). \end{aligned} \tag{11.21}$$

if p_1 exceeded the critical value $v^* = c + 2F/q$, a value which yields zero profits because

$$-F + \left[\left(c + \frac{2F}{q} - c \right) \left(\frac{q}{2} \right) \right] = 0. \tag{11.22}$$

If the incumbent is willing to charge a lower p_1, though, and accept lower first-period profits, he may be able to deter entry, and if he can, the higher second-period profits can more than make up for his sacrifice. Thus, nonstrategic behavior is not an equilibrium for this game. Instead, consider the strategy profile below.

A Limit Pricing Equilibrium

Incumbent: $p_2 = v$. For particular values a and b to be specified later,

$$p_1 = \begin{cases} v & \text{if } v < a, \\ a & \text{if } v \in [a, b], \\ v & \text{if } v > b. \end{cases}$$

Rival: Enter if $p_1 > a$. Otherwise, stay out. If the rival observes $p_1 \in (a, b]$, his out-of-equilibrium belief is that $v = h(v)/\int_{p_1}^{b} h(v)dv$, the expected value of v if it lies between p_1 and b.

The rival's equilibrium payoff if he enters is

$$\begin{aligned} \pi_{rival,enter} &= -F + (Ep_2(v) - c) \left(\frac{q}{2} \right), \\ &= -F + \left(\int_a^b p_2(v) \left(\frac{h(v)}{\int_a^b h(v)dv} \right) dv - c \right) \left(\frac{q}{2} \right), \end{aligned} \tag{11.23}$$

where the density for the expectation $Ep_2(v)$ is $h(v)/\int_a^b h(v)dv$ instead of just $h(v)$ because it is conditional on v being between a and b, rather than v taking any of its possible values. The payoff in (11.23) equals zero in equilibrium for the entry-deterring values of a and b. If the incumbent deters entry, his payoff is

$$\pi_{incumbent}(no\ entry) = (p_1 - c)q + (v - c)q \tag{11.24}$$

and if he does not it is

$$\pi_{incumbent}(entry) = (v - c)q + (p_2(v) - c) \left(\frac{q}{2} \right). \tag{11.25}$$

The incumbent's advantage from limit pricing is the difference between these when $p_1 = a$, which is

$$[(a - c)q + (v - c)q] - \left[(v - c)q + (p_2(v) - c) \left(\frac{q}{2} \right) \right]. \tag{11.26}$$

This advantage is declining in v, and b is defined as the value at which it equals zero. Choosing $v = b$ in expression (11.26), equating to zero, and solving for a yields

$$a = \frac{c + p_2(b)}{2}. \tag{11.27}$$

We now have two equations (equation [11.27] and $\pi_{rival} = 0$ using expression [11.23]) for two unknowns (a and b). Now let us return to the special assumptions in equations (11.18) and (11.19). Given our specific assumptions on $h(v)$ and $p_2(v)$, $p_2(b) = b/2$ and $h(v) = 1/d$, so from (11.27),

$$a = \frac{b}{4}. \tag{11.28}$$

Since $\int_a^b h(v)dv = \int_a^b (1/d)dv = (b/d) - (a/d)$,

$$\begin{aligned}
\pi_{rival} &= -F + \left(\int_a^b \left(\frac{v}{2}\right) \left(\frac{1/d}{(b/d) - (a/d)} \right) dv \right) \left(\frac{q}{2}\right), \\
&= -F + \Big|_{v=a}^{b} \left(\frac{v^2}{4(b-a)} \right) \left(\frac{q}{2}\right), \\
&= -F + \left(\frac{b^2}{4(b-a)} - \frac{a^2}{4(b-a)} \right) \left(\frac{q}{2}\right), \\
&= -F + \left(b^2 - \frac{b^2}{16} \right) \left(\frac{q}{8(b - (b/4))} \right),
\end{aligned} \tag{11.29}$$

so

$$b = \left(\frac{16}{5}\right) \left(\frac{2F}{q}\right). \tag{11.30}$$

where $b < d$ because of our special assumption that $d > 6.4F/q$. From equation (11.28),

$$a = \left(\frac{4}{5}\right) \left(\frac{2F}{q}\right). \tag{11.31}$$

It is useful to compare the limits a and b with v^*, the value of v for which entry yields a payoff of zero to the rival. That value, which we found above using equation (11.22), is

$$v^* = \frac{2F}{q}. \tag{11.32}$$

Thus, $a < v^* < b$. This makes sense. The values of a and b were chosen so that, in effect, v^* was the expected value of v on $[a, b]$, and the rival would be deterred even though he did not know the exact value of v. If $a \geq v^*$, then the expected value of v on $[a, b]$ would be greater than v^*, and the rival would feel safe in entering if the incumbent charged $p_1 = a$. Thus, to use limit pricing the incumbent must charge strictly less than the monopoly price appropriate if the duopoly market would yield zero profits to an rival. This kind of signal jamming reduces the information that reaches the rival compared to nonstrategic behavior, since the rival learns the precise value of v only if v is less than a or greater than b.

Note, however, that limit pricing would not work if this were a truthful announcement game, in which the incumbent could remain silent but could not lie to the rival.

The incumbent would then tell the rival the value of v whenever it was v^* or less – which means that limit pricing for values in the interval $[v^*, b]$ would no longer be effective.

*11.7 Countersignalling

Feltovich, Harbaugh, & To (2002) construct a model to explain a situation commonly observed: that although the worst types in a market do not signal, the best types do not signal either, and their very lack of signalling can be a sign of their confidence in their high quality (**countersignalling**). What is crucial in such situations is that the highest-quality types have an alternative way to convey their quality to the market besides costly signalling. We will illustrate that here with a model of banking.

Countersignalling

PLAYERS
Banks and depositors.

THE ORDER OF PLAY

0 There is a continuum of length 1 of banks. Nature chooses the solvency θ_i of each bank using cumulative distribution $F(\theta)$ on the support $[-10, 10]$ with $F(0) = 0.2$.

1 Bank i chooses to spend s_i on its building, where it must spend at least $\bar{s} = 9$ to operate at all, and otherwise must exit.

2 Depositors on a continuum of length $D = 1$ observe $\hat{\theta} = \theta_i + u_i$, where the u_i values are chosen independently and take values of -5, 0, and $+5$ with equal probability. The banks do not observe $\hat{\theta}$ in move (1).

3 Each depositor chooses a bank.

PAYOFFS
Bankers further from insolvency have cheaper access to capital, so if bank i attracts as many depositors as other active banks its payoff is

$$\pi_i = \frac{D}{B} - \left(\frac{s_i}{10 + \theta_i}\right), \tag{11.33}$$

where B is the interval of banks that attract depositors. If bank i attracts no depositors, its payoff is $-(s_i/(10 + \theta_i))$.
A depositor's payoff is 1 if he picks a bank with solvency θ at or above 0, and 0 if $\theta < 0$.

Equilibrium

Banks with solvency $\theta \in [-10, 0]$ choose $s = 0$. They do not signal, and attract no depositors.

Banks with solvency $\theta \in [0,5)$ choose $s = s^* = 12.5$ and attract depositors.
Banks with solvency $\theta \in [5,10]$ choose $s = 9$, and attract depositors.
Depositors choose banks with either $\hat{\theta} \geq 0$ or $s = 12.5$ or both. Out of equilibrium, they believe that no signal except 12.5 conveys information about a bank's solvency (passive conjectures).

Each bank has three reasonable choices: $s = 0$, $s = 9$, and $s = 12.5$. In equilibrium, $B = 1 - F(0) = 0.8$, because the solvent banks and only those banks enter.

A bank with $\theta \geq 5$ needn't worry about appearing insolvent, because its lowest possible observed solvency is $\hat{\theta} = \theta - 5 \geq 0$. Banks in the interval [5, 10] will not spend more than the $s = 9$ necessary to enter because they attract all depositors anyway. They will enter with $s = 9$, though, because the least-profitable of those banks will then have a payoff of $D/B - (9/(10+5)) = 11/12 > 0$.

A bank with $\theta \in [0,5)$ might be unlucky and have $u = -5$ and $\hat{\theta} < 0$, so signalling can be useful. The self-selection constraint requires that banks with $\theta < 0$ not signal. Thus, the crucial signal level, s^*, requires that the type of insolvent bank for which the signal is cheapest be unwilling to signal. We need $\pi_{\theta=0}(s=0) = \pi_{\theta=0}(s=s^*)$, so

$$0 = \frac{D}{B} - \left(\frac{s^*}{10+0}\right). \tag{11.34}$$

Solving equation (11.34) for s^* and setting $B = 0.8$ and $D = 1$ yields

$$s^* = 12.5. \tag{11.35}$$

Alternatively, a bank with $\theta \in [0,5)$ might enter with $s = 9$ and hope that $u = 0$ of $u = +5$ rather than $u = -5$. This has probability 2/3, so in that case the payoff of the bank in that interval for which signalling is most costly is is negative:

$$\pi(s = K|\theta = 0) = \left(\frac{2}{3}\right)\left(\frac{D}{B}\right) - \left(\frac{9}{10+0}\right) = \frac{5}{6} - \left(\frac{9}{10}\right) < 0. \tag{11.36}$$

The insolvent banks with $\theta \in [-10,0)$ choose $s = 0$ and a payoff of zero in equilibrium, but they also have the option to enter with $s = 12.5$, or with $s = 9$ in the hope that $u = +5$ and $\hat{\theta} \geq 0$. The expected payoffs from these strategies are

$$\pi(s = K|\theta < 0) < \left(\frac{1}{3}\right)\left(\frac{D}{B}\right) - \left(\frac{9}{10+0}\right) = \frac{5}{12} - \left(\frac{9}{10}\right) < 0 \tag{11.37}$$

and

$$\pi(s = s^*|\theta < 0) < \left(\frac{D}{B}\right) - \left(\frac{12.5}{10+0}\right) = \frac{5}{4} - \left(\frac{12.5}{10}\right) = 0. \tag{11.38}$$

As always, there also exists a nonsignalling perfect Bayesian equilibrium, in which depositors ignore any signal and only choose banks with $\hat{\theta} \geq 5$. And there exist other, inefficient, equilibria in which the crucial signal level is $s^* > 10D/(1 - F(0))$. You might also wish to investigate what happens if the number of consumers, D, takes values other than 1.

What is interesting in this model is that unlike in previous models, it is not the highest-quality players that signal, but the middle-quality ones. The highest-quality players have no need to signal, because their quality is so high that information about it reaches the market by a different means – here, the imperfect observation $\hat{\theta}$. The "countersignal" is the absence in the highest quality players of a normal signal. The crucial feature of countersignalling models that gives rise to this is that signalling not be the only way in which information reaches the market.

Another effect that can arise in countersignalling models (though it does not in the simple one here) is that the absence of a signal can have a positive, additional, effect on the uninformed player's estimate of the informed player's quality. In our banking game, that can't happen because if the noisy observation is $\hat{\theta} \geq 5$, depositors have the highest possible opinion of the bank's safety: that it is solvent with probability one. Thus, the high-quality banks abstain from signalling simply to save money, not because the countersignal actually helps them. Suppose, though, that we added new depositors to the model, an amount small enough to be measure zero so they would not affect the equilibrium, and these new depositors had the special features that: (1) they could not observe even $\hat{\theta}$, and (2) their payoffs were 0 not just for insolvent banks but for any bank with $\theta < 7$. These depositors would observe only that some banks have $s = 4$ and some have $s = s^*$. Since they would be looking for especially high-quality banks, they would actually prefer to choose a bank with the lower signal value of $s = 4$, because they could be sure that if $s = s^*$ then $\theta < 7$. Thus, having a smaller building acts as a countersignal, an indicator that this bank has such high quality that it does not need to be ostentatious.

This informational value of the countersignal is what prompted the title of Feltovich, Harbaugh, & To (2002): "Too Cool for School? Signalling and Countersignalling." Low-quality high-school students might do badly in their schoolwork because doing well is too costly a signal. Mid-quality students would do better, to separate themselves from the low-quality students. But high-quality students might also do badly in their schoolwork, as a form of countersignalling. An observable indicator of quality might already show them to be high-quality ("cool"), so they don't need the signal. But they might suppress their schoolwork even though they really would prefer to do well, because a student who does his schoolwork would be considered mid-quality, not high-quality. In such a case, the countersignal is actually costly – the student is refraining from schoolwork purely for strategic reasons. This is not quite signal jamming, because the high-quality students is trying to escape from pooling rather than hide in it, but it shares with signal jamming the idea of costly suppression of an observable variable.

The term "countersignalling" is new, but the idea that the more desirable type might signal less goes as far back as part III, article III of Adam Smith's *Wealth of Nations*, where he lays out a theory of morality and wealth, noting that the poor are often more rigorous than the rich in their morality. Most of his theory is based on the higher utility cost of vice to the poor rather than on information, but he also discusses the observable effects of vice. He suggests that for the urban poor, in the anonymity of the city, belonging to a strict church can signal their morality, while for rich people, the very indulgence in vice can signal wealth, since high expenditure would quickly ruin someone of moderate means. Countersignalling will be a fruitful area for empirical work to test whether it explains signalling by moderate types only in various situations. One fascinating study of quality and information provision is

Jin & Leslie (2003), a study of restaurant responses to government hygiene grade cards for restaurants in Los Angeles, cards whose posting in restaurant windows was optional and later was compulsory.

Notes

N11.1 The informed player moves first: signalling

- The term "signalling" was introduced by Spence (1973). The games in this book take advantage of hindsight to build simpler and more rational models of education than in his original article, which used a rather strange equilibrium concept: a strategy profile from which no worker has incentive to deviate and under which the employer's profits are zero. Under that concept, the firm's incentives to deviate are irrelevant.

  The distinction between signalling and screening has been attributed to Stiglitz & Weiss (1989). The literature has shown wide variation in the use of both terms, and "signal" is such a useful word that it is often used in models that have no signalling of the kind discussed in this chapter. Where confusion might arise, the word "indicator" might be better for an informative variable.
- The applications of signalling are too many to properly list. A few examples are the use of prices in Wilson (1980) and Stiglitz (1987), the payment of dividends in Ross (1977), and bargaining under asymmetric information (section 12.5). Banks (1991) has written a short book surveying signalling models in political science. Empirical papers include Layard & Psacharopoulos (1974) on education and Staten & Umbeck (1986) on occupational diseases. Riley (2001) surveys the signalling literature, and Rochet & Stole (2003) surveys multi-dimensional screening.
- Legal bargaining is one area of application for signalling. See Grossman & Katz (1983). Reinganum (1988) has a nice example of the value of precommitment in legal signalling. In her model, a prosecutor who wishes to punish the guilty and release the innocent wishes, if parameters are such that most defendants are guilty, to commit to a pooling strategy in which his plea bargaining offer is the same whatever the probability that a particular defendant would be found guilty.
- The peacock's tail may be a signal. Zahavi (1975) suggests that a large tail may benefit the peacock because, by hampering him, it demonstrates to potential mates that he is fit enough to survive even with a handicap. By now there is an entire literature on signalling in the natural world; see John Maynard-Smith & David Harper's 2004 book, *Animal Signals.*
- **Advertising:** Advertising is a natural application for signalling. The literature includes Nelson (1974), written before signalling was well known, Kihlstrom & Riordan (1984) and Milgrom & Roberts (1986). I will briefly describe a model based on Nelson's. Firms are one of two types, low quality or high quality. Consumers do not know that a firm exists until they receive an advertisement from it, and they do not know its quality until they buy its product. They are unwilling to pay more than zero for low quality, but any product is costly to produce. This is not a reputation model, because it is finite in length and quality is exogenous.

  If the cost of an advertisement is greater than the profit from one sale but less than the profit from repeat sales, high rates of advertising are associated with high product quality. Only firms with high quality would advertise.

  The model can work even if consumers do not understand the market and do not make rational deductions from the firm's incentives, so it does not have to be a signalling model. If consumers react passively and sample the product of any firm from whom they receive an advertisement, it is still true that the high-quality firm advertises more, because the customers it attracts become repeat customers. If consumers do understand the firms' incentives, signalling reinforces the result. Consumers know that firms which advertise must have high quality, so they are willing to try them. This understanding is important, because if consumers knew that 90 percent of firms were low quality but did not understand that only high-quality firms advertise, they would not respond to the advertisements which they received. This should bring to mind section 6.2's game of PhD Admissions.

- If there are just two workers in the population, the model is different depending on whether:

  1 Each is *High* ability with objective probability 0.5, so possibly both are *High* ability; or
  2 One of them is *High* and the other is *Low*, so only the subjective probability is 0.5.

  The outcomes are different because in case (2) if one worker credibly signals he is *High* ability, the employer knows the other one must be *Low* ability.

Problems

11.1: Is lower ability better? (medium)

Change Education I so that the two possible worker abilities are $a \in \{1, 4\}$.

(a) What are the equilibria of this game? What are the payoffs of the workers (and the payoffs averaged across workers) in each equilibrium?
(b) Apply the Intuitive Criterion (see N6.2). Are the equilibria the same?
(c) What happens to the equilibrium worker payoffs if the high ability is 5 instead of 4?
(d) Apply the Intuitive Criterion to the new game. Are the equilibria the same?
(e) Could it be that a rise in the maximum ability reduces the average worker's payoff? Can it hurt all the workers?

11.2: Productive education and nonexistence of equilibrium (hard)

Change Education I so that the two equally likely abilities are $a_L = 2$ and $a_H = 5$ and education is productive: the payoff of the employer whose contract is accepted is $\pi_{employer} = a + 2s - w$. The worker's utility function remains $U = w - 8s/a$.

(a) Under full information, what are the wages for educated and uneducated workers of each type, and who acquires education?
(b) Show that with incomplete information the equilibrium is unique (except for beliefs and wages out of equilibrium) but unreasonable.

11.3: Price and quality (medium)

Consumers have prior beliefs that Apex produces low-quality goods with probability 0.4 and high-quality with probability 0.6. A unit of output costs 1 to produce in either case, and it is worth 10 to the consumer if it is high quality and 0 if low quality. The consumer, who is risk-neutral, decides whether to buy in each of two periods, but he does not know the quality until he buys. There is no discounting.

(a) What is Apex's price and profit if it must choose one price, p^*, for both periods?
(b) What is Apex's price and profit if it can choose two prices, p_1 and p_2, for the two periods, but it cannot commit ahead to p_2?
(c) What is the answer to part (b) if the discount rate is $r = 0.1$?

(d) Returning to $r = 0$, what if Apex can commit to p_2?
(e) How do the answers to (a) and (b) change if the probability of low quality is 0.95 instead of 0.4? (There is a twist to this question.)

11.4: Signalling with a continuous signal (hard)

Suppose that with equal probability a worker's ability is $a_L = 1$ or $a_H = 5$, and the worker chooses any amount of education $y \in [0, \infty)$. Let $U_{worker} = w - 8y/a$ and $\pi_{employer} = a - w$.

(a) There is a continuum of pooling equilibria, with different levels of y^*, the amount of education necessary to obtain the high wage. What education level, y^*, and wage, $w(y)$, are paid in the pooling equilibria, and what is a set of out-of-equilibrium beliefs that supports them? What are the incentive compatibility constraints?
(b) There is a continuum of separating equilibria, with different levels of y^*. What are the education levels and wages in the separating equilibria? Why are out-of-equilibrium beliefs needed, and what beliefs support the suggested equilibria? What are the self-selection constraints for these equilibria?
(c) If you were forced to predict one equilibrium to be the one played out, which would it be?

11.5: Advertising (medium)

Brydox introduces a new shampoo which is actually very good, but is believed by consumers to be good with only a probability of 0.5. A consumer would pay 11 for high quality and 0 for low quality, and the shampoo costs 6 per unit to produce. The firm may spend as much as it likes on stupid TV commercials showing happy people washing their hair, but the potential market consists of 110 cold-blooded economists who are not taken in by psychological tricks. The market can be divided into two periods.

(a) If advertising is banned, will Brydox go out of business?
(b) If there are two periods of consumer purchase, and consumers discover the quality of the shampoo if they purchase in the first period, show that Brydox might spend substantial amounts on stupid commercials.
(c) What is the minimum and maximum that Brydox might spend on advertising, if it spends a positive amount?

11.6: Game theory books (easy)

In the preface, I explain why I listed competing game theory books by saying, "only an author quite confident that his book compares well with possible substitutes would do such a thing, and you will be even more certain that your decision to buy this book was a good one."

(a) What is the effect of on the value of the signal if there is a possibility that I am an egotist who overvalues his own book?
(b) Is there a possible nonstrategic reason why I would list competing game theory books?
(c) If all readers were convinced by the signal of providing the list and so did not bother to even look at the substitute books, then the list would not be costly even to the author of a bad book, and the signal would fail. How is this paradox to be resolved? Give a verbal explanation.
(d) Provide a formal model for part (c).

11.7: Salesman clothing (medium)

Suppose a salesman's ability might be either $x = 1$ (with probability θ) or $x = 4$, and that if he dresses well, his output is greater, so that his total output is $(x + 2s)$ where s equals 1 if he dresses well and 0 if he dresses badly. The utility of the salesman is $U = w - (8s/x)$, where w is his wage. Employers compete for salesmen.

(a) Under full information, what will the wage be for a salesman with low ability?
(b) Show the self selection contraints that must be satisfied in a separating equilibrium under incomplete information.
(c) Find all the equilibria for this game if information is incomplete.

11.8: Signal jamming in politics (hard)

A congressional committee has already made up its mind that tobacco should be outlawed, but it holds televised hearings anyway in which experts on both sides present testimony. Explain why these hearings might be a form of signalling, where the audience to be persuaded is congress as a whole, which has not yet made up its mind. You can disregard any effect the hearings might have on public opinion.

11.9: Crazy predators (hard) (adapted from Gintis [2000], problem 12.10)

Apex has a monopoly in the market for widgets, earning profits of m per period, but Brydox has just entered the market. There are two periods and no discounting. Apex can either *Prey* on Brydox with a low price or accept *Duopoly* with a high price, resulting in profits to Apex of $-p_a$ or d_a and to Brydox of $-p_b$ or d_b. Brydox must then decide whether to stay in the market for the second period, when Brydox will make the same choices. If, however, Professor Apex, who owns 60 percent of the company's stock, is crazy, he thinks he will earn an amount $p^* > d_a$ from preying on Brydox (and he does not learn from experience). Brydox initially assesses the probability that Apex is crazy at θ.

(a) Show that under the following condition, the equilibrium will be separating, that is, Apex will behave differently in the first period depending on whether the Professor is crazy or not:

$$-p_a + m < 2d_a. \tag{11.39}$$

(b) Show that under the following condition, the equilibrium can be pooling, that is, Apex will behave the same in the first period whether the Professor is crazy or not:

$$\theta \geq \frac{d_b}{p_b + d_b}. \tag{11.40}$$

(c) If neither two condition (11.39) nor (11.40) apply, the equilibrium is hybrid, that is, Apex will use a mixed strategy and Brydox may or may not be able to tell whether the Professor is crazy at the end of the first period. Let α be the probability that a sane Apex preys on Brydox in the first period, and let β be the probability that Brydox stays in the market in the second period after observing that Apex chose *Prey* in the first period. Show that the equilibrium values of α

and β are

$$\alpha = \frac{\theta p_b}{(1-\theta)d_b}, \tag{11.41}$$

$$\beta = \frac{-p_a + m - 2d_a}{m - d_a}. \tag{11.42}$$

(d) Is this behavior related to any of the following phenomenon? – Signalling, Signal Jamming, Reputation, Efficiency Wages.

11.11: Monopoly quality (medium)

A consumer faces a monopoly. He initially believes that the probability that the monopoly has a high-quality product is H, and that a high-quality monopoly would be able to send him an advertisement at zero cost. With probability (1 − H), though, the monopoly has low quality, and it would cost the firm A to send an ad. The firm does send an ad, offering the product at price P. The consumer's utility from a high-quality product is $X > P$, but from a low quality product it is 0. The production cost is C for the monopolist regardless of quality, where $C < P - A$. If the consumer does not buy the product, the seller does not incur the production cost.

You may assume that the high-quality firm always sends an ad, that the consumer will not buy unless he receives an ad, and that P is exogenous.

(a) Draw the extensive form for this game.
(b) What is the equilibrium if H is sufficiently high?
(c) If H is low enough, the equilibrium is in mixed strategies. The high-quality firm always advertises, the low quality firm advertises with probability M, and the consumer buys with probability N. Show using Bayes' Rule how the consumer's posterior belief R that the firm is high-quality changes once he receives an ad.
(d) Explain why the equilibrium is not in pure strategies if H is too low (but H is still positive).
(e) Find the equilibrium probability of M. (You don't have to figure out N.)

Signalling Marriageability: A Classroom Game for Chapter 11

Each student is a Man or a Woman with a random and secret wealth level W distributed between 0 and 100. The wealth levels if there are ten people are 0, 10, 20, 30, 50, 60, 80, 90, 100, and 100. The instructor will choose from these values, repeating them if there are more than ten students, and omitting some values if the number of students is not divisible by ten.

A person's wealth level is secret, because the society has a taboo on telling your wealth level to someone else. Nonetheless, everybody is very interested in wealth because everyone's objective is to marry someone with high wealth.

Each year, each student first simultaneously writes down how much to spend on clothes that year. Then, in whatever order it happens, students pick someone else to pair up with temporarily. Both actions are publicly revealed – you may show people your scoresheet. In the fifth year, the pairings become permanent: marriage.

Your payoff is a concave function the original wealth minus clothing expenditures of you and your spouse (if you have one). Clothing has no value in itself. Thus, if i is married to j his payoff is, letting C_i and C_j denote the cumulative clothing purchases over the five periods,

$$U_i = log(W_i - C_i + W_j - C_j).$$

Table 11.1 shows some of the possible payoffs from this function.

Table 11.1 Marriage values

Remaining family wealth	0	1	2	5	10	25	50	100	150	200
Utility	$-\infty$	0	0.7	1.6	2.3	3.2	3.9	4.6	5.0	5.3

The first time you play the game, the only communication allowed is "Will you pair with me?" and "Yes" or "No." These pairing are not commitments, and can be changed even within the period.

If there is time, the game will be played over with new wealths and with unlimited communication.