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10 Mechanism Design and Post-Contractual Hidden Knowledge

Production Game VIII: Mechanism Design

Players

The principal and the agent.

The Order of Play

1 The principal offers the agent a wage contract of the form $w(q, m)$, where q is output and m is a message to be sent by the agent.
2 The agent accepts or rejects the principal's offer.
3 Nature chooses the state of the world s, according to probability distribution $F(s)$, where the state s is *good* with probability 0.5 and *bad* with probability 0.5. The agent observes s, but the principal does not.
4 If the agent accepted, he exerts effort e unobserved by the principal, and sends message $m \in \{good, bad\}$ to him.
5 Output is $q(e, s)$, where $q(e, good) = 3e$ and $q(e, bad) = e$, and the wage is paid.

Payoffs Agent rejects: $\pi_{agent} = \bar{U} = 0$ and $\pi_{principal} = 0$.
Agent accepts: $\pi_{agent} = U(e, w, s) = w - e^2$ and $\pi_{principal} = V(q - w) = q - w$.

Production Game VII, adverse selection version: two participation constraints and two incentive compatibility constraints. PG VIII, moral hazard with hidden info: one participation constraint.

The first-best is unchanged from Production Game VII: $e_g = 1.5$ and $q_g = 4.5$, $e_b = 0.5$ and $q_b = 0.5$.

Also unchanged is that the principal must solve the problem:

$$\underset{q_g, q_b, w_g, w_b}{Maximize} \; [0.5(q_g - w_g) + 0.5(q_b - w_b)], \qquad (1)$$

where the agent is paid under one of two forcing contracts, (q_g, w_g) if he reports $m = good$ and (q_b, w_b) if he reports $m = bad$, where producing the wrong output for a given contract results in boiling in oil.

The self-selection constraints are the same as in Production Game VII.

$$\pi_{agent}(q_g, w_g|good) = w_g - \left(\frac{q_g}{3}\right)^2 \geq \pi_{agent}(q_b, w_b|good) = w_b - \left(\frac{q_b}{3}\right)^2 \qquad (2)$$

$$\pi_{agent}(q_b, w_b|bad) = w_b - q_b^2 \geq \pi_{agent}(q_g, w_g|bad) = w_g - q_g^2. \qquad (3)$$

The single participation constraint is

$$0.5\pi_{agent}(q_g, w_g|good) + 0.5\pi_{agent}(q_b, w_b|bad) = 0.5\left(w_g - \left(\frac{q_g}{3}\right)^2\right) + 0.5\left(w_b - q_b^2\right) \geq 0 \qquad (4)$$

This single participation constraint is binding, since the principal wants to pay the agent as little as possible.

The good state's self-selection constraint will be binding. In the good state the agent will be tempted to take the easier contract appropriate for the bad state (due to the "single-crossing property" to be discussed in a later section) and so the principal has to increase the agent's payoff from the good-state contract to yield him at least as much as in the bad state. He does not want to increase the surplus any more than necessary, though, so the good state's self-selection constraint will be exactly satisfied.

This gives us two equations,

$$\begin{aligned} &0.5\left(w_g - \left(\frac{q_g}{3}\right)^2\right) + 0.5\left(w_b - q_b^2\right) = 0 \\ &w_g - \left(\frac{q_g}{3}\right)^2 = w_b - \left(\frac{q_b}{3}\right)^2 \end{aligned} \tag{5}$$

Solving them out yields $w_b = \frac{5}{9}q_b^2$ and $w_g = \frac{1}{9}q_g^2 + \frac{4}{9}q_b^2$.

Returning to the principal's maximization problem in (1) and substituting for w_b and w_g, we can rewrite it as

$$\underset{q_g, q_b}{Maximize} \quad \pi_{principal} = \left[0.5\left(q_g - \frac{q_g^2}{9} - \frac{4q_b^2}{9}\right) + 0.5\left(q_b - \frac{5q_b^2}{9}\right)\right] \tag{6}$$

with no constraints. The first-order conditions are

$$\frac{\partial \pi_{principal}}{\partial q_g} = 0.5\left(1 - \left[\frac{2}{9}\right] q_g\right) = 0, \tag{7}$$

so $q_g = 4.5$, and

$$\frac{\partial \pi_{principal}}{\partial q_b} = 0.5\left(-\frac{8q_b}{9}\right) + 0.5\left(1 - \frac{10q_b}{9}\right) = 0, \tag{8}$$

so $q_b = \frac{9}{18} = .5$. We can then find the wages that satisfy the constraints, which are $w_g \approx 2.36$ and $w_b \approx 0.14$.

As in Production Game VII, in the good state the effort is at the first-best level while in the bad state it is less.

The agent does not earn informational rents, because at the time of contracting he has no private information. In Production Game VII the wages were $w_g' \approx 2.32$ and $w_b' \approx 0.07$. Both wages are higher in Production Game VIII, but so is effort in the bad state. The principal in Production Game VIII can (a) come closer to the first-best when the state is bad, and (b) reduce the rents to the agent.

Observable but Nonverifiable Information

If the state or type is public information, then it is straightforward to obtain the first-best using forcing contracts. What if the state is observable by both principal and agent, but is not public information?

We say that the variable s is **nonverifiable** if contracts based on it cannot be enforced.

Maskin (1977) suggested a matching scheme to achieve the first-best which would take the following two-part form for Production Game VIII:

(1) Principal and agent simultaneously send messages m_p and m_a to the court saying whether the state is good or bad. If $m_p \neq m_a$, then no contract is chosen and both players earn zero payoffs. If $m_p = m_a$, the court enforced part (2) of the scheme.

(2) The agent is paid the wage $(w|q)$ with either the good-state forcing contract $(2.25|4.5)$ or the bad-state forcing contract $(0.25|0.5)$, depending on his report m_a, or is boiled in oil if he the output is inappropriate to his report.

Usually this kind of scheme has multiple equilibria, however, perverse ones in which both players send false message which match and inefficient actions result. Here, in a perverse equilibrium the principal and agent would always send the message $m_p = m_a = bad$. Even when the state was actually good, the payoffs would be

$(\pi_{principal}(good) = 0.5 - 0.25 > 0$ and $\pi_{agent}(good) = 0.25 - (0.5)^2 = 0$,

Neither player would have incentive to deviate unilaterally and drive payoffs to zero.

Perhaps a bigger problem than the multiplicity of equilibria is renegotiation due to players' inability to commit to the mechanism.

Suppose the equilibrium says that both players will send truthful messages, but the agent deviates and reports $m_a = bad$ even though the state is good. The court will say that the contract is nullified.

But agent could negotiate a new contract with the principal.

The Maskin scheme is like the Holmstrom Teams contract, where if output was even a little too small, it was destroyed rather than divided among the team members.

Solution: a third party who would receive the output if it was t oo small.

Unravelling: Information Disclosure when Lying Is Prohibited

There is another special case in which hidden information can be forced into the open: when the agent is prohibited from lying and only has a choice between telling the truth or remaining silent.

In Production Game VIII, this set-up would give the agent two possible message sets. If the state were good, the agent's message would be taken from $m \in \{good, silent\}$. If the state were *bad*, the agent's message would be taken from $m \in \{bad, silent\}$.

The agent would have no reason to be silent if the true state were bad (which means low output would be excusable), so his message then would be *bad*. But then if the principal hears the message *silent* he knows the state must be good– *good* and *silent* both would occur only when the state was good. So the option to remain silent is worthless to the agent.

Suppose Nature uses the uniform distribution to assign the variable s some value in the interval $[0, 10]$ and the agent's payoff is increasing in the principal's estimate of s.

Assume the agent cannot lie but he can conceal information. Thus, if $s = 2$, he can send the uninformative message $m \geq 0$ (equivalent to no message), or the message $m \geq 1$, or $m = 2$, but not $m \geq 4.36$.

When $s = 2$ the agent might as well send a message that is the exact truth: "$m = 2$."

If he were to choose the message "$m \geq 1$" instead, the principal's first thought might be to estimate s as the average value in the interval $[1, 10]$, which is 5.5.

But the principal would realize that no agent with a value of s greater than 5.5 would want to send the message "$m \geq 1$" if 5.5 was the resulting deduction. This realization restricts the possible interval to [1, 5.5], which in turn has an average of 3.25.

But then no agent with $s > 3.25$ would send the message "$m \geq 1$."

The principal would continue this process of logical **unravelling** to conclude that $s = 1$.

MODEL REPETITION: Nature uses the uniform distribution to assign the variable s some value in the interval $[0, 10]$ and the agent's payoff is increasing in the principal's estimate of s. The agent cannot lie but he can conceal information.

In this model, no news is bad news.

The agent would therefore not send the message "$m \geq 1$" and he would be indifferent between "$m = 2$" and "$m \geq 2$" because the principal would make the same deduction from either message.

ANOTHER APPROACH

The equilibrium is either fully separating or has some pooling.

If it is fully separating, the agent's type is revealed, so it might as well be $m = s$.

If it had some pooling, then two types with $s_2 > s_1$ would be pooled together and the principal's estimate of s would be the average in the pool. Type s_2 would therefore deviate to $m = s_2$ to reveal his type. So the equilibrium must be perfectly separating.

Where would this logic break down? —

either unpunishable lying or genuine ignorance.

The Revelation Principle: We Can Restrict Attention to Direct Mechanisms

Let w be the agent's wage, q be output, m be his message, and s be his type. ALLOW COMMITMENT TO CONTRACTS.

The Revelation Principle. *For every contract $w(q,m)$ that leads to lying (that is, to $m \neq s$), there is a contract $w^*(q,m)$ with the same outcome for every s but no incentive for the agent to lie.*

A **direct mechanism**— agents tell the truth in equilibrium— can be found equivalent to any **indirect mechanism** in which they lie.

Suppose we are trying to design a mechanism to make people with higher incomes pay higher taxes, but anyone who makes $70,000 a year can claim he makes $50,000 and we do not have the resources to catch him.

We could design a mechanism in which higher reported incomes pay higher taxes, but reports of $50,000 would come from both people who truly have that income and people whose income is $70,000.

The revelation principle says that we can rewrite the tax code to set the tax to be the same for taxpayers earning $70,000 and for those earning $50,000, and the same amount of taxes will be collected without anyone having incentive to lie.

The Crawford-Sobel Sender-Receiver Game

Players

The sender and the receiver.

The Order of Play

0 Nature chooses the sender's type to be $t \sim U[0, 10]$.
1 The sender chooses message $m \in [0, 10]$.
2 The receiver chooses action $a \in [0, 10]$.

Payoffs

The payoffs are quadratic loss functions in which each player has an ideal point and wants a to be close to that ideal point.

$$\begin{aligned} \pi_{sender} &= \alpha - (a - [t+1])^2 \\ \pi_{receiver} &= \alpha - (a - t)^2 \end{aligned} \tag{9}$$

Suppose the receiver believed that the sender always sent $m = t$ and so chooses $a = m$. Would the sender indeed be willing to tell the truth?

No. The sender would not always report $m = 10$, because his ideal point is $a = t + 1$, rather than a being as big as possible. If, however, the sender thinks the receiver will believe him, he will deviate to reporting $m = t + 1$, always exaggerating his type slightly.

Pooling Equilibrium 1
Sender: Send $m = 10$ regardless of t.
Receiver: Choose $a = 5$ regardless of m.
Out-of-equilibrium belief: If the sender sends $m < 10$, the receiver uses passive conjectures and still believes that $t \sim U[0, 10]$.

Pooling Equilibrium 2
Sender: Send m using a mixed-strategy distribution independent of t that has the support $[0, 10]$ with positive density everywhere.
Receiver: Choose $a = 5$ regardless of m.
Out-of-equilibrium belief: Unnecessary, since any message might be observed in equilibrium.

In each of these two equilibria, the sender's action conveys no information and is ignored by the receiver. The sender is happy about this if it happens that $t = 4$, and the receiver is if $t = 5$, but averaging over all possible t, both their payoffs are lower than if the sender could commit to truthtelling.

Partial Pooling Equilibrium 3
Sender: Send $m = 0$ if $t \in [0, 3]$ or $m = 10$ if $t \in [3, 10]$.
Receiver: Choose $a = 1.5$ if $m < 3$ and $a = 6.5$ if $m \geq 3$
Out-of-equilibrium belief: If m is something other than 0 or 10, then $t \sim U[0, 3]$ if $m \in [0, 3)$ and $t \sim U[3, 10]$ if $a \in [3, 10]$.

In effect, the Sender has reduced his message space to two messages, LOW (=0) and HIGH (=10), in Equilibrium 3.

The receiver's optimal strategy in a a partially pooling equilibrium is to choose his action to equal the expected value of the type in the interval the sender has chosen. Thus, if $m = 0$, the receiver will choose $a = x/2$ and if $m = 10$ he will choose $a = (x + 10)/2$.

Partial Pooling Equilibrium 3
Sender: Send $m = 0$ if $t \in [0,3]$ or $m = 10$ if $t \in [3,10]$.
Receiver: Choose $a = 1.5$ if $m < 3$ and $a = 6.5$ if $m \geq 3$
Out-of-equilibrium belief: If m is something other than 0 or 10, then $t \sim U[0,3]$ if $m \in [0,3)$ and $t \sim U[3,10]$ if $a \in [3,10]$.

The receiver's equilibrium response determines the sender's payoffs from his two messages. The payoffs between which he chooses are:

$$\begin{aligned} \pi_{sender,m=0} &= \alpha - \left([t+1] - \frac{x}{2}\right)^2 \\ \pi_{sender,m=10} &= \alpha - \left(\frac{10+x}{2} - [t+1]\right)^2 \end{aligned} \tag{10}$$

There exists a value x such that if $t = x$, the sender is indifferent between $m = 0$ and $m = 10$, but if t is lower he prefers $m = 0$ and if t is higher he prefers $m = 10$. To find x, equate the two payoffs in expression (10) and simplify to obtain

$$[t+1] - \frac{x}{2} = \frac{10+x}{2} - [t+1]. \tag{11}$$

We set $t = x$ at the point of indifference, and solving for x yields $x = 3$.

In the Crawford-Sobel Sender-Receiver Game, the receiver cannot commit to the way he reacts to the message, so this is not a mechanism design problem.

Nor is the sender punished for lying, so the unravelling argument for truthtelling does not apply.

Nor do the players' payoffs depend directly on the message, which might permit the signalling we will study in Chapter 11 to operate.

Instead, this is a **cheap-talk game**, so called because of these very absences: m does not affect the payoff directly, the players cannot commit to future actions, and lying brings no directly penalty.

10.5: Price Discrimination

Pigou was a contemporary of Keynes at Cambridge who usefully divided price discrimination into three types in 1920.

1 Interbuyer Price Discrimination.

2 Interquantity Price Discrimination or **Nonlinear Pricing**.

3 Perfect Price Discrimination.

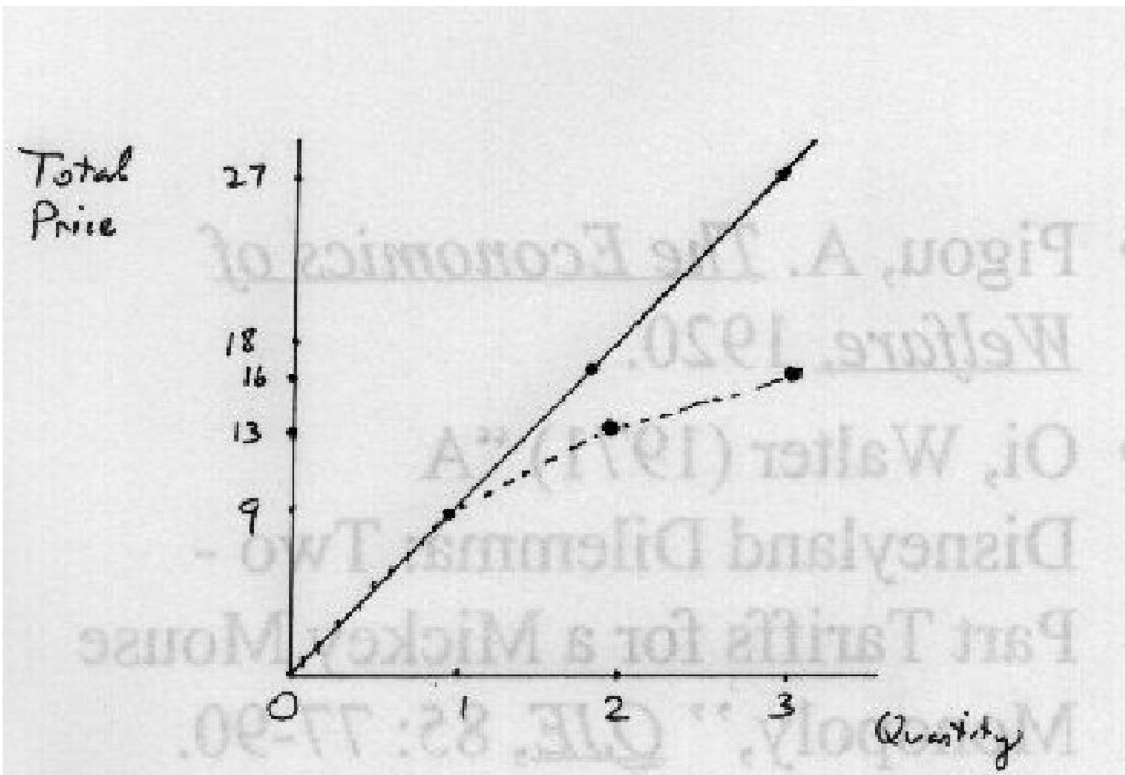


Figure 3: Linear and Nonlinear Pricing

Varian's Nonlinear Pricing Game

Players

One seller and one buyer.

The Order of Play

0 Nature assigns the buyer a type, s. The buyer is "unenthusiastic" with utility function u or "valuing" with utility function v, with equal probability. The seller does not observe Nature's move, but the buyer does.
1 The seller offers mechanism $\{w_m, q_m\}$ under which the buyer can announce his type as m and buy amount q_m for lump sum w_m.
2 The buyer chooses a message m or rejects the mechanism entirely and does not buy at all.

Payoffs

The seller has a constant marginal cost of c, so his payoff is

$$w_u + w_v - c \cdot (q_u + q_v). \tag{12}$$

The buyers' payoffs are $\pi_u = u(q_u) - w_u$ and $\pi_v = v(q_v) - w_v$ if q is positive, and 0 if $q = 0$, with $u', v' > 0$ and $u'', v'' < 0$. The marginal willingness to pay is greater for the valuing buyer: for any q,

$$u'(q) < v'(q) \tag{13}$$

The marginal willingness to pay is greater for the valuing buyer: for any q,

$$u'(q) < v'(q) \tag{14}$$

Condition (14) is an example of **the single-crossing property**, which we will discuss at the end of this section. Combined with the assumption that $v(0) = u(0) = 0$, it also implies that

$$u(q) < v(q) \tag{15}$$

for any value of q.

Perfect Price Discrimination

The game would allow perfect price discrimination if the seller did know which buyer had which utility function. He can then just maximize profit subject to the participation constraints for the two buyers:

$$\overset{Maximize}{w_u, w_v, q_u, q_v} \quad w_u + w_v - c \cdot (q_u + q_v). \tag{16}$$

subject to

$$\begin{aligned} (a) \;\; & u(q_u) - w_u \geq 0 \;\; \text{and} \\ (b) \;\; & v(q_v) - w_v \geq 0. \end{aligned} \tag{17}$$

The constraints will be satisfied as equalities, since the seller will charge all that the buyers will pay. Substituting for w_u and w_v into the maximand, the first order conditions become

$$\begin{aligned} (a) \;\; & u'(q_u^*) - c = 0 \quad \text{and} \\ (b) \;\; & v'(q_v^*) - c = 0. \end{aligned} \tag{18}$$

Thus, the seller will choose quantities so that each buyer's marginal utility equals the marginal cost of production, and will choose prices so that the entire consumer surpluses are eaten up: $w^*(q_u^*) = u(q_u^*)$ and $w^*(q_v^*) = v(q_v^*)$. Figure 4 shows this for the unenthusiastic buyer.

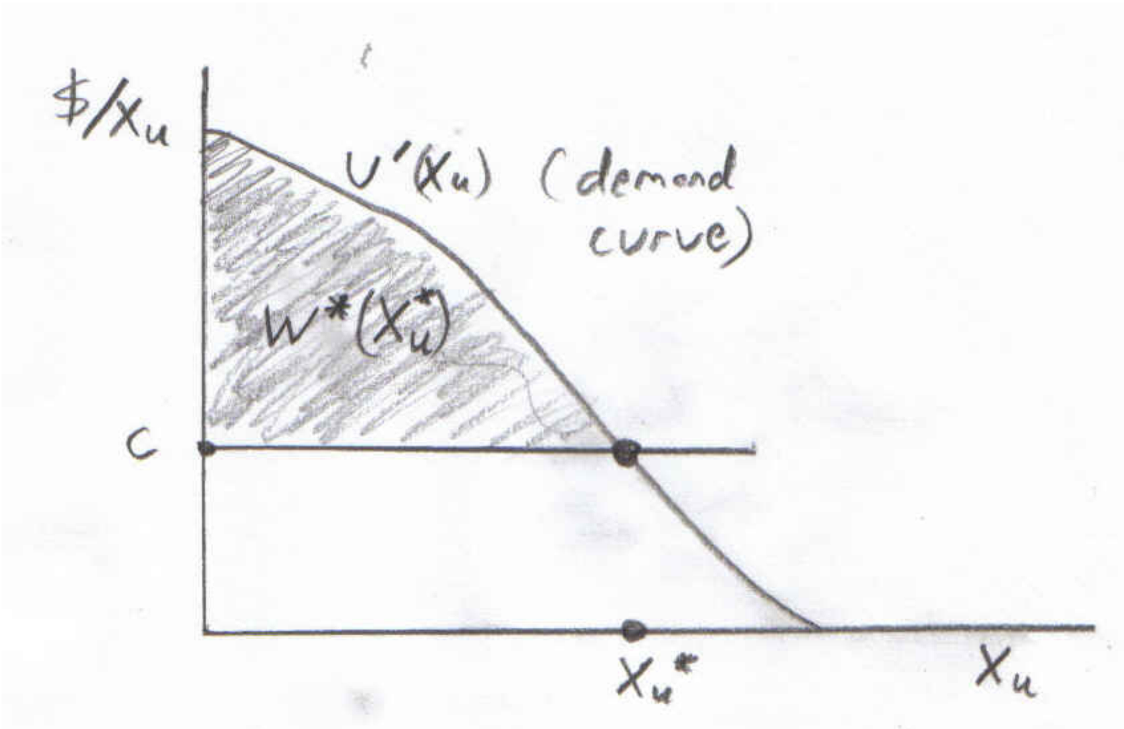


Figure 4: Perfect Price Discrimination

Interbuyer Price Discrimination

$$\underset{q_u, q_v, p_u, p_v}{Maximize} \quad p_u q_u + p_v q_v - c \cdot (q_u + q_v), \tag{19}$$

subject to the participation constraints

$$\begin{aligned} &u(q_u) - p_u q_u \geq 0 \quad \text{and} \\ &v(q_v) - p_v q_v \geq 0 \end{aligned} \tag{20}$$

and the incentive compatibility constraints

$$\begin{aligned} &q_u = argmax[u(q_u) - p_u q_u] \quad \text{and} \\ &q_v = argmax[v(q_v) - p_v q_v]. \end{aligned} \tag{21}$$

This should remind you of moral hazard. It is very like the problem of a principal designing two incentive contracts for two agents to induce appropriate effort levels given their different disutilities of effort.

The agents will solve their quantity choice problems in (21), yielding

$$u'(q_u) - p_u = 0 \quad \text{and}$$
$$v'(q_v) - p_v = 0. \tag{22}$$

Thus, we can simplify the original problem in (19) to

$$\underset{q_u,\, q_v}{\textit{Maximize}} \quad u'(q_u)q_u + v'(q_v)q_v - c \cdot (q_u + q_v), \tag{23}$$

subject to the participation constraints

$$u(q_u) - u'(q_u)q_u \geq 0 \quad \text{and}$$
$$v(q_v) - v'(q_v)q_v \geq 0. \tag{24}$$

The participation constraints will not be binding. If they were, then $u(q)/q = u'(q)$, but since $u'' < 0$ there is diminishing utility of consumption and the average utility, $U(q)/q$, will be greater than the marginal utility, $u'(q)$. Thus we can solve problem (23) as if there were no constraints. The first-order conditions are

$$\begin{aligned} u''(q_u)q_u + u' &= c \quad \text{and} \\ v''(q_v)q_v + v' &= c. \end{aligned} \tag{25}$$

This is just the 'marginal revenue equals marginal cost condition that any monopolist uses, but one for each buyer instead of one for the entire market.

Nonlinear Pricing

$$\underset{q_u, q_v, w_u, w_v}{Maximize} \quad w_u + w_v - c \cdot (q_u + q_v), \tag{26}$$

subject to the participation constraints,

$$\begin{array}{ll} (a) & u(q_u) - w_u \geq 0 \quad \text{and} \\ (b) & v(q_v) - w_v \geq 0, \end{array} \tag{27}$$

and the self-selection constraints,

$$\begin{array}{ll} (a) & u(q_u) - w_u \geq u(q_v) - w_v \\ \\ (b) & v(q_v) - w_v \geq v(q_u) - w_u. \end{array} \tag{28}$$

Not all of these constraints will be binding. If neither type had a binding participation constraint, however, the principal would be losing a chance to increase his profits. In a mechanism design problem like this, what always happens is that the contracts are designed so that one type of agent is pushed down to his reservation utility.

Suppose the optimal contract is in fact separating, and also that both types accept a contract. At least one type will have a binding participation constraint. Since the valuing consumer gets more consumer surplus from a given w and q than an unenthusiastic consumer, it must be the unenthusiastic consumer who is driven down to zero surplus for (w_u, q_u). The valuing consumer would get positive surplus from accepting that same contract, so his participation constraint is not binding. To persuade the valuing consumer to accept (w_v, q_v) instead, the seller must give him that same positive surplus from it. The seller will not be any more generous than he has to, though, so the valuing consumer's self-selection constraint will be binding.

Rearranging our two binding constraints and setting them out as equalities yields:

$$w_u = u(q_u) \tag{29}$$

and

$$w_v = w_u - v(q_u) + v(q_v) \tag{30}$$

This allows us to reformulate the seller's problem from (26) as

$$\underset{q_u,\, q_v}{Maximize} \quad u(q_u) + u(q_u) - v(q_u) + v(q_v) - c \cdot (q_u + q_v), \tag{31}$$

which has the first-order conditions

$$\begin{aligned} &(a) \;\; u'(q_u) - c + [u'(q_u) - v'(q_u)] = 0 \\ &(b) \;\; v'(q_v) - c = 0 \end{aligned} \tag{32}$$

The first-order conditions in (32) could be solved for exact values of q_u and q_v if we chose particular functional forms, but they are illuminating even if we do not.

Equation (32b) tells us that the valuing type of buyer buys a quantity such that his last unit's marginal utility exactly equals the marginal cost of production; his consumption is at the efficient level.

The unenthusiastic type, however, buys less than his first-best amount.

Using the single-crossing property, assumption (14b), $u'(q) < v'(q)$, which implies from (32a) that $u'(q_u) - c > 0$ and the unenthusiastic type has not bought enough to drive his marginal utility down to marginal cost.

The intuition is that the seller must sell less than first-best optimal to the unenthusiastic type so as not to make that contract too attractive to the valuing type. On the other hand, making the valuing type's contract more valuable to him actually helps separation, so q_v is chosen to maximize social surplus.

The single-crossing property has another important implication. Substituting from first-order condition (32b) into first-order condition (32a) yields

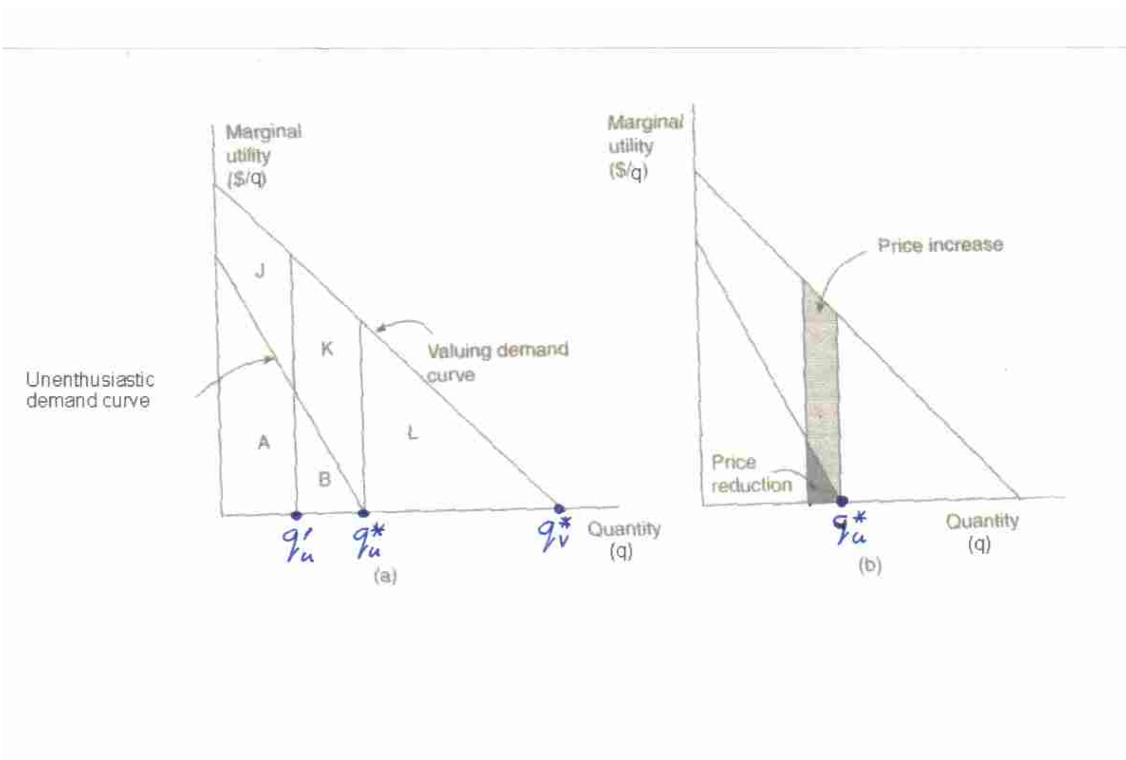
$$[u'(q_u) - v'(q_v)] + [u'(q_u) - v'(q_u)] = 0 \qquad (33)$$

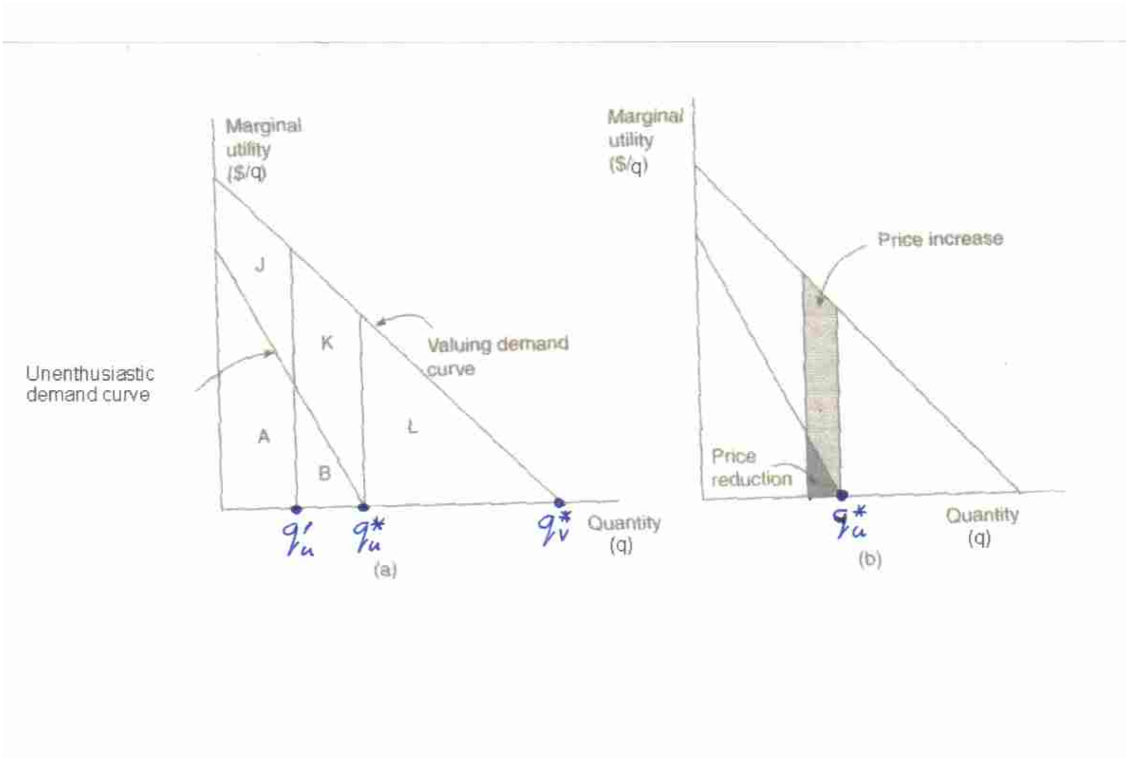
The second term in square brackets is negative by the single- crossing property. Thus, the first term must be positive. But since the single-crossing property tells us that $[u'(q_u) - v'(q_u)] < 0$, it must be true, since $v'' < 0$, that if $q_u \geq q_v$ then $[u'(q_u) - v'(q_v)] < 0$ – that is, that the first term is negative. We cannot have that without contradiction, so it must be that $q_u < q_v$.

The unenthusiastic buyer buys strictly less than the valuing buyer. This accords with our intuition, and also lets us know that the equilibrium is separating, not pooling (though we still have not proven that the equilibrium involves both players buying a positive amount, something hard to prove elegantly since one player buying zero would be a corner solution to our maximization problem).

A Graphical Approach to the Same Problem

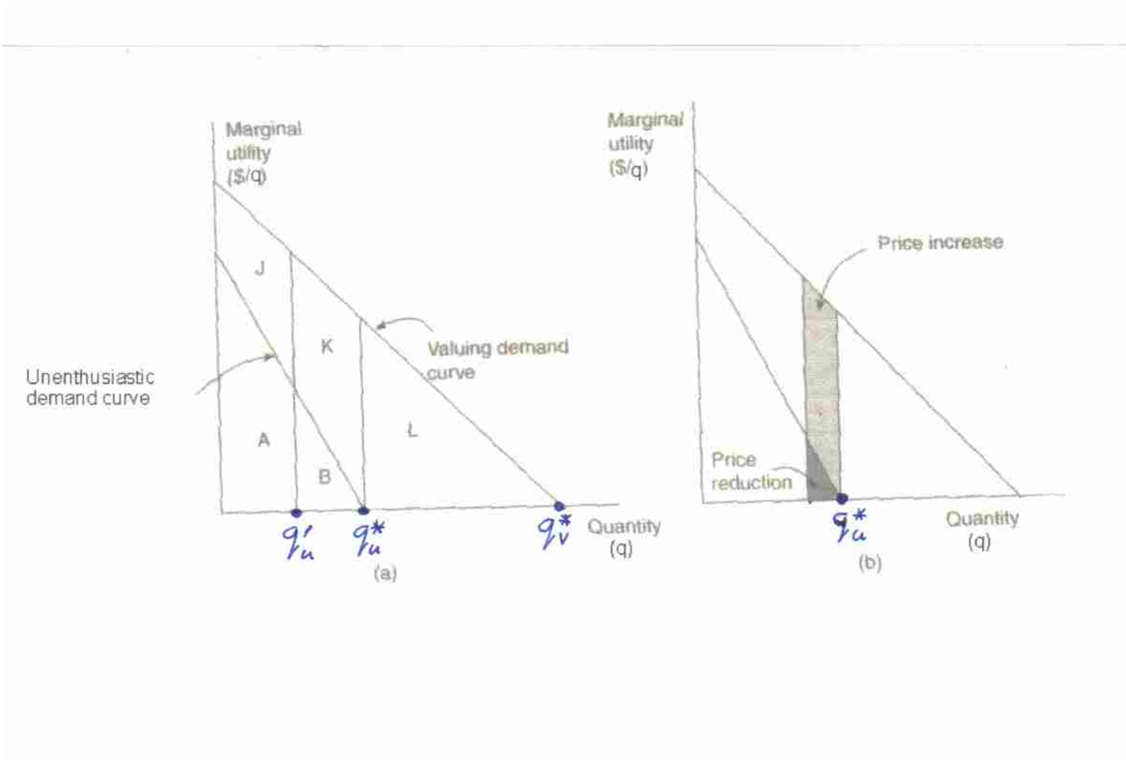
Under perfect price discrimination, the seller would charge $w_u = A + B$ and $w_v = A + B + J + K + L$ to the two buyers for quantities q_u^* and q_v^*, as shown in Figure 5a. An attempt to charge $w_u^* = A + B$ and $w_v^* = A + B + J + K + L$, however, would simply lead to both buyers choosing to buy q_u^*, which would yield the valuing buyer a payoff of $J + K$ rather than the zero he would get as a payoff from buying q_v^*. The seller's payoff from this pooling equilibrium (which is the best pooling contract possible for him, since it drives the unenthusiastic type to a payoff of zero) is $2(A + B)$.

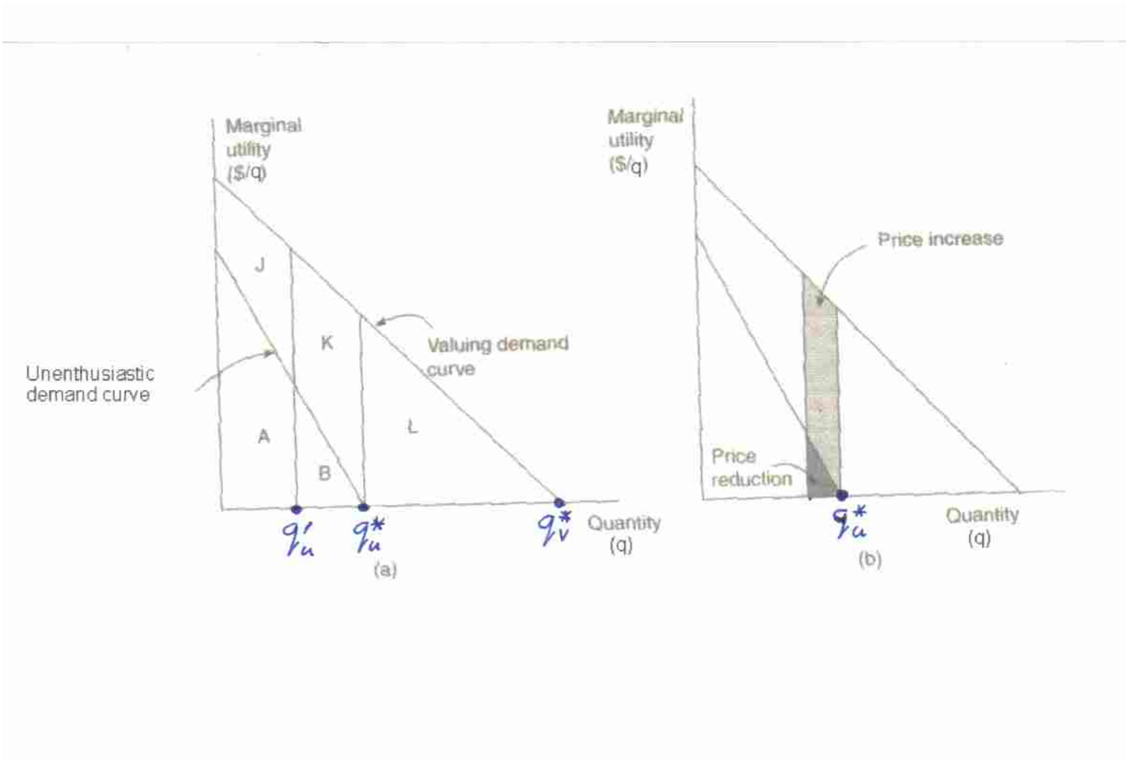




The seller could separate the two buyers by charging $w_u^* = A + B$ for q_u^* and $w_v^* = A + B + L$ for q_v^*, since the unenthusiastic buyer would have no reason to switch to the greater quantity, and that would increase his profits over pooling by amount L.

The seller would do even better to slightly reduce the quantity sold to the unenthusiastic buyer, to below q_u^*, and reduce the price to him by the amount of the dark shading. He could then sell q_v^* to the valuing buyer and raise the price to him by the light shaded area. The valuing buyer will not be tempted to buy the smaller quantity at the lower price, and the seller will have gained profit by, loosely speaking, increasing the size of the L triangle.





Our profit-maximizing mechanism is shown in Figure 5a as $w'_u = A$ for q'_u and $w^*_v = A + B + K + L$ for q^*_v.

Unenthusiastic buyer: binding participation constraint, inefficiently low consumption, because $w'_u = A = u(q'_u)$.

Valuing buyer: nonbinding participation constraint, because $w^*_v = A + B + K + L < v(q^*_v) = A + B + J + K + L$; he is left with a surplus of J. Efficient consumption: q^*_v. B binding self-selection constraint, because he is indifferent between buying q'_u and q^*_v.

His choice is between a payoff of $\pi_v(U) = (A+J) - A$ and $\pi_v(V) = (A+B+J+K+L) - (A+B+K+L)$.

Thus, the diagram replicates the algebraic conclusions.

The Single-Crossing Property

When we say that Buyer V's demand is stronger than Buyer U's, however, there are two things we might mean:

1. Buyer V's *average demand* is stronger: $\frac{v(q)}{q} > \frac{u(q)}{q}$. Buyer V would pay more for quantity q than Buyer U would.

2. Buyer V's *marginal demand* is stronger: $v'(q) > u'(q)$. Buyer V would pay more for an additional unit than Buyer U would.

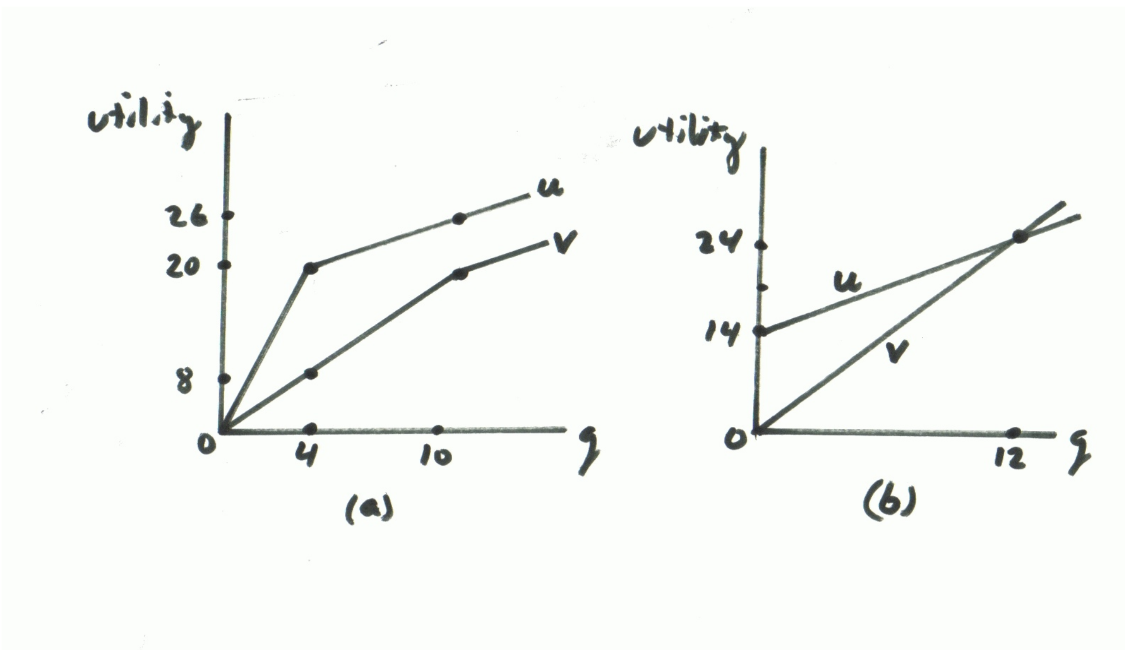


Figure 6: Marginal versus Average Demand

Figure 7a depicts functions which satisfy the assumptions of Varian's Nonlinear Pricing Game: $u = \sqrt{q}$ and $v = 2\sqrt{q}$. The two curves satisfy the single-crossing property, condition (14), because $v'(q) > u'(q)$ for all q and $u(0) = 0$ and $v(0) = 0$.

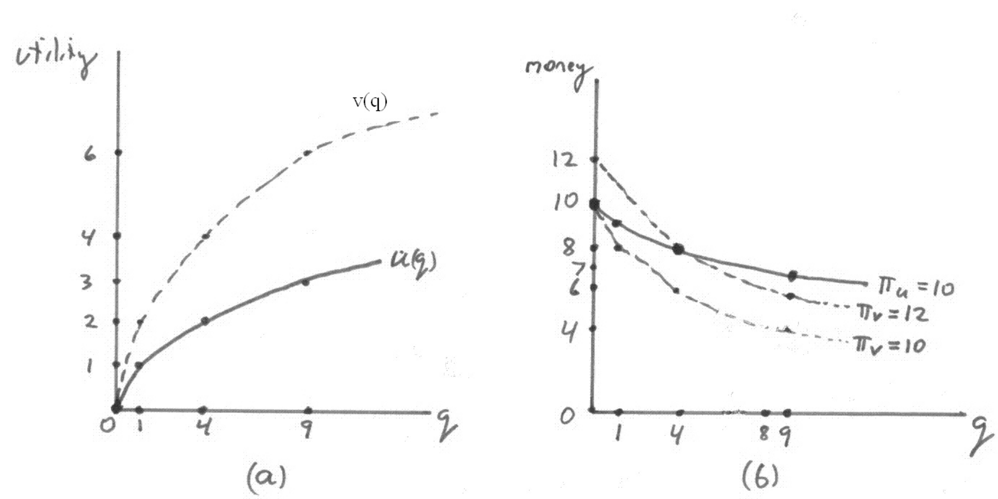
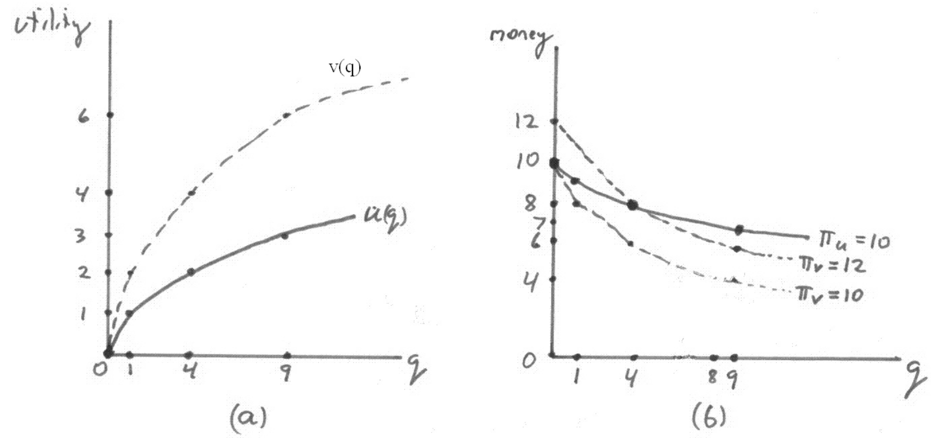


Figure 7: Two Depictions of the Single Crossing Property

Another way to write the payoff functions would have been as $\pi_u(q, money) = money + u(q)$, where $money = wealth - w(q)$.

One comparison is between the curves for which $\pi = 10$, which both pass through the point (0, 10) in $(q, money)$ space.



The $\pi_u = 10$ indfference curve then descends more slowly than the $\pi_v = 10$ curve because the commodity is not so valued by Buyer U.

Another comparison is between the two curves which contain the point (4,8), which are $\pi_u = 10$ and $\pi_v = 12$.

These two curves also cross only once, at that point.

If you pick any one indifference curve for Buyer U and any one for Buyer V, those curves will cross either not at all, or once.

*10.6 Rate-of-Return Regulation and Government Procurement

The central idea in both government procurement and regulation of natural monopolies is that the government is trying to induce a private firm to efficiently provide a good to the public while covering the cost of production.

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Suppose the government wants a firm to provide cable television service to a city.

The firm knows more about its costs before agreeing to accept the franchise (adverse selection), discovers more after accepting it and beginning operations (moral hazard with hidden knowledge), and exerts greater or smaller effort to keep costs low (moral hazard with hidden actions).

The government wants to be generous enough to induce the firm to accept the franchise in the first place but no more generous than necessary.

The government might auction off the right to provide the service,

might allow the firm a maximum price (a **price cap**),

or might agree to compensate the firm to varying degrees for different levels of cost (**rate-of- return regulation**).

The first version of the model will be one in which the government can observe the firm's type and so the first-best can be attained.

Procurement I: Full Information

Players

The government and the firm.

The Order of Play

0 Nature assigns the firm expensive problems with the project, which add costs of x, with probability θ. A firm is thus "normal", with type N and $s = 0$, or "expensive", with type X and $s = x$. The government and the firm both observe the type.

1 The government offers a contract $\{w(m) = c(m) + p(m), c(m)\}$ which pays the firm its observed cost c and a profit p if it announces its type to be m and incurs cost $c(m)$, and pays the firm zero otherwise.

2 The firm accepts or rejects the contract.

3 If the firm accepts, it chooses effort level e, unobserved by the government.

4 The firm finishes the missile at a cost of $c = \bar{c} + s - e$, which is observed by the government, plus an additional unobserved cost[1] of $f(e - \bar{c})$. The government reimburses $c(m)$ and pays $p(m)$.

[1]The reader may ask why this disutility is specified as $f(e - \bar{c})$ rather than just $f(e)$. The reason is that we will later find an equilibrium cost level of $(\bar{c} - e^*)$, which would be negative if $c_0 = 0$.

Payoffs
Both firm and government are risk-neutral and both receive payoffs of zero if the firm rejects the contract. If the firm accepts, its payoff is

$$\pi_{firm} = p - f(e - \bar{c}) \tag{34}$$
$$\tag{35}$$

where $f(e-\bar{c})$, the cost of effort, is increasing and convex, so $f' > 0$ and $f'' > 0$. Assume for technical convenience that f is increasingly convex, so $f''' > 0$.

The government's payoff is

$$\pi_{government} = B - (1+t)c - tp - f, \tag{36}$$

where B is the benefit of the missile and t is the deadweight loss from the taxation needed for government spending. This is substantial. Hausman & Poterba (1987) estimate the loss to be around \$0.30 for each \$1 of tax revenue raised at the margin for the United States.

In Procurement I, whether the firm has expensive problems is observed by the government, which can therefore specify a contract conditioned on the type of the firm.

The government pays p_N to a normal firm with the cost c_N, p_X to an expensive firm with the cost c_X, and $p = 0$ to a firm that does not achieve its appropriate cost level

The expensive firm exerts effort $e = \bar{c} + x - c_X$, achieves $c = c_X$, generating unobserved effort disutility $f(e - \bar{c}) = f(x - c_X)$, so its participation constraint, that type X's payoff from reporting that it is type X, is:

$$\begin{aligned} \pi_X(X) &\geq 0 \\ p_X - f(x - c_X) &\geq 0. \end{aligned} \tag{37}$$

Similarly, in equilibrium the normal firm exerts effort $e = \bar{c} - c_N$, so its participation constraint is

$$\begin{aligned} \pi_N(N) &\geq 0 \\ p_N - f(-c_N) &\geq 0 \end{aligned} \tag{38}$$

The incentive compatibility constraints are trivial here: the government can use a forcing contract that pays a firm zero if it generates the wrong cost for its type, since types are observable.

To make a firm's payoff zero and reduce the deadweight loss from taxation, the government will provide prices that do no more than equal the firm's disutility of effort. Since there is no uncertainty, we can invert the cost equation and write it as $e = \bar{c} + x - c$ or $e = \bar{c} - c$. The prices will be $p_X = f(e - \bar{c}) = f(x - c_X)$ and $p_N = f(e - \bar{c}) = f(-c_N)$.

Suppose the government knows the firm has expensive problems. Substituting the price p_X into the government's payoff function, equation (36), yields

$$\pi_{government} = B - (1+t)c_X - tf(x - c_X) - f(x - c_X). \tag{39}$$

Since $f'' > 0$, the government's payoff function is concave, and standard optimization techniques can be used. The first-order condition for c_X is

$$\frac{\partial \pi_{government}}{\partial c_X} = -(1+t) + (1+t)f'(x - c_X) = 0, \tag{40}$$

so

$$f'(x - c_X) = 1. \tag{41}$$

Equation(41) is the crucial efficiency condition for effort. Since the argument of f is $(e - \bar{c})$, whenever $f' = 1$ the effort level is efficient. At the optimal effort level, the marginal disutility of effort equals the marginal reduction in cost because of effort. This is the first-best efficient effort level, which we will denote by $e^* \equiv e : \{f'(e - \bar{c}) = 1\}$.

If we derived the first-order condition for the normal firm we would find $f'(-c_N) = 1$ in the same way, so $c_N = c_X - x$. Also, if the equilibrium disutility of effort is the same for both firms, then both must choose the same effort, e^*, though the normal firm can reach a lower cost target with that effort. The cost targets assigned to each firm are $c_X = \bar{c} + x - e^*$ and $c_N = \bar{c} - e^*$. Since both types must exert the same effort, e^*, to achieve their different targets, $p_X = f(e^* - \bar{c}) = p_N$. The two firms exert the same efficient effort level and are paid the same price to compensate for the disutility of effort. Let us call this price level p^*.

The assumption that B is sufficiently large can now be made more specific: it is that $B - (1+t)c_X - tf(e^* - \bar{c}) - f(e^* - \bar{c}) \geq 0$, which requires that $B - (1+t)(\bar{c} + x - e^*) - (1+t)p^* \geq 0$. If that were not true, then the government would not want to build the missile at all if the firm had an expensive cost function, as we will not treat of here.