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*15 Entry

*15.1 Innovation and Patent Races

How do firms come to enter particular industries? Of the many potential products that might be produced, firms choose a small number, and each product is only produced by a few firms. Most potential firms choose to remain potential, not actual. Information and strategic behavior are especially important in borderline industries in which only one or two firms are active in production.

This chapter begins with a discussion of innovation with the complications of imitation by other firms and patent protection by the government. Section 15.2 looks at a different way to enter a market: by purchasing an existing firm, something that also provides help against moral hazard on the part of company executives. Section 15.3 analyzes a more traditional form of entry deterrence, predatory pricing, using a Gang of Four model of a repeated game under incomplete information. Section 15.4 returns to a simpler model of predatory pricing, but shows how the ability of the incumbent to credibly engage in a price war can actually backfire by inducing entry for buyout.

Market Power as a Precursor of Innovation

Market power is not always inimical to social welfare. Although restrictive monopoly output is inefficient, the profits it generates encourage innovation, an important source of both additional market power and economic growth. The importance of innovation, however, is diminished because of imitation, which can so severely diminish its rewards as to entirely prevent it. An innovator generally incurs some research cost, but a discovery instantly imitated can yield zero net revenues. Table 15.1 shows how the payoffs look if the firm that innovates incurs a cost of 1 but imitation is costless and results in Bertrand competition. Innovation is a dominated strategy.

Table 15.1 Imitation with Bertrand pricing

		Brydox	
		Innovate	*Imitate*
Apex	*Innovate*	-1,-1	$-1, 0$
	Imitate	$0, -1$	**0,0**

Payoffs to: (Apex, Brydox)

Under different assumptions, innovation occurs even with costless imitation. The key is whether duopoly profits are high enough for one firm to recoup the entire costs of innovation. If they are, the payoffs are as shown in table 15.2, a version of Chicken. Although the firm that innovates pays the entire cost and keeps only half the benefit, imitation is not dominant. Apex imitates if Brydox innovates, but not if Brydox imitates. If Apex could move first, it would bind itself not to innovate, perhaps by disbanding its research laboratory.

Table 15.2 Imitation with profits in the product market

		Brydox	
		Innovate	*Imitate*
Apex	*Innovate*	1,1	**1,2**
	Imitate	**2,1**	0,0

Payoffs to: (Apex, Brydox)

Without a first-mover advantage, the game has two pure strategy Nash equilibria, (*Innovate*, *Imitate*) and (*Imitate*, *Innovate*), and a symmetric equilibrium in mixed strategies in which each firm innovates with probability 0.5. The mixed-strategy equilibrium is inefficient, since sometimes both firms innovate and sometimes neither.

History might provide a focal point or explain why one player moves first. Japan was for many years incapable of doing basic scientific research, and does relatively little even today. The United States therefore had to innovate rather than imitate in the past, and today continues to do much more basic research.

Much of the literature on innovation compares the relative merits of monopoly and competition. One reason a monopoly might innovate more is because it can capture more of the benefits, capturing the entire benefit if perfect price discrimination is possible (otherwise, some of the benefit goes to consumers). In addition, the monopoly avoids a second inefficiency: entrants innovating solely to steal the old innovator's rents without much increasing consumer surplus. The welfare aspects of innovation theory – indeed, all aspects – are intricate, and the interested reader is referred to the surveys by Kamien & Schwartz (1982) and Reinganum (1989).

Patent Races

One way that governments respond to imitation is by issuing patents: exclusive rights to make, use, or sell an innovation. If a firm patents its discovery, other firms cannot imitate, or even use the discovery if the make it independently. Research effort therefore has a discontinuous payoff: if the researcher is the first to make a discovery, he receives the patent; if he is second, nothing. Patent races are examples of the tournaments discussed in section 8.2 except that if no player exerts any effort, none of them will get the reward. Patents are also special because they lose their value if consumers find a substitute and stop buying the patented product. Moreover, the effort in tournaments is usually exerted over a fixed time period, whereas research usually has an endogenous time period, ending when the discovery is made. Because of this endogeneity, we call the competition a **patent race.**

We will consider two models of patents. On the technical side, the first model shows how to derive a continuous mixed strategies probability distribution, instead of just the single number derived in chapter 3. On the substantive side, it shows how patent races lead to inefficiency.

Patent Race for a New Market

Players
Three identical firms, Apex, Brydox, and Central.

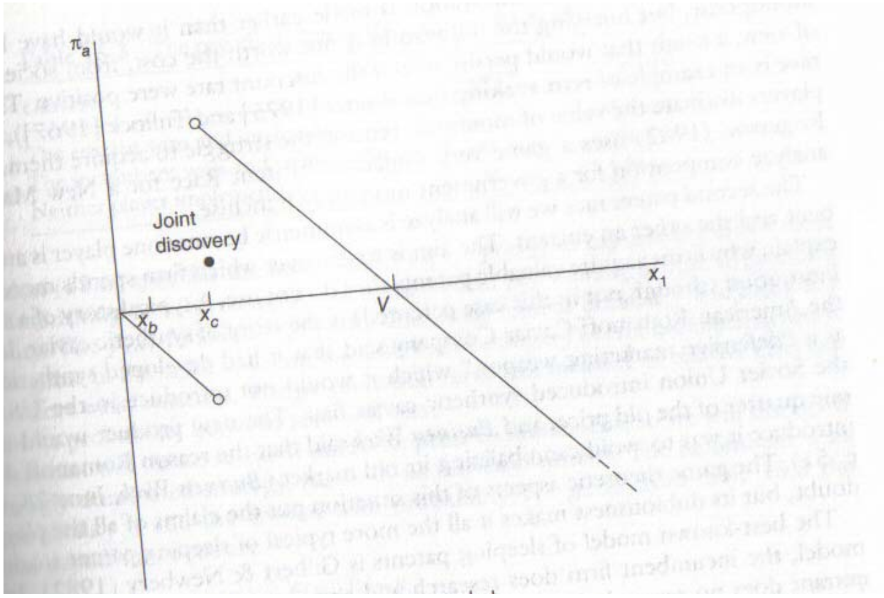
The Order of Play
Each firm simultaneously chooses research spending $x_i \geq 0$, $(i = a, b, c)$.

Payoffs
Firms are risk neutral and the discount rate is zero. Innovation occurs at time $T(x_i)$ where $T' < 0$. The value of the patent is V, and if several players innovate simultaneously they share its value.

$$\pi_i = \begin{cases} V - x_i & \text{if } T(x_i) < T(x_j), \ \ (\forall j \neq i) \quad \text{(Firm } i \text{ gets the patent)} \\ \frac{V}{1+m} - x_i & \text{if } T(x_i) = T(x_k), \quad \text{(Firm } i \text{ shares the patent with } m = 1 \text{ or } 2 \text{ other firms)} \\ -x_i & \text{if } T(x_i) > T(x_j) \text{ for some } j \quad \text{(Firm } i \text{ does not get the patent)} \end{cases}$$

The game does not have any pure strategy Nash equilibria, because the payoff functions are discontinuous. A slight difference in research by one player can make a big difference in the payoffs, as shown in figure 15.1 on the next page for fixed values of x_b and x_c. The research levels shown in figure 15.1 are not equilibrium values. If Apex chose any research level x_a less than V, Brydox would respond with $x_a + \varepsilon$ and win the patent. If Apex chose $x_a = V$, then Brydox and Central would respond with $x_b = 0$ and $x_c = 0$, which would make Apex want to switch to $x_a = \varepsilon$.

Figure 15.1 The payoffs in Patent Race for a New Market



There does exist a symmetric mixed strategy equilibrium. We will derive $M_i(x)$, the cumulative density function for the equilibrium mixed strategy, rather than the density function itself. The probability with which firm i chooses a research level less than or equal to x will be $M_i(x)$. In a mixed-strategy equilibrium a player is indifferent between any of the pure strategies among which he is mixing. Since we know that the pure strategies $x_a = 0$ and $x_a = V$ yield zero payoffs, if Apex mixes over the support $[0, V]$ then the expected payoff for every strategy mixed between must also equal zero. The expected payoff from the pure strategy x_a is the expected value of winning minus the cost of research. Letting x stand for nonrandom and X for random variables, this is

$$V \cdot Pr(x_a \geq X_b, x_a \geq X_c) - x_a = 0, \tag{1}$$

which can be rewritten as

$$V \cdot Pr(X_b \leq x_a) Pr(X_c \leq x_a) - x_a = 0, \tag{2}$$

or

$$V \cdot M_b(x_a) M_c(x_a) - x_a = 0. \tag{3}$$

We can rearrange equation (15.3) to obtain

$$M_b(x_a) M_c(x_a) = \frac{x_a}{V}. \tag{4}$$

If all three firms choose the same mixing distribution M, then

$$M(x) = \left(\frac{x}{V}\right)^{1/2} \text{ for } 0 \leq x \leq V. \tag{5}$$

What is noteworthy about a patent race is not the nonexistence of a pure strategy equilibrium but the overexpenditure on research. All three players have expected payoffs of zero, because the patent value V is completely dissipated in the race. As in Brecht's

Threepenny Opera, "When all race after happiness/Happiness comes in last."[1] To be sure, the innovation is made earlier than it would have been by a monopolist, but hurrying the innovation is not worth the cost, from society's point of view, a result that would persist even if the discount rate were positive. The patent race is an example of **rent seeking** (see Posner [1975] and Tullock [1967]), in which players dissipate the value of monopoly rents in the struggle to acquire them. Indeed, Rogerson (1982) uses a game very similar to "Patent Race for a New Market" to analyze competition for a government monopoly franchise.

The second patent race we will analyze is asymmetric because one player is an incumbent and the other an entrant. The aim is to discover which firm spends more and to explain why firms acquire valuable patents they do not use. A typical story of a sleeping innovation (though not in this case patented) is the story of synthetic caviar. In 1976, the American Romanoff Caviar Company said that it had developed synthetic caviar as a "defensive marketing weapon" which it would not introduce in the US unless the Soviet Union introduced synthetic caviar first. The new product would sell for one quarter of the old price, and *Business Week* said that the reason Romanoff did not introduce it was to avoid cannibalizing its old market (*Business Week*, June 28, 1976, p. 51). The game theoretic aspects of this situation put the claims of all the players in doubt, but its dubiousness makes it all the more typical of sleeping patent stories.

The best-known model of sleeping patents is Gilbert & Newbery (1982). In that model, the incumbent firm does research and acquires a sleeping patent, while the entrant does no research. We will look at a slightly more complicated model which does not reach such an extreme result.

Patent Race for an Old Market

Players
An incumbent and an entrant.

The Order of Play
1 The firms simultaneously choose research spending x_i and x_e, which result in research achievements $f(x_i)$ and $f(x_e)$, where $f' > 0$ and $f'' < 0$.
2 Nature chooses which player wins the patent using a function g that maps the difference in research achievements to a probability between zero and one.

$$Prob(\text{incumbent wins patent}) = g[f(x_i) - f(x_e)], \qquad (6)$$

where $g' > 0$, $g(0) = 0.5$, and $0 \leq g \leq 1$.
3 The winner of the patent decides whether to spend Z to implement it.

Payoffs
The old patent yields revenue y and the new patent yields v. The payoffs are shown in table 15.3.

[1] Act III, scene 7 of the *Threepenny Opera*, translated by John Willett (Berthold Brecht, *Collected Works*, London: Eyre Methuen (1987).

Table 15.3 The payoffs in Patent Race for an Old Market

Outcome	$\pi_{incumbent}$	$\pi_{entrant}$
The entrant wins and implements	$-x_i$	$v - x_e - Z$
The incumbent wins and implements	$v - x_i - Z$	$-x_e$
Neither player implements	$y - x_i$	$-x_e$

Equation (15.6) specifies the function $g[f(x_i) - f(x_e)]$ to capture the three ideas of (a) diminishing returns to inputs, (b) rivalry, and (c) winning a patent race as a probability. The $f(x)$ function represents dimishing returns because f increases at a decreasing rate in the input x. Using the difference between $f(x)$ for each firm makes it relative effort which matters. The $g(\cdot)$ function turns this measure of relative effective input into a probability between zero and one.

The entrant will do no research unless he plans to implement, so we will disregard the strongly dominated strategy, ($x_e > 0$, *no implementation*). The incumbent wins with probability g and the entrant with probability $1 - g$, so from table 15.3 the expected payoff functions are

$$\pi_{incumbent} = (1 - g[f(x_i) - f(x_e)])(-x_i) + g[f(x_i) - f(x_e)]Max\{v - x_i - Z, y - x_i\} \quad (7)$$

and

$$\pi_{entrant} = (1 - g[f(x_i) - f(x_e)])(v - x_e - Z) + g[f(x_i) - f(x_e)](-x_e). \quad (8)$$

On differentiating and letting f_i and f_e denote $f(x_i)$ and $f(x_e)$ we obtain the first order conditions

$$\frac{d\pi_i}{dx_i} = -(1 - g[f_i - f_e]) - g' f_i'(-x_i) + g' f_i' Max\{v - x_i - Z, y - x_i\} - g[f_i - f_e] = 0 \quad (9)$$

and

$$\frac{d\pi_e}{dx_e} = -(1 - g[f_i - f_e]) + g' f_e'(v - x_e - Z) - g[f_i - f_e] + g' f_e' x_e = 0. \quad (10)$$

Equating (15.9) and (15.10), which both equal zero, we obtain

$$-(1-g) - g' f_i' x_i + g' f_i' Max\{v - x_i - Z, y - x_i\} - g = -(1-g) + g' f_e'(v - x_e - Z) - g + g' f_e' x_e, \quad (11)$$

which simplifies to

$$f_i'[x_i + Max\{v - x_i - Z, y - x_i\}] = f_e'[v - x_e - Z + x_e], \quad (12)$$

or

$$\frac{f_i'}{f_e'} = \frac{v - Z}{Max\{v - Z, y\}}. \quad (13)$$

We can use equation (15.13) to show that different parameters generate two qualitatively different outcomes.

Outcome 1. *The entrant and incumbent spend equal amounts, and each implements if successful.*
This happens if there is a big gain from patent implementation, that is, if

$$v - Z \geq y, \tag{14}$$

so that equation (15.13) becomes

$$\frac{f_i'}{f_e'} = \frac{v - Z}{v - Z} = 1, \tag{15}$$

which implies that $x_i = x_e$.

Outcome 2. *The incumbent spends more and does not implement if he is successful (he acquires a sleeping patent).*
This happens if the gain from implementation is small, that is, if

$$v - Z < y, \tag{16}$$

so that equation (15.13) becomes

$$\frac{f_i'}{f_e'} = \frac{v - Z}{y} < 1, \tag{17}$$

which implies that $f_i' < f_e'$. Since we assumed that $f'' < 0$, f' is decreasing in x, and it follows that $x_i > x_e$.

This model shows that the presence of another player can stimulate the incumbent to do research he otherwise would not, and that he may or may not implement the discovery. The incumbent has at least as much incentive for research as the entrant because a large part of a successful entrant's payoff comes at the incumbent's expense. The benefit to the incumbent is the maximum of the benefit from implementing and the benefit from stopping the entrant, but the entrant's benefit can only come from implementing. Contrary to the popular belief that sleeping patents are bad, here they can help society by eliminating wasteful implementation.

*15.2 Takeovers and Greenmail

The Free Rider Problem

Game theory is well suited to modelling takeovers because the takeover process depends crucially on information and includes a number of sharply delineated actions and events. Suppose that under its current mismanagement, a firm has a value per share of v, but no shareholder has enough shares to justify the expense of a proxy fight to throw out the current managers, although doing so would raise the value to $(v + x)$. An outside bidder makes a tender offer conditional upon obtaining a majority. Any bid p between v and $(v+x)$ can make both the bidder and the shareholders better off. But do the shareholders accept such an offer?

We will see that they do not. Quite simply, the only reason the bidder makes a tender offer is that the value would rise higher than his bid, so no shareholder should accept his bid.

The Free Rider Problem in Takeovers
(Grossman & Hart [1980])

Players
A bidder and a continuum of shareholders, with amount m of shares.

The Order of Play
1 The bidder offers p per share for the m shares.
2 Each shareholder decides whether to accept the bid (denote by θ the fraction that accept).
3 If $\theta \geq 0.5$, the bid price is paid out, and the value of the firm rises from v to $(v+x)$ per share.

Payoffs
If $\theta < 0.5$, the takeover fails, the bidder's payoff is zero, and the shareholder's payoff is v per share. Otherwise,

$$\pi_{bidder} = \begin{cases} \theta m(v+x-p) & \text{if } \theta \geq 0.5. \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_{shareholder} = \begin{cases} p & \text{if the shareholder accepts.} \\ v+x & \text{if the shareholder rejects.} \end{cases}$$

Bids above $(v+x)$ are dominated strategies, since the bidder could not possibly profit from them. But if the bid is any lower, an individual shareholder should hold out for the new value of $(v+x)$ rather than accepting p. To be sure, when they all do that, the offer fails and they end up with v, but no individual wants to accept if he thinks the offer will succeed. The only equilibria are the many strategy combinations that lead to a failed takeover, or a bid of $p = (v+x)$ accepted by a majority, which succeeds but yields a payoff of zero to the bidder. If organizing an offer has even the slightest cost, the bidder would not do it.

The free rider problem is clearest where there is a continuum of shareholders, so that the decision of any individual does not affect the success of the tender offer. If there were, instead, nine players with one share each, then in one asymmetric equilibrium five of them tender at a price just slightly above the old market price and four hold out. Each of the five tenderers knows that if he held out, the offer would fail and his payoff would be zero. This is an example of the discontinuity problem of section 8.6.

In practice, the free rider problem is not quite so severe even with a continuum of shareholders. If the bidder can quietly buy a sizeable number of shares without driving up the price (something severely restricted in the United States by the Williams Act), then his capital gains on those shares can make a takeover profitable even if he makes nothing from shares bought in the public offer. Dilution tactics such as freeze-out mergers also help the bidder (see Macey & McChesney [1985]). In a freeze-out, the bidder buys 51 percent of the shares and merges the new acquisition with another firm he owns, at a price below its full value. If dilution is strong enough, the shareholders are willing to sell at a price less than $v + x$.

Still another takeover tactic is the two-tier tender offer, a nice application of the Prisoner's Dilemma. Suppose the underlying value of the firm is 30, which is the initial stock price. A monopolistic bidder offers a price of 10 for 51 percent of the stock and 5 for the other 49 percent, conditional upon 51 percent tendering. It is then a dominant strategy to tender, even though all the shareholders would be better off refusing to sell.

Greenmail

Greenmail occurs when managers buy out some shareholders at an inflated stock price to stop them from taking over. Opponents of greenmail explain this using the Corrupt Managers model. Suppose that a little dilution is possible, or the bidder owns some shares to start with, so he can take over the firm but would lose most of the gains to the other shareholders. The managers are willing to pay the bidder a large amount of greenmail to keep their jobs, and both manager and bidder prefer greenmail to an actual takeover, despite the fact that the other shareholders are considerably worse off.

Managers often use what we might call the Noble Managers model to justify greenmail. In this model, current management knows the true value of the firm, which is greater than both the current stock price and the takeover bid. They pay greenmail to protect the shareholders from selling their mistakenly undervalued shares.

The Corrupt Managers model faces the objection that it fails to explain why the corporate charter does not prohibit greenmail. The Noble Managers model faces the objection that it implies either that shareholders are irrational or that stock prices rise after greenmail because shareholders know that the greenmail signal (giving up the benefits of a takeover) is more costly for a firm which really is not worth more than the takeover bid.

Shleifer & Vishny (1986) have constructed a more sophisticated model in which greenmail is in the interest of the shareholders. The idea is that greenmail encourages potential bidders to investigate the firm, eventually leading to a takeover at a higher price than the initial offer. Greenmail is costly, but for that very reason it is an effective signal that the manager thinks a better offer could come along later. (Like Noble Managers, this assumes that the manager acts in the interests of the shareholders.) I will present a numerical example in the spirit of Shleifer & Vishny rather than following them exactly, since their exposition is not directed towards the behavior of the stock price.

The story behind the model is that a manager has been approached by a bidder, and he must decide whether to pay him greenmail in the hopes that other bidders – "white knights" – will appear. The manager has better information than the market as a whole about the probability of other bidders appearing, and some other bidders can only appear after they undertake costly investigation, which they will not do if they think the takeover price will be bid up by competition with the first bidder. The manager pays greenmail to encourage new bidders by getting rid of their competition.

Greenmail to Attract White Knights

(Shleifer & Vishny [1986])

Players

The manager, the market, and bidder Brydox. (Bidders Raider and Apex do not make decisions.)

The Order of Play

Figure 15.2 shows the game tree. After each time t, the market picks a share price p_t.

0 Unobserved by any player, Nature picks the state to be (A), (B), (C), or (D), with probabilities 0.1, 0.3, 0.1, and 0.5, unobserved by any player.

1 Unless the state is (D), the Raider appears and offers a price of 15. The manager's information partition becomes $\{(A), (B,C), (D)\}$; everyone else's becomes $\{(A,B,C), (D)\}$.

2 The manager decides whether to pay greenmail and extinguish the Raider's offer at a cost of 5 per share.

3 If the state is (A), Apex appears and offers a price of 25 if greenmail was paid, and 30 otherwise.

4 If the state is (B), Brydox decides whether to buy information at a cost of 8 per share. If he does, then he can make an offer of 20 if the Raider has been paid greenmail, or 27 if he must compete with the Raider.

5 Shareholders accept the best offer outstanding, which is the final value of a share. If no offer is outstanding, the final value is 5 if greenmail was paid, 10 otherwise.

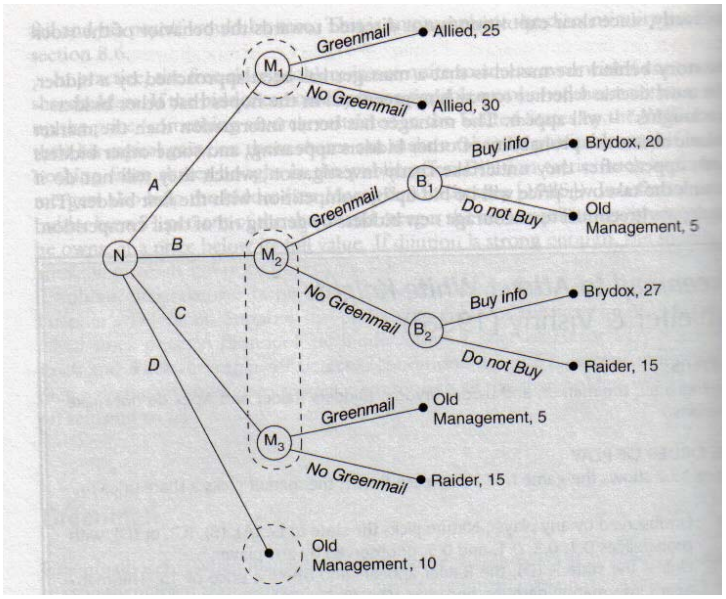
Payoffs

The manager maximizes the final value.

The market minimizes the absolute difference between p_t and the final value.

If he buys information, Brydox receives 23 $(= 31-8)$ minus the value of his offer; otherwise he receives zero.

Figure 15.2 The game tree for Greenmail to Attract White Knights



The payoffs specify that the manager should maximize the final value of the firm, rather than a weighted average of the prices p_0 through p_5. This assumption is reasonable because the only shareholders to benefit from a high value of p_t are those that sell their stock at t. The manager cannot say: "The stock is overvalued: Sell!", because the market would learn the overvaluation too, and refuse to buy.

The prices 15, 20, 27, and 30 are assumed to be the results of blackboxed bargaining games between the manager and the bidders. Assuming that the value of the firm to Brydox is 31 ensures that he will not buy information if he foresees that he would have to compete with the Raider. Since Brydox has a dominant strategy – buy information if the Raider has been paid greenmail and not otherwise – our focus will be on the market price and the decision of whether to pay greenmail. This model is also not designed to answer the question of why the Raider appears. His behavior is exogenous. As the model stands, his expected profit is positive since he is sometimes paid greenmail, but if he actually had to buy the firm he would regret it in states B and C, since the final value of the firm would be 10.

We will see that in equilibrium the manager pays greenmail in states (B) and (C), but not in (A) or (D). Table 15.4 shows the equilibrium path of the market price.

Table 15.4 The equilibrium price in Greenmail to Attract White Knights

State	Probability	p_0	p_1	p_2	p_3	p_4	p_5	Final management
(A)	0.1	14.5	19	30	30	30	30	Allied
(B)	0.3	14.5	19	16.25	16.25	20	20	Brydox
(C)	0.1	14.5	19	16.25	16.25	5	5	Old management
(D)	0.5	14.5	10	10	10	10	10	Old management

The market's optimal strategy amounts to estimating the final value. Before the market receives any information, its prior beliefs estimate the final value to be 14.5 (= 0.1[30] + 0.3[20]+ 0.1[5] + 0.5[10]). If state (D) is ruled out by the arrival of the Raider, the price rises to 19 (= 0.2[30] + 0.6[20]+ 0.2[5]). If the Raider does not appear, it becomes common knowledge that the state is (D), and the price falls to 10.

If the state is (A), the manager knows it and refuses to pay greenmail in expectation of Apex's offer of 30. Observing the lack of greenmail, the market deduces that the state is (A), and the price immediately rises to 30.

If the state is (B) or (C) the manager does pay greenmail and the market, ruling out (A), uses Bayes's Rule to assign probabilities of 0.75 to (B) and 0.25 to (C). The price falls from 19 to 16.25 (= 0.75[20] + 0.25[5]).

It is clear that the manager should not pay greenmail in states (A) or (D), when the manager knows that Brydox is not around to investigate. What if the manager deviates in the information set (B,C) and refuses to pay greenmail? The market would initially believe that the state was (A), so the price would rise to $p_2 = 30$. But the price would fall again after Apex failed to make an offer and the market realized that the manager had deviated. Brydox would refuse to enter at time 3, and the Raider's offer of 15 would be accepted. The payoff of 15 would be less than the expected payoff of 16.25 from paying greenmail.

The model does not say that greenmail is always good for the shareholders, only that it can be good *ex ante*. If the true state turns out to be (C), then greenmail was a mistake, *ex post*, but since state (B) is more likely, the manager is correct to pay greenmail in information set (B,C). What is noteworthy is that greenmail is optimal even though it drives down the stock price from 19 to 16.25. Greenmail communicates the bad news that Apex is not around, but makes the best of that misfortune by attracting Brydox.

*15.3 Predatory Pricing: The Kreps-Wilson Model

One traditional form of monopolization and entry deterrence is predatory pricing, in which the firm seeking to acquire the market charges a low price to drive out its rival. We have looked at predation already in chapters 4, 5 and 6 in the "Entry Deterrence" games. The major problem with entry deterrence under complete information is the chainstore paradox. The heart of the paradox is the sequential rationality problem faced by an incumbent who wishes to threaten a prospective entrant with low post-entry prices. The incumbent can respond to entry in two ways. He can collude with the entrant and share the profits, or he can fight by lowering his price so that both firms make losses. We have seen that the incumbent would not fight in a perfect equilibrium if the game has complete information. Foreseeing the incumbent's accommodation, the potential entrant ignores the threats.

In Kreps & Wilson (1982a), an application of the gang of four model of chapter 6, incomplete information allows the threat of predatory pricing to successfully deter entry. A monopolist with outlets in N towns faces an entrant who can enter each town. In our adaption of the model, we will start by assuming that the order in which the towns can be entered is common knowledge, and that if the entrant passes up his chance to enter a town, he cannot enter it later. The incomplete information takes the form of a small probability that the monopolist is "strong" and has nothing but *Fight* in his action set: he is an uncontrolled manager who gratifies his passions in squelching entry instead of maximizing profits.

Predatory Pricing
(Kreps & Wilson [1982a])

Players
The entrant and the monopolist.

The Order of Play
0 Nature chooses the monopolist to be *Strong* with low probability θ and $Weak$, with high probability $(1-\theta)$. Only the monopolist observes Nature's move.
1 The entrant chooses *Enter* or *Stay Out* for the first town.
2 The monopolist chooses *Collude* or *Fight* if he is weak, *Fight* if he is strong.
3 Steps (1) and (2) are repeated for towns 2 through N.

Payoffs
The discount rate is zero. Table 15.5 gives the payoffs per period, which are the same as in table 4.1.

Table 15.5 Predatory Pricing

		Weak incumbent	
		Collude	*Fight*
Entrant	*Enter*	**40,50**	$-10, 0$
	Stay out	$0, 100$	**0,100**

Payoffs to: (Entrant, Incumbent)

In describing the equilibrium, we will denote towns by names such as i_{30} and i_5, where the numbers are to be taken purely ordinally. The entrant has an opportunity to enter town i_{30} before i_5, but there are not necessarily 25 towns between them. The actual gap depends on θ but not N.

Part of the Equilibrium for Predatory Pricing

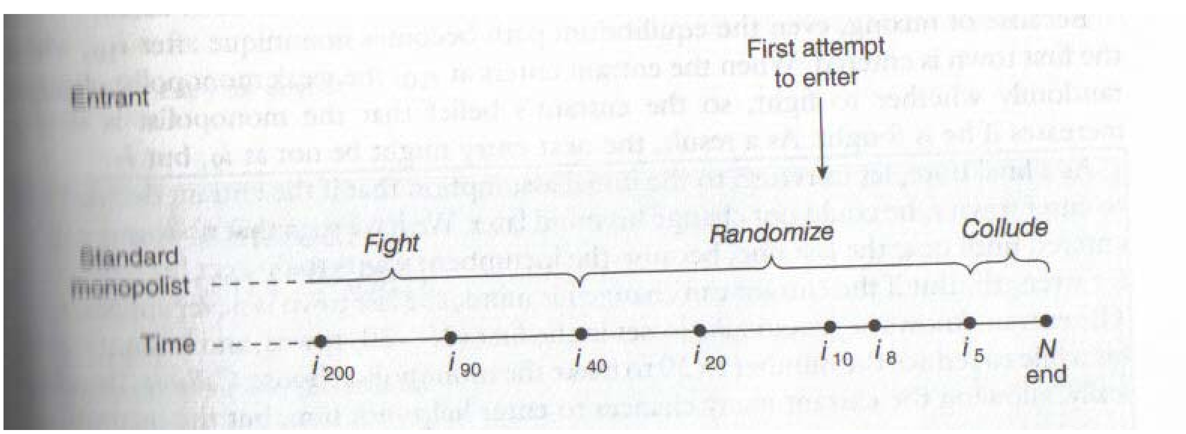
Entrant: Enter first at town i_{-10}. If entry has occurred before i_{10} and been answered with *Collude*, enter every town after the first one entered.

Strong monopolist: Always fight entry.

Weak monopolist: Fight any entry up to i_{30}. Fight the first entry after i_{-30} with a probability $m(i)$ that diminishes until it reaches zero at i_5. If *Collude* is ever chosen instead, always collude thereafter. If *Fight* was chosen in response to the first attempt at entry, increase the mixing probability $m(i)$ in subsequent towns.

This description, which is illustrated by figure 15.3, only covers the equilibrium path and small deviations. Note that out-of-equilibrium beliefs do not have to be specified (unlike in the original model of Kreps and Wilson), since whenever a monopolist colludes, in or out of equilibrium, Bayes's Rule says that the entrant must believe him to be *Weak*.

Figure 15.3 The equilibrium in Predatory Pricing



The entrant will certainly stay out until i_{30}. If no town is entered until i_5 and the monopolist is *Weak*, then entry at i_5 is undoubtedly profitable. But entry is attempted at i_{10}, because since $m(i)$ is diminishing in i, the weak monopolist probably would not fight even there.

Out of equilibrium, if an entrant were to enter at i_{90}, the weak monopolist would be willing to fight, to maintain i_{10} as the next town to be entered. If he did not, then the entrant, realizing that he could not possibly be facing a strong monopolist, would enter every subsequent town from i_{89} to i_1. If no town were entered until i_5, the weak monopolist would be unwilling to fight in that town, because too few towns are left to protect. If a town between i_{30} and i_5 has been entered and fought over, the monopolist raises the mixing probability that he fights in the next town entered, because he has a more valuable reputation to defend. By fighting in the first town he has increased the belief that he is strong and increased the gap until the next town is entered.

What if the entrant deviated and entered town i_{20}? The equilibrium calls for a mixed strategy response beginning with i_{30}, so the weak monopolist must be indifferent between fighting and not fighting. If he fights, he loses current revenue but the entrant's posterior belief that he is strong rises, rising more if the fight occurs late in the game. The entrant knows that in equilibrium the weak monopolist would fight with a probability of, say, 0.9 in town i_{20}, so fighting there would not much increase the belief that he was strong, but if he fought in town i_{13}, where the mixing probability has fallen to 0.2, the belief would rise much more. On the other hand, the gain from a given reputation diminishes as fewer towns remain to be protected, so the mixing probability falls over time.

The description of the equilibrium strategies is incomplete because describing what happens after unsuccessful entry becomes rather intricate. Even in the simultaneous-move games of chapter 3, we saw that games with mixed strategy equilibria have many different possible realizations. In repeated games like Predatory Pricing, the number of possible realizations makes an exact description very complicated indeed. If, for example, the entrant entered town i_{20} and the monopolist chose $Fight$, the entrant's belief that he was strong would rise, pushing the next town entered to i_{-8} instead of i_{10}. A complete description of the strategies would say what would happen for every possible history of the game, which is impractical at this book's level of detail.

Because of mixing, even the equilibrium path becomes nonunique after i_{10}, when the first town is entered. When the entrant enters at i_{10}, the weak monopolist chooses randomly whether to fight, so the entrant's belief that the monopolist is strong increases if he is fought. As a result, the next entry might be not at i_9, but i_7.

As a final note, let us return to the initial assumption that if the entrant decided not to enter town i, he could not change his mind later. We have seen that no towns will be entered until near the last one, because the incumbent wants to protect his reputation for strength. But if the entrant can change his mind, the last town is never approached. The entrant knows he would take losses in the first $(N - 30)$ towns, and it is not worth his while to reduce the number to 30 to make the monopolist choose *Collude*. Paradoxically, allowing the entrant many chances to enter helps not him, but the incumbent.

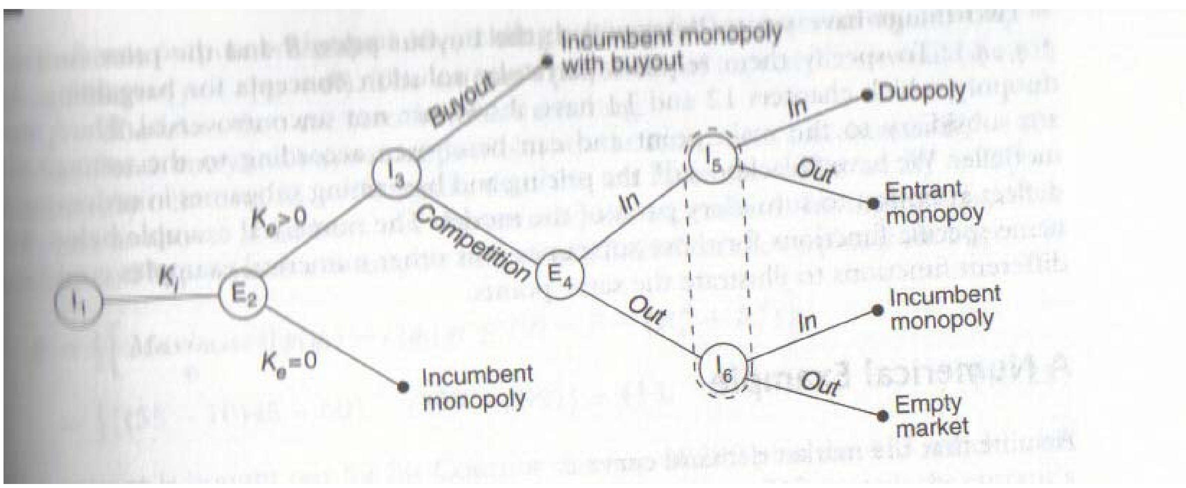
15.4 *Entry for Buyout

The previous section suggested that predatory pricing might actually be a credible threat if information were slightly incomplete, because the incumbent might be willing to makes losses fighting the first entrant to deter future entry. This is not the end of the story, however, because even if entry costs exceed operating revenues, entry might still be profitable if the entrant is bought out by the incumbent.

To see this most simply, let us start by thinking about how entry might be deterred under complete information. The incumbent needs some way to precommit himself to unprofitable post-entry pricing. Spence (1977) and Dixit (1980) suggest that the incumbent could enlarge his initial capacity to make the post-entry price naturally drop to below average cost. The post-entry price would still be above average variable cost, so having already sunk the capacity cost the incumbent fights entry without further expense. The entrant's capacity cost is not yet sunk, so he refrains from entry.

In the model with the extensive form of figure 15.4, the incumbent has the additional option of buying out the entrant. An incumbent who fights entry bears two costs: the loss from selling at a price below average total cost, and the opportunity cost of not earning monopoly profits. He can make the first a sunk cost, but not the second. The entrant, foreseeing that the incumbent will buy him out, enters despite knowing that the duopoly price will be less than average total cost. The incumbent faces a second perfectness problem, for while he may try to deter entry by threatening not to buy out the entrant, the threat is not credible.

Figure 15.4 Entry for Buyout



Entry for Buyout
(Rasmusen [1988a])

Players
The incumbent and the entrant.

The Order of Play
1 The incumbent selects capacity K_i.
2 The entrant decides whether to enter or stay out, choosing a capacity $K_e \geq 0$.
3 If the entrant picks a positive capacity, the incumbent decides whether to buy him out at price B.
4 If the entrant has been bought out, the incumbent selects output $q_i \leq K_i + K_e$.
5 If the entrant has not been bought out, each player decides whether to stay in the market or exit.
6 If a player has remained in the market, he selects the output $q_i \leq K_i$ or $q_e \leq K_e$.

Payoffs
Each unit of capacity costs a, the constant marginal cost is c, a firm that stays in the market incurs fixed cost F, and there is no discounting. There is only one period of production.
If no entry occurs, $\pi_{inc} = [p(q_i) - c]q_i - aK_i - F$ and $\pi_{ent} = 0$.
If entry occurs and is bought out, $\pi_{inc} = [p(q_i) - c]q_i - aK_i - B - F$ and $\pi_{ent} = B - aK_e$.
Otherwise,

$$\pi_{incumbent} = \begin{cases} [p(q_i, q_e) - c]q_i - aK_i - F & \text{if the incumbent stays.} \\ -aK_i & \text{if the incumbent exits.} \end{cases}$$

$$\pi_{entrant} = \begin{cases} [p(q_i, q_e) - c]q_e - aK_e - F & \text{if the entrant stays.} \\ -aK_e & \text{if the entrant exits.} \end{cases}$$

Two things have yet to be specified: the buyout price B and the price function $p(q_i, q_e)$. To specify them requires particular solution concepts for bargaining and duopoly, which chapters 12 and 14 have shown are not uncontroversial. Here, they are subsidiary to the main point and can be chosen according to the taste of the modeller. We have "blackboxed" the pricing and bargaining subgames in order not to deflect attention to subsidiary parts of the model. The numerical example below will name specific functions for those subgames, but other numerical examples could use different functions to illustrate the same points.

A Numerical Example

Assume that the market demand curve is

$$p = 100 - q_i - q_e. \tag{18}$$

Let the cost per unit of capacity be $a = 10$, the marginal cost of output be $c = 10$, and the fixed cost be $F = 601$. Assume that output follows Cournot behavior and the bargaining solution splits the surplus equally, in accordance with the Nash bargaining solution and Rubinstein (1982).

If the incumbent faced no threat of entry, he would behave as a simple monopolist, choosing a capacity equal to the output which solved

$$\underset{q_i}{Maximize} \;\; (100 - q_i)q_i - 10q_i - 10q_i. \tag{19}$$

Problem (15.19) has the first-order condition

$$80 - 2q_i = 0, \tag{20}$$

so the monopoly capacity and output would both equal 40, yielding a net operating revenue of 1,399 ($= [p - c]q_i - F$), well above the capacity cost of 400.

We will not go into details, but under these parameters the incumbent chooses the same output and capacity of 40 even if entry is possible but buyout is not. If the potential entrant were to enter, he could do no better than to choose $K_e = 30$, which costs 300. With capacities $K_i = 40$ and $K_e = 30$, Cournot behavior leads the two firms to solve

$$\underset{q_i}{Maximize} \;\; (100 - q_i - q_e)q_i - 10q_i \;\; s.t. \;\; q_i \leq 40 \tag{21}$$

and

$$\underset{q_e}{Maximize}\ (100 - q_i - q_e)q_e - 10q_e \ \ s.t.\ \ q_e \leq 30, \tag{22}$$

which have first order conditions

$$90 - 2q_i - q_e = 0 \tag{23}$$

and

$$90 - q_i - 2q_e = 0. \tag{24}$$

The Cournot outputs both equal 30, yielding a price of 40 and net revenues of $R_i^d = R_e^d = 299$ $(= [p - c]q_i - F)$. The entrant's profit net of capacity cost would be -1 $(= R_e^d - 30a)$, less than the zero from not entering.

What if both entry and buyout are possible, but the incumbent still chooses $K_i = 40$? If the entrant chooses $K_e = 30$ again, then the net revenues would be $R_e^d = R_i^d = 299$, just as above. If he buys out the entrant, the incumbent, having increased his capacity to 70, produces a monopoly output of 45. Half of the surplus from buyout is

$$\begin{aligned} B &= 1/2\left[\underset{q_i}{Maximize}\ \{[p(q_i) - c]q_i | q_i \leq 70\} - F - (R_e^d + R_i^d)\right] \\ &= 1/2[(55 - 10)45 - 601 - (299 + 299)] = 413. \end{aligned} \tag{25}$$

The entrant is bought out for his Cournot revenue of 299 plus the 413 which is his share of the buyout surplus, a total buyout price of 712. Since 712 exceeds the entrant's capacity cost of 300, buyout induces entry which would otherwise have been deterred. Nor can the incumbent deter entry by picking a different capacity. Choosing any K_i greater than 30 leads to the same Cournot output of 60 and the same buyout price of 712. Choosing K_i less than 30 allows the entrant to make a profit even without being bought out.

Realizing that entry cannot be deterred, the incumbent would choose a smaller initial capacity. A Cournot player whose capacity is less than 30 would produce right up to capacity. Since buyout will occur, if a firm starts with a capacity less than 30 and adds one unit, the marginal cost of capacity is 10 and the marginal benefit is the increase (for the entrant) or decrease (for the incumbent) in the buyout price. If it is the entrant who adds a unit of capacity, the net revenue R_e^d rises by at least $(40 - 10)$, the lowest possible Cournot price minus the marginal cost of output. Moreover, R_i^d falls because the entrant's extra output lowers the market price, so under our bargaining solution the buyout price rises by more than 15 $(= \frac{40-10}{2})$ and the entrant should add extra capacity up to $K_e = 30$. A parallel argument shows why the incumbent should build a capacity of at least 30. Increasing the capacities any further leaves the buyout price unchanged, because the duopoly net revenues are unaffected, so both firms choose exactly 30.

The industry capacity equals 60 when buyout is allowed, but after the buyout only 45 is used. Industry profits in the absence of possible entry would have been 999 $(= 1,399 - 400)$, but with buyout they are 824 $(= 1,424 - 600)$, so buyout has decreased industry profits by 175. Consumer surplus has risen from 800 $(= 0.5[100 - p(q|K = 40)][q|K = 40])$ to 1,012.5 $(= 0.5[100 - p(q|K = 60)][q|K = 60])$, a gain of 212.5, so buyout raises total welfare in this example. The increase in output outweighs the inefficiency of the entrant's investment in capacity, an outcome that depends on the particular parameters chosen.

The model is a tangle of paradoxes. The central paradox is that the ability of the incumbent to destroy industry profits after entry ends up hurting him rather than helping because it increases the buyout price. This has a similar flavor to the "judo economics" of Gelman & Salop (1983): the incumbent's very size and influence weighs against him. In the numerical example, allowing the incumbent to buy out the entrant raised total welfare, even though it solidified monopoly power and resulted in wasteful excess capacity. Under other parameters, the effect of excess capacity dominates, and allowing buyout would lower welfare – but only because it encourages entry, of which we usually approve. Adding more potential entrants would also have perverse effects. If the incumbent's excess capacity can deter one entrant, it can deter any number. We have seen that a single entrant might enter anyway, for the sake of the buyout price. But if there are many potential entrants, it is easier to deter entry. Buying out a single entrant would not do the incumbent much good, so he would only be willing to pay a small buyout price, and the small price would discourage any entrant from being the first. The game becomes complicated, but clearly the multiplicity of potential entrants makes entry more difficult for any of them.

Notes

N15.1 Innovation and patent races

- The idea of the patent race is described by Barzel (1968), although his model showed the same effect of overhasty innovation even without patents.

- Reinganum (1985) has shown that an important element of patent races is whether increased research hastens the arrival of the patent or just affects whether it is acquired. If more research hastens the innovation, then the incumbent might spend less than the entrant because the incumbent is enjoying a stream of profits from his present position that the new innovation destroys.

- **Uncertainty in innovation.** Patent Race for an Old Market, is only one way to model innovation under uncertainty. A more common way is to use continuous time with discrete discoveries and specifies that discoveries arrive as a Poisson process with parameter $\lambda(X)$, where X is research expenditure, $\lambda' > 0$, and $\lambda'' < 0$, as in Loury (1979) and Dasgupta & Stiglitz (1980). Then

$$\begin{array}{ll} Prob(\text{invention at } t) & = \lambda e^{-\lambda(X)t}; \\ Prob(\text{invention before } t) & = 1 - e^{-\lambda(X)t}. \end{array} \qquad (26)$$

A little algebra gives us the current value of the firm, R_0, as a function of the innovation rate, the interest rate, the post-innovation value V_1, and the current revenue flow R_0. The return on the firm equals the current cash flow plus the probability of a capital gain.

$$rV_0 = R_0 - X + \lambda(V_1 - V_0), \qquad (27)$$

which implies

$$V_0 = \frac{\lambda V_1 + R_0 - X}{\lambda + r}. \qquad (28)$$

Expression (15.28) is frequently useful.

- A common theme in entry models is what has been called the **fat-cat effect** by Fudenberg & Tirole (1986a, p. 23). Consider a two-stage game, in the first stage of which an incumbent firm chooses its advertising level and in the second stage plays a Bertrand subgame with an entrant. If the advertising in the first stage gives the incumbent a base of captive customers who have inelastic demand, he will choose a higher price than the entrant. The incumbent has become a "fat cat." The effect is present in many models. In section 14.3's Hotelling Pricing Game a firm located so that it has a large "safe" market would choose a higher price. In section 5.5's Customer Switching Costs a firm that has old customers locked in would choose a higher price than a fresh entrant in the last period of a finitely repeated game.

N15.2 Predatory Pricing: the Kreps-Wilson Model

- For other expositions of this model see pages 77-82 of Martin (1993) 239-243 of Osborne & Rubinstein (1994).
- Kreps & Wilson (1982a) do not simply assume that one type of monopolist always chooses *Fight*. They make the more elaborate but primitive assumption that his payoff function makes fighting a dominant strategy. Table 15.6 shows a set of payoffs for the strong monopolist which generate this result.

Table 15.6 Predatory Pricing with a dominant strategy

		Strong Incumbent	
		Collude	*Fight*
Entrant	*Enter*	20,10	$-10, 40$
	Stay out	$0, 100$	**0,100**

Payoffs to: (Entrant, Incumbent)

Under the Kreps-Wilson assumption, the strong monopolist would actually choose to collude in the early periods of the game in some perfect Bayesian equilibria. Such an equilibrium could be supported by out-of-equilibrium beliefs that the authors point out are absurd: if the monopolist fights in the early periods, the entrant believes he must be a weak monopolist.

Problems

15.1: Crazy Predators (adapted from Gintis [forthcoming], Problem 12.10.)
Apex has a monopoly in the market for widgets, earning profits of m per period, but Brydox has just entered the market. There are two periods and no discounting. Apex can either *Prey* on Brydox with a low price or accept *Duopoly* with a high price, resulting in profits to Apex of $-p_a$ or d_a and to Brydox of $-p_b$ or d_b. Brydox must then decide whether to stay in the market for the second period, when Brydox will make the same choices. If, however, Professor Apex, who owns 60 percent of the company's stock, is crazy, he thinks he will earn an amount $p^* > d_a$ from preying on Brydox (and he doesn not learn from experience). Brydox initially assesses the probability that Apex is crazy at θ.

15.1a Show that under the following condition, the equilibrium will be separating, i.e., Apex will behave differently in the first period depending on whether the Professor is crazy or not:

$$-p_a + m < 2d \tag{29}$$

15.1b Show that under the following condition, the equilibrium can be pooling, i.e., Apex will behave the same in the first period whether the Professor is crazy or not:

$$\theta \geq \frac{d_b}{p_b + d_b} \tag{30}$$

15.1c If neither of the two conditions above applies, the equilibrium is hybrid, i.e., Apex will use a mixed strategy and Brydox may or may not be able to tell whether the Professor is crazy at the end of the first period. Let α be the probability that a sane Apex preys on Brydox in the first period, and let β be the probability that Brydox stays in the market in the second period after observing that Apex chose Prey in the first period. Show that equilibrium values of α and β are:

$$\alpha = \frac{\theta d_b}{(1-\theta)p_b} \tag{31}$$

$$\beta = \frac{-p_a + m - 2d_a}{m - d_a} \tag{32}$$

15.1d Is this behavior related to any of the following phenomenon: signalling, signal jamming, reputation, efficiency wages?

15.2: Rent Seeking

I mentioned that Rogerson (1982) uses a game very similar to "Patent Race for a New Market" to analyze competition for a government monopoly franchise. See if you can do this too. What can you predict about the welfare results of such competition?

15.3: A Patent Race

See what happens in Patent Race for an Old Market when specific functional forms and parameters are assumed. Set $f(x) = log(x)$, $g(y) = 0.5(1 + y/(1 + y)$ if $y \geq 0$, $g(y) = 0.5(1 + y/(1 - y)$ if $y \leq 0$, $y = 2$, and $z = 1$. Figure out the research spending by each firm for the three cases of (a) $v = 10$, (b) $v = 4$, (c) $v = 2$ and (d) $v = 1$.

15.4: Entry for Buyout

Find the equilibrium in Entry for Buyout if all the parameters of the numerical example are the same except that the marginal cost of output is $c = 20$ instead of $c = 10$.