G601 Final, 14 December 2006 ANSWERS

This is a closed-book test, except that you may use one double-sided page of notes. Answer each question as best you can. If you get lost in solving equations, write down in words what you are trying to do and what you think would come out of the mathematical analysis. Each part of each question is worth 5 points.

Scores out of 70: 35, 36, 36, 44, 45, 46, 48, 50, 50, 53, 54,59, 60, 61. A, A-: 50+, B, B+: 44-49, B- 35-36.

1. Firms Apex and Brydox are thinking of entering the China market. They only have resources to enter one market each. The cost of entry is 10. If they both enter the Beijing market, they each earn revenues of 11 (which could be in units of 10's of millions of dollars). If they both enter Shanghai, they each earn revenues of 10. If one enters in each market, the firm that enters Shanghai gets a revenue of $x + 10$ and the one that enters Beijing gets a revenue of $y + 10$, where $0 < 1 < x < y$. The firms make their entry decisions simultaneously.

(a) What are two pure-strategy equilibria for this game?

Answer: (Apex-Beijing, Brydox-Shanghai) and (Brydox-Beijing, Apex-Shanghai)

(b) What is a symmetric mixed-strategy equilibrium for this game ?

<u>*Answer*</u>. Let each firm enter Beijing with probability β. Apex's payoffs from his two pure strategies must be equal, so

$$\pi(Beijing) = \beta(11-10)+(1-\beta)(y+10-10) = \pi(Shanghai) = \beta(x+10-10)+(1-\beta)(10-10) \quad (1)$$

so

$$\beta + (1-\beta)y = \beta x \quad (2)$$

sp $\beta - \beta y - \beta x = -y$ and

$$\beta = \frac{y}{x+y-1} \quad (3)$$

(c) Suppose Apex's cost of entry is 10 with probability θ and 14 with probability $(1-\theta)$. Find one pure strategy equilibrium.

<u>*Answer*</u>. This question was ambiguous. It matters whether Apex knows his own cost of entry (what I was assuming) or not. I gave full credit for a correct answer under either premise.

If Apex knows his cost, then one equilibrium is (Brydox-Beijing, Apex-10-Shanghai, Apex-14-Stay out).

If θ is low enough there is also an equilibrium of the form

(Brydox-Shanghai, Apex-10-Beijing, Apex-14-Stay out)

This requires that Brydox not deviate to Beijing, i.e.,

$$\pi(Brydox, Shanghai) = (x+10-10) \geq \pi(Brydox, Beijing) = \theta(11-10)+(1-\theta)(y+10-10),$$

that is, that: $x \geq theta + y - \theta y$, $x + y \geq \theta(y-1)$ which means

$$\theta \leq \frac{x+y}{y-1}.$$

If Apex does not know his cost, he uses his expected cost, $K = 10\theta + 14(1-\theta)$.

If $K > y$, the equilibrium is (Apex-OUT, Brydox-Beijing).

If $x < K < y$, the equilibria are (Apex-OUT, Brydox-Beijing) and (Apex-Beijing, Brydox-Shanghai).

If $10 \leq K < x$, the equilibria are (Apex-Shanghai, Brydox-Beijing) and (Apex-Beijing, Brydox-Shanghai).

2. A manager offers a worker a wage contract based on the worker's output, which the manager will observe. The manager does not, however, know the state of the market, which is $s = simple$ with probability .5 and $s = tricky$ with probability .5, nor the agent's effort, e. The agent will know the state later, after the contract is signed but before choosing his effort.

Output is $q = 1 + e^2/3$ if effort is e and the state is simple, but it is $q = 2e$ if the state is tricky. Both players are risk-neutral, with reservation payoffs of zero. The manager offers a take-it-or-leave-it contract to the worker. The worker's payoff function is $w - e^2$.

(a) If the manager could observe the state of the world and condition the contract on it, what would effort be?

<u>*Answer*</u>. The agent's expected payoff is always zero. The agent needs to be paid $w = e^2$ to exert effort e. Thus, the manager's payoff in the simple state is $1 + e^2/3 - e^2 = 0$. Maximizing this has the corner solution $e(simple) = 0$.

The manager's payoff in the tricky state is $2e - e^2 = 0$. This has FOC $2 - 2e = 0$ so $e(tricky) = 1$.

(b) If he couldn't, what constraints must the optimal contract satisfy? Write them out as inequalities or equalities.

<u>*Answer*</u>. The manager must choose an output and a wage for each state, using a contract that will induce the worker to tell him the true state. He solves the problem,

$$\underset{q_s, q_t, w_s, w_t}{Maximize} \; [0.5(q_s - w_s) + 0.5(q_t - w_t)]. \tag{4}$$

Expressing effort as a function of output, $e_s = \sqrt{3(1-q_s)}$ and $e_t = q_t/2$.

The participation constraint is

$$0.5\pi_{agent}(q_s, w_s|simple) + 0.5\pi_{agent}(q_t, w_t|tricky) \geq 0 \tag{5}$$

or

$$0.5(w_s - [\sqrt{3(1-q_s)}]^2) + .5(w_t - [q_t/2]^2) \geq 0. \tag{6}$$

Note that there is just one of these, but it is a function of two wages and outputs, not just one.

In the simple state, the agent must choose the simple-state contract,so

$$\pi_{agent}(q_s, w_s|simple) \geq \pi_{agent}(q_s, w_s|simple) \tag{7}$$

$$w_s - \left(\sqrt{3(1-q_s)}\right)^2 \geq w_t - \left(\sqrt{3(1-q_t)}\right)^2 \tag{8}$$

$$w_s - 3(1-q_s) \geq w_t - 3(1-q_t) \tag{9}$$

In the tricky state he must choose the tricky-state contract, so

$$\pi_{agent}(q_b, w_t|tricky) = w_t - (q_t/2)^2 \geq \pi_{agent}(q_s, w_s|tricky) = w_s - (q_s/2)^2. \tag{10}$$

(b) NOT INCLUDED: Which constraints are binding in equilibrium?

(d) NOT INCLUDED ON THE TEST. What are the wages in each state in terms of the output?

3. Answer the following short questions.

(a) Derive the Lerner rule for the percentage price markup ($\frac{P-C'}{P}$) as a function of the price elasticity of demand.

Answer. Profit is $P(Q)Q - C(Q)$. Maximizing with respect to Q yields

$$P + P'Q - C' = 0,$$

so

$$P - C' = -P'Q \quad \frac{P-C'}{P} = \frac{-P'Q}{P} = elasticity$$

(b) What is the hold-up problem?

Answer. To create surplus, one player must take a costly action before contracting with the other. When they bargain over the contract, they treat that cost as sunk, and hence the first player is not fully compensated for it. As a result, he will not do enough of the costly action.

(c) Show how if there are two types of consumers, high and low valuers, a move from a single monopoly price to price discrimination with two prices can raise welfare.

Answer. Suppose there are 6 high valuers, with reservation price 10, and 2 low valuers, with reservation price 3 and cost of zero. The monopoly price will be 10, for profit of 60. The discrimination prices will be 10 and 3, for profit of 90. Consumer surplus is zero in either case, so welfare has risen.

(d) Your regression output looks like the following. Construct a table to show the results, including all the vertical and horizontal lines that you would include in a typeset version. The variables are return on capital, the square of assets, and the log of sales for a firm.

```
regress OpIn04Cap  AssetSq

      Source |       SS       df       MS              Number of obs =    3074
-------------+------------------------------           F(  2,  3071) =   42.01
       Model |  69.5767432     2  34.7883716           Prob > F      =  0.0000
    Residual |  2542.94466  3071  .828051014           R-squared     =  0.0266
-------------+------------------------------           Adj R-squared =  0.0260
       Total |  2612.52141  3073  .850153403           Root MSE      =  .90997

------------------------------------------------------------------------------
   OpIn04Cap |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     AssetSq |   2.998617   .8489725     3.53   0.000     1.334005    4.663228
       _cons |   .1449039   .0859417     1.69   0.092    -.0236052    .3134129
------------------------------------------------------------------------------
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```
regress OpIn04Cap    LnSales04

.   Source |       SS       df       MS              Number of obs =    3074
-------------+------------------------------           F(  2,  3071) =   46.54
       Model |  76.8597115     2  38.4298558           Prob > F      =  0.0000
    Residual |   2535.6617  3071  .825679484           R-squared     =  0.0294
-------------+------------------------------           Adj R-squared =  0.0288
       Total |  2612.52141  3073  .850153403           Root MSE      =  .90867

------------------------------------------------------------------------------
   OpIn04Cap |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   LnSales04 |   .1118174     .02421     4.62   0.000     .0643479    .1592869
       _cons |  -.5443425   .1542445    -3.53   0.000    -.8467754   -.2419096
------------------------------------------------------------------------------
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Answer. Remember to say what it is you are putting in parentheses. Do not include the F-statistic for the test that all the coefficients are jointly different from zero. I can't recall ever seeing a paper in which that mattered (usually the constant alone would make the F-test show significance, since usually the constant is non-zero). Where that kind of F-test is used is when you want to test the joint significance of a *subset* of the variables, and for that you don't get an automatic number from the regression program; you have to specify the subset.

Table 1: Profit Regressions

	(a)	(b)
Constant	0.14*	-0.54***
	(1.69)	(3.53)
Assets squared	2.99***	
	(3.53)	
Log (sales)		0.11***
		(4.62)
R^2	.03	.03

Note: $n = 3,074$. The dependent variable is return on equity. t-statistics are in parentheses. Stars indicate significance at the 10% (*) and 1% (***) levels.

4. A company is selling widgets of quality x to consumers. Consumers think that x is low quality, a value of 3, with probability .2; and high quality, $x = 6$, with probability .8. The company's constant marginal cost is 1. The company can also make zingers to bundle together with the widgets. (Widget and Zinger are both nonsense names, not real products.) The quality of zingers is y, which is observable by the consumers before purchase. A zinger of quality y costs the company $3y/x$ to make. zingers are worthless to the public. The company will sell the widgets, or widget-zinger bundles, at price p.

(a) Is there an equilibrium in which the company does not make zingers regardless of the value of x? If so, describe it fully and show why no deviation would increase any player's payoff.

<u>*Answer*</u>. Yes.

Low Company: no zingers $p = .2(3) + .8(6) = .6 + 4.8 = 5.4$.
High company: no zingers, $p = .2(3) + .8(6) = .6 + 4.8 = 5.4$.
Consumers: Pay up to 5.4. If they observe a non-equilibrium price or a zinger bundled, they keep their initial .2 belief that quality is low.

It is crucial for this problem to remember what an "equilibrium" is. It gives the action taken at each possible node. That means it must say what the company would do if Nature makes it Low-quality, what the company would do if Nature made it high-quality, and what the consumers do for any price they might observe, plus any out-of-equilibrium beliefs the consumer might need.

(b) There exist equilibria in which the company makes zingers only if x is high. What is the most efficient such equilibrium?

Answer.

Low Company: no zingers $p = 3$.
High company: zingers of $y = 3$, $p = 6$.
Consumers: Pay up to 6 if there is a zinger with $y \geq 3$; otherwise, pay up to 3. If they observe a non-equilibrium price they keep their initial .2 belief that quality is low. If they observe zinger quality of less than 3 they believe the quality is low. If they observe zinger quality of more than 3, they believe quality is high.

Again, remember what an equilibrium is. It is not the same as an equilibrium outcome, and not the same as picking one of the many realized outcomes that might happen in equilibrium. Thus, it is wrong to say that the most efficient outcome is for the company to be lucky and have high quality. An equilibrium must say what the company would do if it were low-quality too.

The self-selection constraint is

$$\pi(low, y = 0) = 6 - 1 - (3)(y^*)/(3) = 5 - y^* \geq \pi(high, y = y^*) = 3 - 1 = 2 \qquad (11)$$

The lowest such zinger quality is $y^* = 3$. The high seller will choose that, since it satisfies the high IC constraint (5 - 3/2 ¿2):

$$\pi(high, y = y^*) = 6 - 1 - (3)(y^*)/(6) = 5 - y^*/2 \geq \pi(high, y = 0) = 3 - 1 = 2 \qquad (12)$$

(c) (NOT USED ON THE TEST) Are there equilibria in which the company makes zingers regardless of the value of x? If so, describe one of them fully and show why no deviation would increase any player's payoff.

Answer. Yes. $p = 5.4$. $y = 1$. Out of eq, a deviator is a Low.

The incentive compatibility constraints are:

$$\pi(low, y = 1) = 5.4 - 1 - (3)(1)/(3) = 3.4 \geq \pi(high, y = 0) = 3 - 1 = 2 \qquad (13)$$

and

$$\pi(high, y = 1) = 5.4 - 1 - (3)(1)/(6) = 3.9 \geq \pi(high, y = 0) = 3 - 1 = 2 \qquad (14)$$

5. It costs a firm c_1 per unit to produce high quality and c_0 to produce low quality, with $0 < c_0 < c_1$. Consumer value high quality at $\theta > c_1$ and low quality at 0. All consumers observe the prices of all firms, but no consumer observes the quality before choosing to visit one of the firms. Proportion α of the consumers are informed, and after they pick one firm to visit, they observe its quality before deciding whether to purchase. Proportion $(1 - \alpha)$ are uninformed, and must decide without observing quality.

(a) Suppose there is just one seller in the market. Under what conditions will there be a pure-strategy equilibrium in which it chooses high quality?

Answer. Suppose the price is p. The payoff per consumer from the equilibrium high quality is

$$p - c_1 \qquad (15)$$

The monopolist's payoff per consumer from deviating to low quality is

$$(1-\alpha)(p-c_0) \tag{16}$$

Equating these yields $p-c_1=(1-\alpha)(p-c_0)$, so $p-(1-\alpha)p=-(1-\alpha)c_0+c_1$ so we need

$$p \geq p^* = \frac{c_1 - c_0 + \alpha c_0}{\alpha} = c_0 + \frac{c_1 - c_0}{\alpha} \tag{17}$$

In any such equilibrium, the monopolist will choose the price to be θ, the most the consumers will pay for high quality. Thus, for this equilibrium to be possible requires that

$$\theta \geq p^* = c_0 + \frac{c_1 - c_0}{\alpha} \tag{18}$$

That requires that $c_1 - c_0$ be small or α be big relative to θ. Note that my condition relates exogenous parameters such as θ and *alpha*; p is endogenous, so a condition for existence should not be stated to depend on it.

(b) Suppose there are N firms competing. First, they simultaneously choose prices, which every player observes. Second, they simultaneously choose qualities, but nobody observes a firm's quality except the firm itself. Third, each consumer visits one firm.

There will not be an equilibrium in which all the firms charge $p=c_1$ and quality is high. Why not?

Answer. In such a strategy profile, the equilibrium payoff of a firm is zero. One firm could deviate and choose low quality, in which case its payoff per consumer would be $(c_1-c_0)>0$, so the deviation would be profitable.

(c) Continue with the model of N firms. Under what conditions will there be a pure-strategy equilibrium in which all the firms choose high quality and all the consumers buy the product? What will be the equilibrium price?

Answer. The payoff per consumer from high quality is

$$p - c_1 \tag{19}$$

The payoff per consumer from low quality is

$$(1-\alpha)(p-c_0) \tag{20}$$

Equating these yields $p-c_1=(1-\alpha)(p-c_0)$, so $p-(1-\alpha)p=-(1-\alpha)c_0+c_1$ so we need

$$p \geq p^* = \frac{c_1 - c_0 + \alpha c_0}{\alpha} = c_0 + \frac{c_1 - c_0}{\alpha} \tag{21}$$

Competition will force the price down to exactly p^*. For this to be an equilibrium, however, it must be that $p^* \leq \theta$.

So: in equilibrium all firms choose $p = p^*$ and high quality, and all consumers buy. Each firm's profit per customer is $(c_0 + \frac{c_1 - c_0}{\alpha} - c_1) > 0$.

The equilibrium strategies are a bit more complicated. All firms choose $p = p^*$. If any firm deviates and chooses $p < p^*$, it chooses low quality. If any firm deviates and chooses $p > p^*$, it chooses low quality. Consumers visit any firm with $p = p^*$, and if there are none, they stay home. Uninformed consumers buy if they visit. Informed consumers buy if they visit if the high quality is high and otherwise do not buy.[1]

I also accepted the true but off-track answer that if all the consumers are informed then quality is high and $p = c_1$.

(d) NOT INCLUDED. If a high-quality equilibrium does exist for the monopoly, does there also exist an equilibrium in which the firm produces low quality?

Answer. No. In such an equilibrium, the firm would not sell to any consumers. It could profitably deviate by producing high quality and charging slightly less than θ, in which case the informed consumers would observe the high quality and buy, giving the seller positive profits. (High quality with just the informed consumers buying would not be an equilibrium either, but I am using it here just to show a profitable deviation.)

[1] There is another equilibrium in which a firm that deviated and chose $p > p^*$ then chooses high quality, and in which if no firms choose $p = p^*$ and some pick $p > p^*$, the consumers all visit the firm or firms with the lowest price. The observed outcome of this other equilibrium is identical to the one described in the text: all firms choose $p = p^*$ and high quality, and all consumers buy.