PROBLEM SET 7 ANSWERS: Answers to Exercises in Jean Tirole's Theory of Industrial Organization

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0.1 (p. 26), 0.2 (p. 41), 1.2 (p. 67) and 1.3 (p.68)

0.1 (p. 26)**

In the text, it is assumed that the supplier's investment reduces his production cost. Assume instead that the ex ante investment affects the quality of the product and thus the value to the buyer. The buyer's *ex post* value is $v(I) = 3I - I^2/2$. Hence, the buyer's surplus in case of trade is $v(I) - p$. The supplier's surplus is then $p - c - I$ (where $c < 1/2$ is now a constant production cost). Suppose that I (and, hence, v) is observable by the buyer; however, it is not verifiable by a court, so that it cannot be specified by a contract.

(i) Determine the efficient amount of investment.

Answer. Our starting point is that trade will occur even if there is no investment. The the total surplus is then $v(I) - p + p - c - I = v(I) - I = 3I - I^2/2 - I$. Maximizing that with respect to I gives the first order condition $3 - I - 1 = 0$, so $I = 2$.

(ii) Suppose that there is no contract and the two parties bargain *ex post* according to the Nash bargaining solution. Is the investment optimal? Point out the externality.

Answer. The surplus viewed from after the investment has been sunk is $v(I) - p + p - c = v(I) - c$. The buyer gets $v(I) - p$ of that and the seller gets $p - c$ of that. Equating those (if they get equal surplus, ignoring the sunk cost I), the price comes out to $p = \frac{v(I)+c}{2}$. Thus, in making his ex ante investment decision the seller maximizes with respect to I

$$\frac{v(I) + c}{2} - c - I = I/2 - c/2 - I^2/4$$

The first order condition is $1/2 - I/2 = 0$, so $I = 1$.

Investment is too low.

The externality is that when the seller invests, he helps not just himself but also the buyer, but he does not get compensated fully for that. He increases not just his share of the surplus, but the buyer's.

(iii) Suppose that the parties sign a contract specifying that the buyer has the right to buy the good at a given price p. Is this contract efficient? What if the supplier has the right to sell at a given price?

Answer. In that case, p would no longer depend on $v(I)$ and thus would not depend on I. This is tricky, though, because if investment is too low, the buyer will refuse to buy the product. Thus, $profit$ depends on I even if $price$ does not.

What is that profit? If the buyer will not buy, it is zero. If $v(I) > p$, which means $3I - I^2/2 > p$, then he will. Solving this last equation (which could be done with the quadratic formula) yields a function $I(p)$ which is the same thing as the $v^{-1}(p)$ that Tirole has in his answer on page 55. I think the $I(p)$ notation is more understandable. The seller's profit is $p - c - I(p)$.

If $p = 4$, the seller's profit is 0 with $I = 0$ or $2 - c$ with $I = 2$, so he will choose $I = 2$ and we get the first best. The buyer's surplus will be zero.

At $p > 4$, the buyer will still buy if the quality is high enough, so rather than make profit go to zero, the seller will choose $I > 2$ and overinvest so long as p is not too high. For high enough p, though, I would have to be so high as to drive social surplus to zero, so even though the seller would get all of the social surplus, he would prefer to choose $I = 0$. Since $v(I)$ is quadratic, the maximum value it can take is 4.5, which happens with $I = 3$, and higher values of I are actually counterproductive. The buyer will never pay more than $p = 4.5$, so $I = 0$ for prices higher than that.

If $p < 4$, but is not *too* low, then the seller can get away with $I < 2$ and the buyer will still buy, so the seller will underinvest.

If p is less than c, however, the seller will choose $I = 0$ and there will be no trade. In fact, even if p is somewhat larger than c this will be true, because the buyer will only pay up to $p = v(I)$, which is less than c if I is small. The seller will accept no less than $c + I$; the buyer will pay no more than $v(I)$. Thus, we need $p \geq c + I_{min}$, where $c + I_{min} = v(I_{min})$. This expands to $c + I_{min} = 3I_{min} - I_{min}^2/2$, $0 = 2I_{min} - I_{min}^2/2 - c$. If, for example, $c = .25$, then $I_{min} = \frac{15}{32}$ and we require $p \geq \frac{23}{32}$.

What if the supplier has the right to sell at price p? Then, his profit does not depend on the amount the buyer is willing to pay at all (it is $p - c$ no matter what) so he will choose $I = 0$ and sell only if $p \geq c$.

Wenbin Wang pointed out several of these points to me.

(iv) What happens if the supplier is given the right to choose the price *ex post*?

Answer. Here, we have not assumed that the buyer *must* buy, as in part (iii). The seller, able to make a take-it-or-leave-it offer because he chooses the price, will get the entire surplus. Thus, his incentives are efficient and he will choose $I = 2$, which maximizes that surplus (and he will choose $p = 4$ so the buyer is barely willing to buy).

0.2 (p. 41) Sole Sourcing vs. Dual Sourcing

Answer. In this exercise, which concerns sole sourcing versus dual sourcing, we build a simple model in which the managers' objective function is such that monetary incentives are rather ineffective. A firm has a project of a given size. The cost of the project is $C = \beta - e$. The variable β is random on $[\underline{\beta}, \overline{\beta}]$ with expectation $E\beta$. The variable e denotes the effort exerted by the manager assigned to the project. A manager has utility function $U(w, e) = u(w) - \Phi(e)$, where $\Phi' > 0, \Phi'' > 0, \Phi'(0) > 0$, and

$$\begin{aligned} u(w) &= -\infty \; if \quad w < \overline{w} \\ &= \overline{u} + \lambda(w - \overline{w}) \quad if \quad w \geq \overline{w} \end{aligned}$$

Thus, $\overline{w}$ can be interpreted as a subsistence wage, λ is a "small" positive parameter; mathematically $\lambda \leq \Phi'(e)$ for all e. The manager is infinitely risk averse over the states of nature β, so he is only interested in his utility in the worst state of nature: $min_\beta U(w, e)$. The manager learns β after signing the contract and before choosing e. The principal observes C but not β or e. So the wage structure is a function of C,$w(C)$, and the manager's objective function can be written

$$\overset{min}{\beta} \left(\overset{max}{e} \left(u(w(\beta - e)) - \Phi(e) \right) \right).$$

Let U_0 denote the manager's reservation utility, and let $e^* > 0$ be defined by $u(\overline{w}) - \Phi(e^*) = U_0$. The principal wants to minimize the expected cost of the project.

(i) Show that if β and/or e were observable by the principal, the optimal contract would yield $w = \overline{w}$ and $e = e^*$ for all β, and that the expected cost of the project is $\overline{w} + E\beta - e^*$.

Answer. (p. 55) The principal will want to drive down the agent to the reservation utility as cheaply as possible. He can do that by choosing $U(w, e) = u(w) - \Phi(e) = U_0$ in the very worst state of nature. He wants to minimize the cost of the project, $w + \beta - e$ subject to that participation constraint. He has to pay at least $\overline{w}$ or the agent will reject the contract. If he pays exactly that, the participation constraint solves to $e = e^*$.

Why not choose e higher? The reason is that paying any more would require a higher wage, and the marginal disutility of effort $\Phi'(e)$, is bigger, by assumption for all e, than the marginal utility of income, λ.

The expected cost of the project with this wage and effort will then be $\overline{w} + E\beta - e^*$.

(ii) Under asymmetric information, show that the optimal contract is

$$\begin{aligned} w \quad &= \overline{w}\, l \quad if w \leq \overline{\beta} - e^* \\ &< \overline{w} \qquad otherwise \end{aligned}$$

and that the expected cost of the project is $\overline{w} + \overline{\beta} - e^*$.

Answer. Now the principal can only observe C, not e or β. How will the agent respond to the contract above? He will accept it and, since there is no incentive for beating the cost target, he will choose the lowest effort that will yield a cost of $C = \overline{\beta} - e^*$. That effort solves $\beta - e = \overline{\beta} - e^*$. Thus, the expected cost is always $\overline{w} + \overline{\beta} - e^*$ under this contract.

Why is this contract optimal? That's a lot tougher to show. Let's follow Tirole in looking at the constraints. What we have here is a mechanism design problem. Suppose we have a contract $w(\beta)$ that could pay an agent more if he admits the state is β and chooses higher effort than he would have to choose if he pretended it was state $\overline{\beta}$ and chose lower effort.

The incentive compatibility constraint in state β requires that the manager prefer choosing to report the true state:

$$U(w(\beta), e(\beta), \beta) = u(w(\beta)) - \Phi(e(\beta)) \geq U(w(\overline{\beta}), e(\overline{\beta}), \beta) = u(w(\overline{\beta})) - \Phi(e(\overline{\beta}) + (\overline{\beta} - \beta))$$

The disutility of effort in the right-hand-side is lower, because the manager can slack off and still attain the target.

We can substitute in for the u function and delete the $\overline{u}$ terms and rearrange to get

$$\lambda(w(\beta) - w(\overline{\beta})) \geq \Phi(e(\beta)) - \Phi(e(\overline{\beta}) - (\overline{\beta} - \beta))$$

Since $\lambda < \Phi'(e(\beta))$, we can rewrite this as

$$\Phi'(e(\beta))(w(\beta) - w(\overline{\beta})) \geq \Phi(e(\beta)) - \Phi(e(\overline{\beta}) - (\overline{\beta} - \beta))$$

and

$$(*) \;\; (w(\beta) - w(\overline{\beta})) \geq \frac{\Phi(e(\beta)) - \Phi(e(\overline{\beta}) - (\overline{\beta} - \beta))}{\Phi'(e(\beta))}.$$

Now we need to use convexity of $\Phi(e)$. You might want to draw a figure to help see this. If the value of e rises by $e(\beta) - [e(\overline{\beta}) - (\overline{\beta} - \beta)]$, then that change of e multiplied by the slope,$\Phi'(e(\beta))$, is less than the actual increase, $\Phi(e(\beta)) - \Phi(e(\overline{\beta}) - (\overline{\beta} - \beta))$:

$$\Phi'(e(\beta))[e(\beta)) - [e(\overline{\beta}) - (\overline{\beta} - \beta)]] \leq \Phi(e(\beta)) - \Phi(e(\beta)) - \Phi(e(\overline{\beta}) - (\overline{\beta} - \beta)),$$

with equality only if $\beta = \overline{\beta}$. As a result, we can rewrite inequality (*) as

$$(**) \;\; w(\beta) - w(\overline{\beta}) \geq e(\beta) - [e(\overline{\beta}) - (\overline{\beta} - \beta)]$$

and

$$w(\beta) - e(\beta) + \beta \geq w(\overline{\beta}) - e(\overline{\beta}) - \overline{\beta}$$

This last expression has the principal's wage plus production cost in state β on the left hand side and the cost of wage plus production cost in state $\overline{\beta}$ on the right, and we see that the principal can do no better than to settle for having the same total costs in each state. He can do that with the constant wage and constant effort in the contract above.

(iii) Suppose that the project can be given to two managers. The cost fo the principal (net of the wage bill) is $min(C_1, C_2)$, where $C_i = \beta - e_i$ and e_i is manager i's effort (i=1,2). That is, β is the same for both managers. Show that an optimal contract is

$$\begin{aligned} w \quad &= \overline{w} \; if \; C_i = C_j \\ &< \overline{w} \; if \; C_i > C_j \\ &= \overline{w} + \Phi'(e^*)(C_j - C_i)/\lambda \; if \; C_i < C_j \end{aligned}$$

It is concluded that the principal prefers dual outsourcing to sole sourcing if and only if $\overline{w} \leq \overline{\beta} - E\beta$. Interpret.

Answer. Under the contract above, an equilibrium is for both managers to choose e^*. If either unilaterally deviates to lower effort, he will get negative infinite utility. Deviating to higher effort yields a higher wage, but the rate of change in the disutility of effort is $\Phi'(e^*)$ and the rate of change in the wage is $\Phi'(e^*)/\lambda$(since more effort reduces C at a rate of 1), and that is too small to make the improvement from deviation positive.

The maximization problem for the manager set out in Tirole uses essentially the same logic.

With the yardstick contract, the manager's expected wage plus production costs is $2\overline{w} + E\beta - e^*$. With the single-manager contract, the principal doesn't pay as much in wages but the production cost is higher; his total cost is $\overline{w} + \overline{\beta} - e^*$. Taking the difference in those two total payoffs, the principal prefers yardstick competition iff $\overline{w} \leq \overline{\beta} - E\beta$; that is, if the extra wage is less than the average production cost reduction.

1.1 (p. 67)
I misread this and thought it was 1.2, so I prepared an answer for it. I include it now just for reference– you don't have to read through it. It is a heavily algebraic problem.

In a monopolized industry, the demand function has constant elasticity: $q = D(p) = p^{-\epsilon}$ where $\epsilon > 1$ is the elasticity of demand. Marginal cost is constant and equal to c.

(i) Show that a social planner (or a competitive industry) would yield a total welfare of

$$W^c = c^{1-\epsilon}/(\epsilon - 1)$$

Answer. (p. 88) Optimal welfare W^c is consumer surplus plus profit, so it is the integral of consumer benefit from the price to infinity plus $(p - c)$ times quantity demanded:

$$W^c = \stackrel{max}{p} \left(\int_p^\infty x^{-\epsilon} dx + (p-c)p^{-\epsilon} \right)$$

Marginal cost pricing solves this because at $p = c$ the marginal benefit to the consumer equals the marginal cost to the seller. Then,

$$(*)W^c = \left(\int_c^\infty x^{-\epsilon} dx + (c-c)p^{-\epsilon} \right) = \Big|_c^\infty x^{1-\epsilon}/(1-\epsilon) = 0 - c^{1-\epsilon}/(1-\epsilon) = c^{1-\epsilon}/(\epsilon - 1)$$

(ii) Compute the total welfare loss, WL, from monopoly.

Answer. The demand is $q = p^{-\epsilon}$. The monopoly price maximizes $(p-c)p^{-\epsilon} = p^{1-\epsilon} - cp^{-\epsilon}$ so the first order condition is $(1-\epsilon)p^(-\epsilon) + c\epsilon p^{-\epsilon-1} = 0$, so $(1-\epsilon) + c\epsilon p^{-1} = 0$, so $p^{-1} = \frac{\epsilon-1}{c\epsilon}$ and

$$p_m = \frac{c\epsilon}{\epsilon - 1} = \frac{c}{1 - 1/\epsilon}$$

.

Welfare loss is $W^c - W^m$. To get W^m, put the monopoly price into equation (*) above instead of $p = c$.

$$\begin{aligned} W^m &= \left(\int_{p_m}^{\infty} x^{-\epsilon} dx + (c-c){p_m}^{-\epsilon} \right) \\ &= \Big|_{p_m}^{\infty} x^{1-\epsilon}/(1-\epsilon) \\ &= 0 - {p_m}^{1-\epsilon}/(1-\epsilon) \\ &= {p_m}^{1-\epsilon}/(\epsilon - 1) \end{aligned}$$

Substituting in for W^m, we get

$$W^m = (\frac{c}{1 - 1/\epsilon})^{1-\epsilon}/(\epsilon - 1)$$

The welfare loss is $W^c - W^m$ or

$$WL = c^{1-\epsilon}/(\epsilon - 1) - (\frac{c}{1 - 1/\epsilon})^{1-\epsilon}/(\epsilon - 1)$$

Tirole simplifies this to

$$(**) WL = c^{1-\epsilon}/(\epsilon - 1) \left[1 - \left(\frac{2\epsilon - 1}{\epsilon - 1} \right) \left(\frac{\epsilon}{\epsilon - 1} \right)^{-\epsilon} \right]$$

with "some computations", and I'll just trust that he got it right.

(iii) Show that the ratio WL/W^c (relative deadweight loss) increases with ϵ, that WL is nonmonotonic in ϵ, and that the fraction Π^m/W^c of potential consumer surplus that can be captured by the monopolist increases with ϵ. Discuss the result. (Note that the "size" of the market changes with ϵ.)

Answer. First, look at WL/W^c. Using expression (**), we get

$$WL/W^c = \left[1 - \left(\frac{2\epsilon - 1}{\epsilon - 1} \right) \left(\frac{\epsilon}{\epsilon - 1} \right)^{-\epsilon} \right] = 1 - K(\epsilon)$$

If we can show that $K' < 0$, we've shown that the relative loss increases with ϵ. Tirole does this, but as it just involves tricks of algebra-solving, I will not pursue it, nor the nonmonotonicity of WL in ϵ.

Monopoly profit is $(p-c)p^{-\epsilon}$, so it is

$$\begin{aligned} profit &= (\tfrac{c\epsilon}{\epsilon-1} - c) \left(\tfrac{c\epsilon}{\epsilon-1} \right)^{-\epsilon} \\ &= \tfrac{c\epsilon - c\epsilon + c}{\epsilon - 1} \left(\tfrac{c\epsilon}{\epsilon-1} \right)^{-\epsilon} \\ &= \tfrac{c}{\epsilon-1} \left(\tfrac{c\epsilon}{\epsilon-1} \right)^{-\epsilon} \\ &= \left(\tfrac{c^{1-\epsilon}}{(\epsilon-1)^{1-\epsilon}} \right) \epsilon^{-\epsilon} \end{aligned}$$

The ratio of this to $W^c = \frac{c^{1-\epsilon}}{\epsilon - 1}$ is

$$\frac{\Pi_m}{W^c} = \frac{\epsilon^{-\epsilon}}{(\epsilon - 1)^{-\epsilon}}$$

To see how this ratio changes with elasticity, take its log, whose derivative has the same sign as the ratio's:

$$-\epsilon ln(\epsilon) + \epsilon ln(\epsilon - 1) = \epsilon(-ln(\epsilon) + ln((\epsilon - 1) = \epsilon ln(\frac{\epsilon - 1}{\epsilon}),$$

This last expression is increasing in ϵ because the fraction in the log is.

1.2. (p.67)*

Suppose that all consumers have unit demand. They buy 0 or 1 unit of the good produced by the monopolist. They are identical, and they have willingness-to-pay (valuation) $\overline{s}$ for the good. Show that monopoly pricing does not create a welfare loss.

Answer. (p. 88)

The monopolist will charge a price equal to $\overline{s}$ (assuming marginal cost is less than that and the fixed cost is not too big, things which Tirole does not mention except in the solution). All consumers will buy. This the same result as a social planner would choose, the same as would result if price equalled marginal cost. The monopoly price maximizes social surplus because the seller acquires the entire surplus.

If, just to add some complication (since this is a remarkably easy Tirole problem), the seller had a rising marginal cost, it might happen that he would set $p = \overline{s}$ where $p \geq MC(q^*)$ and $p \leq MC(q^* + 1)$ and just sell q^* units even if there were $n > q^*$ consumers. Some consumers would go unserved, but this, too, is the same thing as the social planner would do, since serving more consumers would raise costs more than benefits.

1.3 (p.68)*

A monopolist's marginal cost of supplying a good to consumers is $\overline{c} = c + t$ (where t is a unit commodity tax). Let $p^m(\overline{c})$ denote the corresponding monopoly price.

(i) Compute $dp^m/d(\overline{c})$ for the following demand functions: $p = q^{-1/\epsilon}, p = \alpha - \beta q^\delta, p = a - blnq$.

Answer. (p. 88)

We need the monopoly prices first. It's most convenient to get those by rearranging the functions above as q(p)'s:

Profit is $(p - c)q$, so for $p = q^{-1/\epsilon}$ it is

$$(q^{-1/\epsilon} - c)q = q^{1-1/\epsilon} - cq$$

Maximizing with respect to q yields $(1 - 1/\epsilon)q^{-1/\epsilon} - c = 0$ so

$$q = \left(\frac{c}{1 - 1/\epsilon}\right)^{-\epsilon}.$$

Substituting into p we get

$$p = \frac{c}{1 - 1/\epsilon},$$

so

$$\frac{dp}{dc} = \frac{1}{1 - 1/\epsilon}.$$

For $p = \alpha - \beta q^\delta$, profit is

$$(\alpha - \beta q^\delta - c)q = \alpha q - \beta q^{1+\delta} - cq$$

Maximizing with respect to q yields $(\alpha - \beta(1 + \delta)q^\delta - c = 0$ so

$$q = \left(\frac{\alpha - c}{\beta(1 + \delta)}\right)^{-\delta}.$$

Substituting into p we get

$$p = \alpha - \beta \left(\frac{\alpha - c}{\beta(1+\delta)} \right)$$

so

$$\frac{dp}{dc} = \frac{1}{1+\delta}.$$

For $p = a - blnq$, profit is

$$(a - blnq - c)q = aq - qb * ln(q) - cq$$

Maximizing with respect to q yields

$$(a - bln(q) - \frac{qb}{q} - c = 0so$$

$$q = e^{\frac{a-b-c}{b}}.$$

Substituting into p we get

$$p = a - b\frac{a-b-c}{b} = a - a + b + c = c + b$$

so

$$\frac{dp}{dc} = 1$$

(ii) Sumner (1981) uses an ingenious approach to estimate the elasticity of demand– and thus the degree of monopoly power— in the American cigarette industry. He notes that in the United States commodity taxes— and therefore the generalized cost $\bar{c}$– vary across states. Although data on c are hard to obtain, data on t are readily available. Sumner uses varying levels of taxation across states to estimate the elasticity of demand. Bulow and Pfleider (1983) argue that this method has limited applicability. What do you think?

Answer. The problem lies in the answers to part (i). The constant elasticity demand curve had markup falling with elasticity, but the other two didn't— elasticity didn't matter to the markup for them.

Suppose Sumner found that states with bigger taxes had higher prices at a one-for-one rate. He would conclude that ϵ was very high, if he used constant-elasticity demand curves. But if he used the last of the demand curves in part (i), he should expect to find that one-to-one correspondence even though there was a low elasticity.

Thus, a first step is to figure out the shape of the demand curve. This can be very hard to do, and if the shape changes with price (constant el. for high price, linear for low price, for example) it would be very tough to figure out both the shape of the demand curve and the elasticities.

That is a rather hopeless view. Before I gave up on the Sumner technique I'd want to get a feel for how accurately it was possible to estimate demand curve shapes, and how accurately we could estimate elasticities even if we knew the shape of the demand curve.

I have a different criticism of this method, by the way, though not as serious a one. What if states with higher elasticity also have lower cigarette taxes? After all, rationality suggests that states with bigger losses from a policy will use it less. See my "Observed Choice, Estimation, and Optimism about Policy Changes, " *Public Choice*, (October 1998) 97(1-2): 65- 91.