PROBLEM SET 4 ANSWERS

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Problems: 7.1, 7.2, 8.4, 8.5

7.1. First-Best Solutions in a Principal-Agent Model
Suppose an agent has the utility function of $U = \sqrt{w} - e$, where e can assume the levels 0 or 1. Let the reservation utility level be $\overline{U} = 3$. The principal is risk neutral. Denote the agent's wage, conditioned on output, as $\underline{w}$ if output is 0 and $\overline{w}$ if output is 100. Table 5 shows the outputs.

Table 5: A Moral Hazard Game

Effort	**Probability of Output of** 0	100	Total
Low ($e = 0$)	0.3	0.7	1
High ($e = 1$)	0.1	0.9	1

(a) What would the agent's effort choice and utility be if he owned the firm?

Answer. The agent gets everything in this case. His utility is either

$$U(High) = 0.1(0) + 0.9\sqrt{100} - 1 = 8 \tag{1}$$

or

$$U(Low) = 0.3(0) + 0.7\sqrt{100} - 0 = 7. \tag{2}$$

So the agent chooses high effort and a utility of 8.

(b) If agents are scarce and principals compete for them, what will the agent's contract be under full information? His utility?

Answer. The efficient effort level is $High$, which produces an expected output of 90. The principal's profit is zero, because of competition. Since the agent is risk averse, he should be fully insured in equilibrium: $\overline{w} = \underline{w} = 90$ But he should get this only if his effort is high. Thus, the contract is w=90 if effort is high, w=0 if effort is low. The agent's utility is 8.5 $(= \sqrt{90} - 1$, rounded).

(c) If principals are scarce and agents compete to work for them, what would the contract be under full information? What will the agent's utility and the principal's profit be in this situation?

Answer. The efficient effort level is high. Since the agent is risk averse, he should be fully insured in equilibrium: $\overline{w} = \underline{w} = w$. The contract must satisfy a participation constraint for the agent, so $\sqrt{w} - 1 = 3$. This yields $w = 16$, and a utility of 3 for the agent. The actual contract specified a wage of 16 for high effort and 0 for low effort. This is incentive compatible, because the agent would get only 0 in utility if he took low effort. The principal's profit is 74 (= 90-16).

(d) Suppose that $U = w - e$. If principals are the scarce factor and agents compete to work for principals, what would the contract be when the principal cannot observe effort? (Negative wages are allowed.) What will be the agent's utility and the principal's profit be in this situation?

Answer. The contract must satisfy a participation constraint for the agent, so U = 3. Since effort is 1, the expected wage must equal 4. One way to produce this result is to allow the agent to keep all the output, plus 4 extra for his labor, but to make him pay the expected output of 90 for this privilege ("selling the store"). Let $\overline{w} = 14$ and $\underline{w} = -86$ (other contracts also work). Then expected utility is 3 $(= 0.1(-86) + 0.9(14) - 1 = -8.6 + 12.6 - 1)$. Expected profit is 86 $(= 0.1(0 - -86) + 0.9(100 - 14) = 8.6 + 77.4)$.

7.2. The Principal-Agent Problem

Suppose the agent has a utility function of $U = \sqrt{w} - e$, where e can assume the levels 0 or 7, and a reservation utility of $\overline{U} = 4$. The principal is risk neutral. Denote the agent's wage, conditioned on output, as $\underline{w}$ if output is 0 and $\overline{w}$ if output is 1,000. Only the agent observes his effort. Principals compete for agents. Table 6 shows the output.

Table 6: Output from Low and High Effort

	Probability of Output of		
Effort	0	1000	Total
Low$(e = 0)$	0.9	0.1	1
High $(e = 7)$	0.2	0.8	1

(a) What are the incentive compatibility, participation, and zero- profit constraints for obtaining high effort?

<u>Answer</u>. The incentive compatibility constraint is

$$U(e = 7) \geq U(e = 0) \tag{3}$$

so

$$0.2\sqrt{\underline{w}} + 0.8\sqrt{\overline{w}} - 7 \geq 0.9\sqrt{\underline{w}} + 0.1\sqrt{\overline{w}}. \tag{4}$$

The participation constraint is

$$U(e = 7) \geq \overline{U}, \tag{5}$$

so

$$0.2\sqrt{\underline{w}} + 0.8\sqrt{\overline{w}} - 7 \geq 4. \tag{6}$$

The zero-profit constraint is

$$E(wage) = E(output) \tag{7}$$

so

$$0.2\underline{w} + 0.8\overline{w} = 0.2(0) + 0.8(1000) = 800. \tag{8}$$

(b) What would utility be if the wage were fixed and could not depend on output or effort?

<u>Answer</u>. Effort would be low, so the expected value of output would be 100 $(= 0.9(0) + 0.1(1000))$. The wage would be a flat 100, and utility would be 10 $(= \sqrt{100} - 0)$.

(c) What is the optimal contract? What is the agent's utility?

<u>Answer</u>. The participation constraint is not binding, since principals compete for agents. The incentive compatibility constraint can be rewritten, since it is binding, as

$$\sqrt{\overline{w}} - \sqrt{\underline{w}} = 7/0.7 = 10. \tag{9}$$

The zero-profit constraint is also binding, and can be written as:

$$\underline{w} = 4000 - 4\overline{w}. \tag{10}$$

Substituting the zero-profit constraint into the incentive compatibility constraint yields

$$\sqrt{\overline{w}} - \sqrt{4000 - 4\overline{w}} = 10. \tag{11}$$

If $\overline{w} = 900$ and $\underline{w} = 400$, then since $30 - \sqrt{4000 - 3600} = 10$, we have a solution. The agent's utility is 21, from

$$0.2\sqrt{400} + 0.8\sqrt{900} - 7 = 21. \tag{12}$$

(d) What would the agent's utility be under full information? Under asymmetric information, what is the agency cost (the lost utility) as a percentage of the utility the agent receives?

Answer. Under full information, the agent would be perfectly insured and would choose high effort. To see that he would choose high effort, note that his wage would be 100 and his utility would be 10 ($= \sqrt{100} - 0$) under low effort. Under high effort, his wage would be 800 ($= 0.2(0) + 0.8(1000)$), and his utility would be 21.3 ($= \sqrt{800} - 7$). Since the utility under asymmetric information is 21, and the difference is 0.3, the agency cost is 0.3/21, about 1.4 percent.

8.4. Teams

A team of two workers produces and sells widgets for the principal. Each worker chooses high or low effort. An agent's utility is $U = w - 20$ if his effort is high, and $U = w$ if it is low, with a reservation utility of $\overline{U} = 0$. Nature chooses business conditions to be excellent, good, or bad, with probabilities θ_1, θ_2, and θ_3. The principal observes output but not business conditions, as shown in Table 5.

Table 5: Team Output

	Excellent (θ_1)	**Good** (θ_2)	**Bad** (θ_3)
$High, High$	100	100	60
$High, Low$	100	50	20
Low, Low	50	20	0

(a) Suppose $\theta_1 = \theta_2 = \theta_3$. Why is $\{(w(100) = 30, w(not\ 100) = 0), (High, High)\}$ not an equilibrium?

Answer. A worker would deviate. His payoff from $High$ is $\pi(High) = \frac{2}{3}(30) - 20 = 0$, and his payoff from Low is $\pi(Low) = \frac{1}{3}(30) = 10 > 0$.

(b) Suppose $\theta_1 = \theta_2 = \theta_3$. Is it optimal to induce high effort? What is an optimal contract with nonnegative wages?

Answer. High effort by both workers is efficient here. The expected output minus the real cost of labor for HH is $46\frac{2}{3}$ $(= \frac{2}{3}(100) + \frac{1}{3}(60) - 40)$; from HL it is $36\frac{2}{3}$ $(= \frac{1}{3}(100) + \frac{1}{3}(50) + \frac{1}{3}(20) - 20)$; from LL it is $23\frac{1}{3}$ $(= \frac{1}{3}(50) + \frac{1}{3}(20))$. An optimal contract is the boiling- in-oil contract $((w = 60|(q = 60), w = 0|(q \neq 60))$. This satisfies the participation constraint by giving each worker an expected utility of 0 $(= \frac{1}{3}(60) - 20)$, and the incentive compatibility constraint by making a worker's expected utility 0 if he chooses low effort.

(c) Suppose $\theta_1 = 0.5$, $\theta_2 = 0.5$, and $\theta_3 = 0$. Is it optimal to induce high effort? What is an optimal contract (possibly with negative wages)?

Answer. High effort is still efficient. The expected output minus the real cost of labor for (HH) is 60 (=100 - 40); from HL it is 55 $(= \frac{1}{2}(100 + 50) - 20$; from LL it is 35 $(= \frac{1}{2}(50 + 20))$. An optimal contract is $(w = -3000|(q = 100), w = 20|(q \neq 100))$. This satisfies the participation constraint by giving each worker an expected utility of 0 $(= 20 - 20)$, and the incentive compatibility constraint by making a worker's expected utility very negative if he chooses low effort.

(d) Should the principal stop the agents from talking to each other?

Answer. It doesn't matter. If there were multiple equilibria, talk might help the agents coordinate on a preferred equilibrium, but that is not the case here.

8.5. Efficiency Wages and Risk Aversion (see Rasmusen [1992c])

In each of two periods of work, a worker decides whether to steal amount v, and is detected with probability α and suffers legal penalty p if he, in fact, did steal. A worker who is caught stealing can also be fired, after which he earns the reservation wage w_0. If the worker does not steal, his utility in the period is $U(w)$; if he steals, it is $U(w + v) - \alpha p$, where $U(w_0 + v) - \alpha p > U(w_0)$. The worker's marginal utility of income is diminishing: $U' > 0$, $U'' < 0$, and $lim_{x \to \infty} U'(x) = 0$. There is no discounting. The firm definitely wants to deter stealing in each period, if at all possible.

(a) Show that the firm can indeed deter theft, even in the second period, and, in fact, do so with a second-period wage w_2^* that is higher than the reservation wage w_0.

<u>*Answer.*</u> It is easiest to deter theft in the first period, since a high second-period wage increases the penalty of being fired. If w_2 is increased enough, however, the marginal utility of income becomes so low that $U(w_2 + v)$ and $U(w_2)$ become almost identical, and the difference is less than αP, so theft is deterred even in the second period.

(b) Show that the equilibrium second-period wage w_2^* is higher than the first-period wage w_1^*.

<u>*Answer.*</u> We already determined that $w_2 > w_0$. Hence, the worker looks hopefully towards being employed in period 2, and in Period 1 he is reluctant to risk his job by stealing. This means that he can be paid less in Period 1, even though he may still have to be paid more than the reservation wage.