G604 Midterm, 28 Feb. 2006

This is a close-book test, except that you may use one double-sided page of notes. Answer each question as best you can. If you get lost in solving equations, write down in words what you are trying to do and what you think would come out of the mathematical analysis.(40 points total)

I told you not to do problem 2, since I decided the test was too long for 75 minutes. I added 7 points to each test score to account for problem 2.

The scores were: 30-40: 5 (A-, A), 20-29 (B-,B, B+): 4, less than 20 (C, C+) 2.

1. Attached are some accounting statements from Cinergy Corp.

(a) (2 points) What is the company's return on equity and return on assets? You will receive credit even if your arithmetic is very approximate, but show your work.

Answer. The return on equity is net income over equity, which is 400.868 million divided by 4115.922 million dollars, or 9.7%.

The return on assets is net income over assets, which is 400.868 million divided by 14982.317 million dollars, or 2.7%.

(b) (2 points) "Depreciation" shows up in the two places I have boxed. What is depreciation, and why does it show up twice?

Answer. Depreciation is how much an asset's value has diminished over time.

It shows up on the income statement because the decline in value is treated as a cost, a flow each year.

It shows up on the balance sheet as "accumulated depreciation" because to see the net value of the plant and equipment one starts with the purchase cost and then subtracts the amount the plant and equipment has depreciated up to this year.

2. (Dropped from the test, because the test was too long.) In the game of Table 1, only Jane observes the value of x, which is either 1 or 5, with equal probability. Before Jane and Mary simultaneously choose $Museum$ or $Carnival$, Jane says *Renoir* or *Popcorn*. Her statement has no direct effect on either player's payoff.

Table 1: Subgame Payoffs

		Mary	
		$Museum$	$Carnival$
Jane:	$Museum$	x, x	$0, 0$
	$Carnival$	$0, 0$	2,2

Payoffs to: (Jane, Mary).

(a) (4 points) Describe a perfect bayesian equilibrium in which Jane sometimes strictly prefers to say *Renoir* and sometimes strictly prefers *Popcorn*. What happens in that equilibrium if she means to say *Renoir* but says *Popcorn* by mistake?

Answer. Jane says popcorn if $x = 1$ and Renoir if $x = 5$. If she says popcorn she and Mary both go to the carnival. If she says Renoir both go to the museum. No out-of-equilibrium beliefs are needed. If Jane says popcorn by mistake, she goes to the carnival.

Table 1: Subgame Payoffs

		Mary	
		Museum	*Carnival*
Jane:	*Museum*	x, x	$0, 0$
	Carnival	$0, 0$	2,2

Payoffs to: (Jane, Mary).

(b) (3 points) Describe a pareto-dominated perfect bayesian equilibrium in which Jane always says *Renoir*, but is indifferent about her statement.

Answer. Jane says *Renoir* for any value of x. She and Mary both go to the carnival regardless of what she says. The payoff is 2, compared to an expected payoff of $.5(1) + .5(5) = 3$ from going to the museum.

3. Lindenberg and Ross (1981) ran a regression of Tobin's q on the Lerner Index for a firm and on the four-firm concentration ratio. The first regression is across both years and firms, and the second and third are just for the year 1972. The last column is the R^2.

Table 5 **Regression Results**

Regressions	Indices	r^2
$\bar{q}_{it} = 1.03 + 3.10\, L_{it}$ (19.59)	$i = 1, \ldots, 246$ $t = 1960, \ldots, 1977$	.08
$\bar{q}_{i,1972} = 1.46 + .27\, CR_{i,1972}$ (0.63)	i indexes manufacturing-sector firms	.01
$\bar{q}_{i,1972} = .19 + 8.28\, L_{i,1972} + .04\, CR_{i,1972}$ (11.27) (0.11)	i indexes manufacturing-sector firms	.29

NOTE.—Numbers in parentheses are t-values.

(a) (4 points) We would expect Tobin's q and the Lerner Index both to be correlated with a firm's profitability. Explain why.

(b) (3 points) What do you conclude from these regressions? Comment on the coefficient size and the R^2 as well as the t-statistics. For this question, you may assume that q is a good measure of profitability.

Answer. t-statistics: L, not concentration, matters.

The R^2 says that the Lerner Index explains a lot more of q in cross-section data than in time-series data, though there is much variance in q unexplained in either case. One reason would be that transitory changes in the Lerner Index across time (because of recessions, for example) might not have as big an effect on q as industry or firm effects, because the value of q depends on all future profits, not just one year's profits.

A low value of R^2 says that omitted variables *uncorrelated with the explanatory variables* explain the left-hand-side variable.

If there are omitted variables correlated with one of the included, right-hand-side, explanatory, variables, then those included variables will mistakenly pick up the effect of the omitted variables, and the R^2 will actually be high, deceptively.

The coefficient size raises a puzzle. Would a Lerner Index of 0 really lead to a q of .19, saying the firm was highly unprofitable? That is what the coefficient value says. No— we would expect a q of 1 then. Also, a coefficient of 8.28 on L seems unreasonably high. Would going from a Lerner Index of 1.1 to 1.2 raise q by .828? Something is wrong in this regression. I wonder if there might be a typo or a coding error.

Be sure you understand the difference between the following three econometric problems: Multicollinearity, Simultaneity, and Serial Correlation.

(c) (2 points) Other regressions have shown concentration related positively to profitability. How could that be reconciled with the results above?

Answer. q might not be a good measure of profitability, especially current profitability. Recall that q is bigger if market value is bigger, which is largely due to expectations of future profitability, not current profitability.

Or, growing industries might have big values for q , but be less concentrated than old industries. Dying industries, in particular, would have low values of q, but might have high values of concentration, as the weaker firms exit first.

Or, the measure of concentration used here, the concentration ratio, might not be very good.

Many profitability measures (e.g., return on equity) have the defect that they do not take risk into account. q actually does, though. A firm with high but variable profits could still have a low q, because its stock, so risky, wouldn't be highly valued by the market.

(d) (2 points) Lindenberg and Ross note that one problem with their regression is that public utilities (e.g., electric companies) have low values for q and high values for L. Can either of these things be attributed to the heavy capital investments that public utilities must make?

Answer. Public utilities have great economies of scale. They produce with a high fixed cost and a low marginal cost. Thus, it is natural for them to have price above marginal cost, which makes the Lerner Index high.

Something that would make q smaller is that public utilities have regulated prices. If regulators do not allow them to have high enough prices, then even if the prices are above marginal cost, it could happen that the price is not high enough to recover the company's fixed costs.

Note that the "market value" in q is not the price at which a company could sell off its assets to be used by some other company, though that could play a part in the market value. The market value is the value of the assets in their best use, which is almost always their current use. Thus, the market value of an electrical generating plant is the value of the profits it creates, not its value as scrap.

4. Consider the "sad loser" sealed-bid auction. The highest bidder wins the object and pays a price equal to his bid. The losers each pay twice their bids. Each player's value v is independently drawn from the same atomless, strictly increasing distribution $F(v)$ on support [0,1] and is known only to himself. You may restrict yourself to considering what happens in a symmetric equilibrium.

(a) (2 points) What is the payoff function for a bidder with value v who bids as if he were a player with value z?

Answer. The equilibrium payoff function for a bidder with value v who pretends he has value z is

$$\pi(v,z) = F(z)^{n-1}(v + p(z)) - 2p(z), \tag{1}$$

or, equivalently,

$$\pi(v,z) = F(z)^{n-1}v - F(z)^{n-1}p(z) - [1 - F(z)^{n-1}][2p(z)]. \tag{2}$$

If our bidder bids $p(z)$, that is the highest bid only if all $(n-1)$ other bidders have $v < z$, a probability of $F(z)$ for each of them. As for the amount he pays, we can think of it as $2p(z)$ with a probability $F(z)^{n-1}$ of getting $p(z)$ refunded because he wins.

(b) (3 points) What is the equilibrium bidding function $p(v)$?

The equilibrium payoff function for a bidder with value v who pretends he has value z is

$$\pi(v, z) = F(z)^{n-1}(v + p(z)) - 2p(z),$$

Answer. We need to find z such that

$$\frac{\partial \pi(v,z)}{\partial z} = (n-1)F(z)^{n-2}f(z)(v+p(z))+F(z)^{n-1}p'(z)-2p'(z) = 0 \quad (3)$$

In equilibrium, our bidder does follow the strategy $p(v)$, so $z = v$ and we can write

$$(n-1)F(v)^{n-2}f(v)(v + p(v)) + F(v)^{n-1}p'(v) - 2p'(v) = 0 \quad (4)$$

so

$$(n-1)F(v)^{n-2}f(v)v = [2-F(v)^{n-1}]p'(v)-(n-1)F(v)^{n-2}f(v)p(v). \quad (5)$$

Now comes the tricky part. We have both $p(v)$ and $p'(v)$ on the right-hand side of equation (5), and we'd like something just in terms of $p(v)$. Looking closely, that right-hand-side is the derivative of a simpler expression:

$$(n-1)F(v)^{n-2}f(v)v = \frac{d}{dv}[2 - F(v)^{n-1}]p(v) \quad (6)$$

We can integrate up on both sides, putting in zero for the constant of integration since $b(0) = 0$.

$$\int_0^v (n-1)F(x)^{n-2}f(x)x dx = [2 - F(v)^{n-1}]p(v) \quad (7)$$

Then it is easy to solve for $p(v)$.

$$p(v) = \frac{\int_0^v (n-1)F(x)^{n-2}f(x)x dx}{2 - F(v)^{n-1}}. \quad (8)$$

Here's another approach, a mechanism deisgn one, that leads to the same answer. Start with a version of the payoff function, which makes a bidder's payoff the difference between the expected benefit from winning the object,$F(z)^{n-1}v$ in a symmetric increasing equilibrium, and the expected payment, which we will denote by $m(z)$.

$$\pi(v,z) = F(z)^{n-1}v - m(z) \tag{9}$$

Differentiating yields

$$\frac{\partial \pi(v,z)}{\partial z} = (n-1)F(z)^{n-2}f(z)v + m'(z) = 0, \tag{10}$$

so, since $z = v$ in equilibrium,

$$m'(v) = (n-1)F(v)^{n-2}f(v)v \tag{11}$$

We can integrate this up to get

$$m(v) = m(0) + \int_0^v (n-1)F(x)^{n-2}f(x)x dx \tag{12}$$

We know that $m(0) = 0$ since if $v = 0$ the bidder won't bid anything and won't pay anything.

If this were the all-pay auction, we'd stop here, since the expected payment equals the bid function ($m(v) = p(v)$) so we'd have

$$p(v) = \int_0^v (n-1)F(x)^{n-2}f(x)x dx \tag{13}$$

But it's not, so we use what we know about the sad loser auction, which is that

$$m(v) = F(v)^{n-1}p(v) + [1 - F(v)^{n-1}][2p(v)] = [2 - F(v)^{n-1}]p(v). \tag{14}$$

Solving for $p(v)$ in (14) and substituting for $m(v)$ from (12), with $m(0) = 0$, we get

$$p(v) = \frac{m(v)}{2 - F(v)^{n-1}} = \frac{\int_0^v (n-1)F(x)^{n-2}f(x)x dx}{2 - F(v)^{n-1}}. \tag{15}$$

(c) (2 points) Is bidding higher than in the conventional all-pay auction, or lower? Why? (Do not confuse this with part (d)).

Answer. Lower. All players but the winner must pay more, so bidders with low values will clearly be more cautious and bid less. Bidders with high values, though, do not pay any less under this auction rule if they win than they would in the conventional all-pay auction, so they have no reason to bid higher either.

Equations (13) and (15) above confirm this reasoning (but were not necessary for you to get full credit). The bids in the sad loser auction are about half those in the all-pay auction, if n is large so $F(v)^{n-1}$ is small. (They would be exactly half if this was a "pay double your bid" auction.)

(d) (2 points) How does revenue from this auction compare with revenue in the ascending auction?

Answer. It is the same, by the Revenue Equivalence Theorem. The Theorem applies here, because a bidder with a value of 0 will have a zero payoff and the winner will be the high valuer.

5. An employer believes that a market is strong with probability α and weak with probability $1 - \alpha$. His agent will know whether it is weak or strong once he accepts the employer's contract and starts doing market research. The employer's revenue is 20 from high sales and 6 from low sales. The agent's disutility of the effort needed to get high sales is 5 for high sales in a strong market, 2 for low sales in a strong market, 18 for high sales in a weak market, and 2 for low sales in a weak market. The agent's reservation utility is zero and his utility is linear in his wage.

(a) (3 points) What payoff function does the employer maximize to find a direct mechanism in a truthtelling equilibrium with a separating contract? Denote the wage by $w(m)$.

Answer. The employer wants high sales in a strong market and low sales in a weak market. To induce the agent to produce high sales in a weak market would require the wage to be too high. It costs the agent 18 in utility to generate the high 20 sales of a weak market, and just 2 in utility to generate the low 6 in sales, so the employer will prefer to have him sell only 6. In a strong market, on the other hand, the surplus from high sales is 20-5 =15 and from low sales it is only 6-2=4.

Thus,

$$Payoff(employer) = \alpha(20 - w(high)) + (1-\alpha)(6 - w(low)) \quad (16)$$

(b) (3 points) Show the participation and incentive compatibility constraints that must be satisfied for a direct mechanism in a truthtelling equilibrium with a separating contract.

Answer. There is one participation constraint:

$$U(accept) = \alpha U(strong) + (1-\alpha)U(weak) \tag{17}$$

$$U(accept) = \alpha(w(strong) - 5) + (1-\alpha)(w(low) - 2) \geq 0.$$

and two incentive compatibility constraints:

$$w(high) - 5 \geq w(low) - 2 \tag{18}$$

and

$$w(low) - 2 \geq w(high) - 18 \tag{19}$$

(c) (3 points) Which constraints are binding in a separating contract? Why?

Answer. The participation constraint is binding. That is because there is no reason for the employer to leave the worker with any surplus. If the worker had ex ante expected surplus, then the employer could just reduce both wages by a constant amount big enough to reduce the worker's payoff to zero.

The incentive compatibility constraint for the strong agent is binding, because the employer must add enough extra wage to make the strong contract at least as attractive for the strong agent as the weak contract, or he will pick the weak contract. $w(high) - 5 = w(low) - 2$, which can be rewritten as $w(high) - w(low) = 7$.

The weak agent, on the other hand, is under no temptation to choose the strong, high-sales contract, because it is harder for him to generate extra sales. If the strong agent is indifferent, the weak agent will strictly prefer the weak contract.

I didn't ask you to actually find the equilibrium contract, but I'll discuss that here. We need to separate the strong and weak wages to satisfy the strong incentive compatibility constraint, so $w(high) - w(low) = 7$. We need to pick k in $w(high) = 7 + k$ and $w(low) = 0 + k$ to satisfy the participation constraint,

$$\alpha(w(high) - 5) + (1 - \alpha)(w(low) - 2) = 0,$$

so

$$\alpha(7 + k - 5) + (1 - \alpha)(k - 2) = 0, . \tag{20}$$

and

$$2\alpha + k\alpha + k - 2 - \alpha k + 2\alpha = 0, \tag{21}$$

and $k = 2 - 4\alpha$. (Note that k is negative if $\alpha > .5$.)