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This is a section that was in the third edition of *Games and Information.* In the fourth edition, I have kept only about a third of it.

10.3: Myerson Mechanism Design

Now let's look at another example, a classic one from Myerson.

Myerson (1991) uses a trading example in Sections 6.4 and 10.3 of his book to illustrate mechanism design. A seller has 100 units of a good. If it is high quality, he values it at 40 dollars per unit; if it is low quality, at 20 dollars. The buyer, who cannot observe quality before purchase, values high quality at 50 dollars per unit and low quality at 30 dollars. For efficiency, all of the good should be transferred from the seller to the buyer. The only way to get the seller to truthfully reveal the quality of the good, however, is for the buyer to say that if the seller admits the quality is bad, he will buy more units than if the seller claims it is good. Let us see how this works out.

Depending on who offers the contract and when it is offered, various games result. We will start with one in which the seller makes the offer, and does so before he knows whether his quality is high or low.

Myerson Trading Game I: Moral Hazard, Seller Offers

Players
A buyer and a seller.

The Order of Play
1 The seller offers the buyer a contract $\{q_h, p_h, T_h, q_l, p_l, T_l\}$ under which the seller will later declare his quality m to be high or low, and the buyer will first pay the lump sum T_l or T_h to the seller (perhaps with the lump sum being negative) and then buy q_l or q_h units of the 100 the seller has available, at price p_l or p_h. The contract is $\{w(m) = q(m)p(m)+T(m), q(m)\}$. Zero is paid if the wrong output is delivered.
2 The buyer accepts or rejects the contract.
3 Nature chooses whether the seller's type s of good is High quality (probability 0.2) or Low (probability 0.8), unobserved by the buyer.
4. If the contract was accepted by both sides, the seller declares his type to be L or H and sells at the appropriate quantity and price as stated in the contract.

Payoffs
If the buyer rejects the contract, $\pi_{buyer} = 0$, $\pi_{seller\ H} = 40 * 100$, and $\pi_{seller\ L} = 20 * 100$.
If the buyer accepts the contract and the seller declares a type that has price p, quantity q, and transfer T, then

$$\pi_{buyer|seller\ H} = -T + (50 - P)q \quad and \quad \pi_{buyer|seller\ L} = -T + (30 - p)q \tag{1}$$

and

$$\pi_{seller\ H} = T + 40(100 - Q) + pq \quad and \quad \pi_{seller\ L} = T + 20(100 - q) + pq. \tag{2}$$

The seller wants to design a contract subject to two sets of constraints. First, the buyer must accept the contract. Thus, the participation constraint is[1]

$$\begin{aligned} 0.8\pi_{buyer|seller\ H}(q_l, p_l, T_l) + 0.2\pi_{buyer|seller\ L}(q_h, p_h, T_h) & \geq 0 \\ 0.8[-T_l + (30 - p_l)q_l] + 0.2[-T_h + (30 - p_h)q_h] & \geq 0 \end{aligned} \qquad (3)$$

There might also be a participation constraint for the seller himself, because it might be that even when he designs the contract that maximizes his payoff, his payoff is no higher than when he refuses to offer a contract. He can always offer the acceptable (if vacuous) null contract, $(q_l = 0, p_l = 0, T_l = 0, q_h = 0, p_h = 0, T_h = 0)$, however, so we do not need to write out the seller's participation constraint separately.

Second, the seller must design a contract that will induce himself to tell the truth later once he discovers his type. This is, of course a bit unusual— the seller is like a principal designing a contract for himself as agent. That is why things are different in this chapter than in the chapters on moral hazard. What is happening is that the seller is trying to sell not just a good, but a contract, and so he must make the contract attractive to the buyer. Thus, he faces incentive compatibility constraints: one for when he is low quality,

$$\begin{aligned} \pi_{seller\ L}(q_l, p_l, T_l) & \geq \pi_{seller\ L}(q_h, p_h, T_h) \\ 20(100 - q_l) + p_l q_l + T_l & \geq 20(100 - q_h) + p_h q_h + T_h, \end{aligned} \qquad (4)$$

and one for when he has high quality,

$$\begin{aligned} \pi_{seller\ H}(q_h, p_h, T_h) & \geq \pi_{seller\ H}(q_l, p_l, T_l) \\ 40(100 - q_h) + p_h q_h + T_h & \geq 40(100 - q_l) + p_l q_l + T_l. \end{aligned} \qquad (5)$$

There is not just one incentive compatibility constraint, but two, one for each type, something different from moral hazard.

To make the contract incentive compatible, the seller needs to set p_h greater than p_l, but if he does that it will be necessary to set q_h less than q_l. Then, the low-quality seller will not be irresistably tempted to pretend his quality is high: he would be able to sell at a higher price, but not as great a quantity.

Since q_h is being set below 100 only to make pretending to be high-quality unattractive, there is no reason to set q_l below 100, so $q_l = 100$. The buyer will accept the contract if $p_l \leq 30$, so the seller should set $p_l = 30$. The low-quality seller's incentive compatibility constraint, inequality (4), will be binding, and thus becomes

$$\begin{aligned} \pi_{seller\ L}(q_l, p_l, T_l) & \geq \pi_{seller\ L}(q_h, p_h, T_h) \\ 20(100 - 100) + 30 * 100 + 0 & = 20(100 - q_h) + p_h q_h + 0. \end{aligned} \qquad (6)$$

[1]Another kind of participation constraint would apply if the buyer had the option to reject purchasing anything, after accepting the contract and hearing the seller's type announcement. That would not make a difference here.

Solving for q_h gives us $q_h = \frac{1000}{p_h - 20}$, which when substituted into the seller's payoff function yields

$$
\begin{aligned}
\pi_s &= 0.8\pi_{seller\ L}(q_l, p_l, T_l) + 0.2\pi_{seller\ H}(q_h, p_h, T_h) \\
&= 0.8[(20)(100 - q_l) + p_l q_l + T_l] + 0.2[(40)(100 - q_h) + p_h q_h + T_h] \\
&= 0.8[(20)(100 - 100) + 30 * 100 + 0] + 0.2[(40)(100 - \tfrac{1000}{p_h - 20}) + p_h(\tfrac{1000}{p_h - 20}) + 0]
\end{aligned}
\tag{7}
$$

Maximizing with respect to p_h subject to the constraint that $p_h \leq 50$ (or else the buyer will turn down the contract) yields the corner solution of $p_h = 50$, which allows for $q_h = 33\frac{1}{3}$.

The participation constraint for the buyer is already binding, so we do not need the transfers T_l and T_h to take away any remaining surplus, as we might in other situations.[2] Thus, the equilibrium contract is

$$
\begin{aligned}
q_l &= 100, p_l = 30, T_l = 0 \\
q_h &= 33\tfrac{1}{3}, p_h = 50, T_h = 0.
\end{aligned}
\tag{8}
$$

This mechanism will not work if further offers can be made after the end of the game. The mechanism is not first-best efficient; if the seller is high-quality, then he only sells $33\frac{1}{3}$ units to the buyer instead of all 100, even though both realize that the buyer's value is 50 and the seller's is only 40. If they could agree to sell the remaining $66\frac{2}{3}$ units, then the mechanism would not be incentive compatible in the first place, though, because then the low-quality seller would pretend to be high-quality, first selling $33\frac{1}{3}$ units and then selling the rest. The importance of commitment is a general feature of mechanisms.

What if it is the buyer who makes the offer?

Myerson Trading Game II: Moral Hazard, Buyer Makes the Offer

The Order of Play
The same as in Myerson Trading Game I except that the buyer makes the contract offer in move (1) and the seller accepts or rejects in move (2).

Payoffs
The same as in Myerson Trading Game I.

The participation constraint in the buyer's mechanism design problem is

$$
0.8\pi_{seller\ L}(q_l, p_l, T_l) + 0.2\pi_{seller\ H}(q_h, p_h, T_h) \quad \geq 0. \tag{9}
$$

The incentive compatibility constraints are just as they were before, since the buyer has to design a mechanism which makes the seller truthfully reveal his type.

[2] The transfers could be used to adjust the prices, too. We could have $q_l = 20$ and $T_l = 1000$ in equation (7) without changing anything important.

As before, the mechanism will set $q_l = 100$, but it will have to make $q_h < 100$ to deter the low-quality seller from pretending he is high- quality. Also, $p_h \geq 40$, or the high-quality seller will pretend to be low-quality.

Suppose $p_h = 40$. The low-quality seller's incentive compatibility constraint, inequality (4), will be binding, and thus becomes

$$\begin{aligned} \pi_{seller\ L}(q_l, p_l, T_l) & \geq \pi_{seller\ H}(q_h, p_h, T_h) \\ 20(100 - 100) + p_l * 100 + 0 & = 20(100 - q_h) + 40q_h + 0. \end{aligned} \tag{10}$$

Solving for q_h gives us $q_h = 5p_l - 100$, which when substituted into the buyer's payoff function yields

$$\begin{aligned} \pi_b & = 0.8\pi_{b|L}(q_l, p_l, T_l) + 0.2\pi_{b|H}(q_h, p_h, T_h) \\ & = 0.8[(30 - p_l)q_l] + 0.2[(50 - p_h)q_h] \\ & = 0.8[(30 - p_l)100] + 0.2[(50 - 40)(5p_l - 100)] \\ & = 2400 - 80p_l + 10p_l - 200 = 2200 - 70p_l \end{aligned} \tag{11}$$

Maximizing with respect to p_l subject to the constraint that $p_l \geq 20$ (or else we would come out with $q_h < 0$ to satisfy incentive compatibility constraint (6)) yields the corner solution of $p_l = 20$, which requires that $q_h = 0$.

Would setting $p_h > 40$ help? No, because that just makes it harder to satisfy the low-quality seller's incentive compatibility constraint. We would continue to have $q_h = 0$, and, of course, p_h does not matter if nothing is sold. And as before, we do not need to make use of transfers to make the participation constraint binding. Thus, the equilibrium contract has p_h take any possible value and

$$\begin{aligned} & (q_l = 100, p_l = 20, T_l = 0 \\ & q_h = 0, T_h = 0). \end{aligned} \tag{12}$$

In the next version of the game, we will continue to let the buyer make the offer, but he makes it at a time when the seller already knows his type. Thus, this will be an adverse selection model.

Myerson Trading Game III: Adverse Selection, The Buyer Makes the Offer

The Order of Play

0. Nature chooses whether the seller's good is high quality (probability 0.2) or low quality (probability 0.8), unobserved by the buyer.
1 The buyer offers the seller a contract $(q_h, p_h, T_h, q_l, p_l, T_l)$ under which the seller will later declare his quality to be high or low, and the buyer will first pays the lump sum T to the

seller (perhaps with $T < 0$) and then buy q units of the 100 the seller has available, at price p.
2 The seller accepts or rejects the contract.
3. If the contract was accepted by both sides, the seller declares his type to be L or H and sells at the appropriate quantity and price as stated in the contract.

Payoffs
The same as in Myerson Trading Games I and II.

The incentive compatibility constraints are unchanged from the previous two versions of the game, but now the participation constraints are different for the two types of seller.

$$\pi_L(q_l, p_l, T_l) \quad \geq 0 \tag{13}$$

and

$$\pi_H(q_h, p_h, T_h) \quad \geq 0. \tag{14}$$

Any mechanism which satisfies these two constraints would also satisfy the single participation constraint in Myerson Trading Game II, since it says that a weighted average of the payoffs of the two sellers must be positive. Thus, any mechanism which maximized the buyer's payoff in Myerson Trading Game II would also maximize his payoff in Myerson Trading Game III, if it satisfied the tougher bifurcated participation constraints. The mechanism we found for the game does satisfy the tougher constraints, so it is the optimal mechanism here too.

This is not a general feature of mechanisms. More generally the optimal mechanism will not have as high a payoff when one player starts the game with superior information, because of the extra constraints on the mechanism.

In the last of our versions of this game, the seller makes the offer, but after he knows his type.

Myerson Trading Game IV

The Order of Play
The same as in Myerson Trading Game III except that in (1) the seller makes the offer and in (2) the buyer accepts or rejects.

Payoffs
The same as in Myerson Trading Games I, II, and III.

The incentive compatibility constraints are the same as in the previous games, and the participation constraint is inequality (**??**), just as in *Myerson Trading Game I.* The big difference now is that unlike in the first three versions, Myerson Trading Game IV has an informed player making the contract offer. As a result, the form of the offer can convey information, and we have to consider out-of- equilibrium beliefs, as in the dynamic games of incomplete information in Chapter 6 (and we will see more of this in the signalling models of Chapter 11). Surprisingly, however, the importance of out-of-equilibrium beliefs does not lead to multiple equilibria. Instead, the equilibrium contract is

M1: $q_L = 100, p_l = 30, T_l = 0,$

$q_h = 33\frac{1}{3}, p_h = 50, T_h = 0,$

This is part of equilibrium under the out-of-equilibrium belief that if the seller offers any other contract, the buyer believes the quality is low.

This is the same equilibrium mechanism as in Myerson Trading Game I. It is interesting to compare it to two other mechanisms, M2 and M3, which satisfy the two incentive compatibility constraints and the participation constraint, but which are not equilibrium choices: [3]

M2: $q_l = 100, p_l = 28, T_l = 0,$

$q_h = 0, p_h = 40, T_h = 800.$

M3: $q_l = 100, p_l = 31\frac{3}{7}, T_l = 0,$

$q_h = 57\frac{1}{7}, p_h = 40, T_h = 0.$

Mechanism M2 is interesting because the buyer expects a positive payoff of (30- 28) (100) = 200 if the seller is low-quality and a negative payoff of 800 if the seller is high-quality, for an overall expected payoff of zero. The contract is incentive compatible because a low-quality seller could not increase his payoff of 28*100 by pretending to be high-quality (he would get 20*100 + 800 instead), and a high- quality seller would reduce his payoff of (40*100 + 800) if he pretended to have low quality. Here, for the first time, we see a positive value for the transfer T_h.

Under mechanism M3, the buyer expects a negative payoff of (30-31 $\frac{1}{7}$) (100) = -11 $\frac{3}{7}$ if the seller is low- quality and a positive payoff of $(57\frac{1}{7})(50-40)$ =11 $\frac{3}{7}$ if the seller is high-quality, for an overall expected payoff of zero. The contract is incentive compatible because a low- quality seller could not increase his payoff of 3,142 $\frac{6}{7}$ (31 $\frac{3}{7}$) (100) by pretending to be high-quality (he would get (57 $\frac{1}{7}$) (40) + (42 $\frac{6}{7}$) (20) instead, which comes to the same figure), and a high-quality seller would reduce his payoff if he pretended to have low quality and sold something he valued at 40 at a price of 31 $\frac{1}{7}$.

In Myerson Trading Game IV, unlike the previous versions of the game, the particular mechanism chosen in equilibrium is not necessarily the one that the player who offers the contract likes best. Instead, an informed offeror— here, the seller— must worry that his offer might make the uninformed receiver believe the offeror's type is undesirable.

Mechanism M1 maximizes the payoff of the average seller, as we found in Myerson Trading Game I, yielding the low-quality seller a payoff of 3,000 and the high- quality seller

[3] xxx M2– hwo about an oo belief that a deviator is a HIGH type? Then no deviation is profitable.

a payoff of 4,333 $(= (33\frac{1}{3})(50) + 66\frac{2}{3}(40))$, for an average payoff of 3,867. If the seller is high-quality, however, he would prefer mechanism M2, which has payoffs of 2800 and 4800 (=800+ 40(100)), for an average payoff of 3200. If the seller is low-quality, he would prefer mechanism M3, which has payoffs of $3,142\frac{6}{7}$ and 4000, for an average payoff of $3,314\frac{6}{7}$.

Suppose that the seller chose M2, regardless of his type. This could not be an equilibrium, because a low-quality seller would want to deviate. Suppose he deviated and offered a contract almost like M1, except that $p_l = 29.99$ instead of 30 and $p_h = 49.99$ instead of 50. This new contract would yield positive expected payoff to the buyer whether the buyer believes the seller is low-quality or high-quality, and so it would be accepted. It would yield higher payoff to the low- quality seller than M2, and so the deviation would have been profitable. Similarly, if the seller chose M3 regardless of his type, a high- quality seller could profitably deviate in the same way.

The *Myerson Trading Game* is a good introduction to the flavor of the algebra in mechanism design problems. For more on this game, in a very different style of presentation, see Sections 6.4 and Chapter 10 of Myerson (1991). We will next go on to particular economic applications of mechanism design.[4]

[4]xxx Think abvout risk sharing too, with risk aversion. Then it makes a big difference when the seller learns his type.