31 January 2006 Eric Rasmusen, Erasmuse@indiana.edu. Http://www.rasmusen.org. Overheads for Parts of Sections 5.1, 5.2, 5.4, 6.4, 6.5 of *Games and Information*

5.4 Product Quality in an Infinitely Repeated Game

Players

An infinite number of potential firms and a continuum of consumers.

The Order of Play

1 An endogenous number n of firms decide to enter the market at cost F.
2 A firm that has entered chooses its quality to be $High$ or Low, incurring the constant marginal cost c if it picks $High$ and zero if it picks Low. The choice is unobserved by consumers. The firm also picks a price p.
3 Consumers decide which firms (if any) to buy from, choosing firms randomly if they are indifferent. The amount bought from firm i is denoted q_i.
4 All consumers observe the quality of all goods purchased in that period.
5 The game returns to (2) and repeats.

Product Quality in an Infinitely Repeated Game

Payoffs

The consumer benefit from a product of low quality is zero, but consumers are willing to buy quantity $q(p) = \sum_{i=1}^{n} q_i$ for a product believed to be high quality, where $\frac{dq}{dp} < 0$.

If a firm stays out of the market, its payoff is zero.

If firm i enters, it receives $-F$ immediately. Its current end-of-period payoff is $q_i p$ if it produces *Low* quality and $q_i(p - c)$ if it produces *High* quality.

The discount rate is $r \geq 0$.

AN EQUILIBRIUM

Firms. $\tilde{n}$ firms enter. Each produces high quality and sells at price $\tilde{p}$. If a firm ever deviates from this, it thereafter produces low quality (and sells at the same price $\tilde{p}$). The values of $\tilde{p}$ and $\tilde{n}$ are given by equations (2) and (6) below.

Buyers. Buyers start by choosing randomly among the firms charging $\tilde{p}$. Thereafter, they remain with their initial firm unless it changes its price or quality, in which case they switch randomly to a firm that has not changed its price or quality.

This strategy profile is a perfect equilibrium.

Each firm is willing to produce high quality and refrain from price-cutting because otherwise it would lose all its customers.

If it has deviated, it is willing to produce low quality because the quality is unimportant, given the absence of customers.

Buyers stay away from a firm that has produced low quality because they know it will continue to do so, and they stay away from a firm that has cut the price because they know it will produce low quality.

The equilibrium must satisfy three constraints: incentive compatibility, competition, and market clearing.

The **incentive compatibility** constraint says that the individual firm must be willing to produce high quality. Given the buyers' strategy, if the firm ever produces low quality it receives a one-time windfall profit, but loses its future profits. The tradeoff is represented by constraint (1), which is satisfied if the discount rate is low enough.

$$\frac{q_i p}{1+r} \leq \frac{q_i(p-c)}{r} \qquad (\textit{incentive compatibility}). \tag{1}$$

Inequality (1) determines a lower bound for the price, which must satisfy

$$\tilde{p} \geq (1+r)c. \tag{2}$$

The second constraint is that competition drives profits to zero, so firms are indifferent between entering and staying out of the market.

$$\frac{q_i(p-c)}{r} = F \qquad (competition) \tag{3}$$

Treating (1) as an equation and using it to replace p in equation (3) gives

$$q_i = \frac{F}{c}. \tag{4}$$

We have now determined p and q_i, and only n remains, which is determined by the equality of supply and demand.

$$nq_i = q(p). \qquad (market\ \ clearing) \tag{5}$$

Combining equations (1), (4), and (5) yields

$$\tilde{n} = \frac{cq([1+r]c)}{F}. \tag{6}$$

Table 2: Prisoner's Dilemmas

(a) Two-Sided (conventional)

		Column		
		Silence		*Blame*
	Silence	5,5	→	-5,10
Row:		↓		↓
	Blame	10,-5	→	**0,0**

Payoffs to: (Row, Column). Arrows show how a player can increase his payoff.

REPEATED GAME IDEAS

The Prisoner's Dilemma: Each prisoner can Blame or be Silent, and in equilibrium, they choose (Blame, Blame). (Tucker)

The Prisoner's Dilemma and Duopoly Collusion: Each firm can choose Low or High price, and in equilibrium they choose (Low, Low). (Bertrand, Cournot both are like that)

The Chainstore Paradox: With finite repetitions, the only perfect equilibrium is (Blame, Blame) in each period. (Selten)

The Folk Theorem: With infinite repetitions, the number of equilibria is infinite, and in any set of T periods, any behavior could be equilibrium behavior. (Aumann)

The Grim Strategy

1 Start by choosing Silence.
2 Continue to choose Silence unless some player has chosen Blame, in which case choose Blame forever.

Tit-for-Tat

1 Start by choosing Silence.
2 Thereafter, in period n choose the action that the other player chose in period $(n-1)$.

6.4 Incomplete Information in the Repeated Prisoner's Dilemma: The Gang of Four Model

One of the most important explanations of reputation is that of Kreps, Milgrom, Roberts & Wilson (1982).

Theorem 6.1: The Gang of Four Theorem
Consider a T-stage, repeated Prisoner's Dilemma, without discounting but with a probability γ of a Tit-for-Tat player. In any perfect bayesian equilibrium, the number of stages in which either player chooses Blame *is less than some number M that depends on γ but not on T.*

The significance of the Gang of Four theorem is that while the players do resort to *Blame* as the last period approaches, the number of periods during which they *Blame* is independent of the total number of periods. Suppose $M = 2,500$. If $T = 2,500$, there might be *Blame* every period. But if $T = 10,000$, there are 7,500 periods without a *Blame* move. For reasonable probabilities of the unusual type, the number of periods of cooperation can be much larger. Wilson (unpublished) has set up an entry deterrence model in which the incumbent fights entry (the equivalent of *Silence* above) up to seven periods from the end, although the probability the entrant is of the unusual type is only 0.008.

To get a feeling for why Theorem 6.1 is correct, consider what would happen in a 10,001 period game with a probability of 0.01 that Row is playing the Grim Strategy of *Silence* until the first *Blame*, and *Blame* every period thereafter.

A best response for Column to a known Grim player is (*Blame* only in the last period, unless Row chooses *Blame* first, in which case respond with *Blame*).

Both players will choose *Silence* until the last period, and Column's payoff will be 50,010 (= (10,000)(5) + 10).

The Grim Strategy is NOT an equilibrium strategy in the complete-information game, though. Row would deviate to *Blame* in the second-to-last period.

Suppose for the moment that if Row is not Grim, he is highly aggressive, and will choose *Blame* every period. If Column follows the strategy just described, the outcome will be (*Blame*, *Silence*) in the first period and *(Blame, Blame*) thereafter, for a payoff to Column of $-5(= -5 + (10,000)(0))$.

If the probabilities of the two outcomes are 0.01 and 0.99, Column's expected payoff from the strategy described is 495.15. If instead he follows a strategy of (*Blame* every period), his expected payoff is just 0.1 $(= 0.01(10) + 0.99(0))$.

Column should risk cooperating with Row even if Row has a 0.99 probability of following a very aggressive strategy.

The aggressive strategy, however, is not Row's best response to Column's strategy. A better response is for Row to choose *Silence* until the second-to- last period, and then to choose *Blame*. Given that Column is cooperating in the early periods, Row will cooperate also.

TIT-FOR-TAT

Tit-for-Tat has three strong points.

1. It never initiates blaming (**niceness**);
2. It retaliates instantly against blaming (**provocability**);
3. It forgives someone who plays *Blame* but then goes back to cooperating (it is **forgiving**).