ODD

Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

PROBLEMS FOR CHAPTER 2: INFORMATION

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This file contains answers to the odd-numbered problems in the gourth edition of *Games and Information* by Eric Rasmusen, which I am working on now and perhaps will come out in 2005. The answers to the even- numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I must ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

PROBLEMS FOR CHAPTER 2: INFORMATION

2.1. The Monty Hall Problem (easy)
You are a contestant on the TV show, "Let's Make a Deal." You face three curtains, labelled A, B and C. Behind two of them are toasters, and behind the third is a Mazda Miata car. You choose A, and the TV showmaster says, pulling curtain B aside to reveal a toaster, "You're lucky you didn't choose B, but before I show you what is behind the other two curtains, would you like to change from curtain A to curtain C?" Should you switch? What is the exact probability that curtain C hides the Miata? *Answer.* You should switch to curtain C, because:

$$\begin{aligned}\text{Prob (Miata behind C} \mid \text{Host chose B)} &= \frac{\text{Prob(Host chose B} \mid \text{Miata behind C)Prob(Miata behind C)}}{\text{Prob(Host chose B)}} \\ &= \frac{(1)(\frac{1}{3})}{(1)(\frac{1}{3})+(\frac{1}{2})(\frac{1}{3})}. \\ &= \frac{2}{3}.\end{aligned}$$

The key is to remember that this is a game. The host's action has revealed more than that the Miata is not behind B; it has also revealed that the host did not want to choose Curtain C. If the Miata were behind B or C, he would pull aside the curtain it was not behind. Otherwise, he would pull aside a curtain randomly. His choice tells you nothing new about the probability that the Miata is behind Curtain A, which remains $\frac{1}{3}$, so the probability of it being behind C must rise to $\frac{2}{3}$ (to make the total probability equal one).

What would be the best choice if curtain B simply was blown aside by the wind, revealing a toaster, and the host, Monty Hall, asked if you wanted to switch to Curtain C? In that case you should be indifferent. Just as easily, Curtain C might have blown aside, possibly revealing a Miata, but though the wind's random choice is informative– your posterior on the probability that the Miata is behind Curtain C rises from 1/3 to 1/2— it does not convey as much information as Monty Hall's deliberate choice.

See http://www.stat.sc.edu/∼west/javahtml/LetsMakeaDeal.html for a Java applet on this subject.

2.3. Cancer Tests (easy) (adapted from McMillan [1992, p. 211])
Imagine that you are being tested for cancer, using a test that is 98 percent accurate. If you indeed have cancer, the test shows positive (indicating cancer) 98 percent of the time. If you do not have cancer, it shows negative 98 percent of the time. You have heard that 1 in 20 people in the population actually have cancer. Now your doctor tells you that you tested positive, but you shouldn't worry because his last 19 patients all died. How worried should you be? What is the probability you have cancer?

Answer. Doctors, of course, are not mathematicians. Using Bayes' Rule:

$$\begin{aligned} Prob(Cancer|Positive) &= \frac{Prob(Positive|Cancer)Prob(Cancer)}{Prob(Positive)} \\ &= \frac{0.98(0.05)}{0.98(0.05)+0.02(0.95)} \\ &\approx 0.72. \end{aligned} \tag{1}$$

With a 72 percent chance of cancer, you should be very worried. But at least it is not 98 percent.

Here is another way to see the answer. Suppose 10,000 tests are done. Of these, an average of 500 people have cancer. Of these, 98% test positive on average— 490 people. Of the 9,500 cancer-free people, 2% test positive on average—190 people. Thus there are 680 positive tests, of which 490 are true positives. The probability of having cancer if you test positive is 490/680, about 72% .

This sort of analysis is one reason why HIV testing for the entire population, instead of for high-risk subpopulations, would not be very informative— there would be more false positives than true positives.

2.5. Joint Ventures (medium)

Software Inc. and Hardware Inc. have formed a joint venture. Each can exert either high or low effort, which is equivalent to costs of 20 and 0. Hardware moves first, but Software cannot observe his effort. Revenues are split equally at the end, and the two firms are risk neutral. If both firms exert low effort, total revenues are 100. If the parts are defective, the total revenue is 100; otherwise, if both exert high effort, revenue is 200, but if only one player does, revenue is 100 with probability 0.9 and 200 with probability 0.1. Before they start, both players believe that the probability of defective parts is 0.7. Hardware discovers the truth about the parts by observation before he chooses effort, but Software does not.

(a) Draw the extensive form and put dotted lines around the information sets of Software at any nodes at which he moves.

Answer. See Figure A2.1. To understand where the payoff numbers come from, see the answer to part (b).

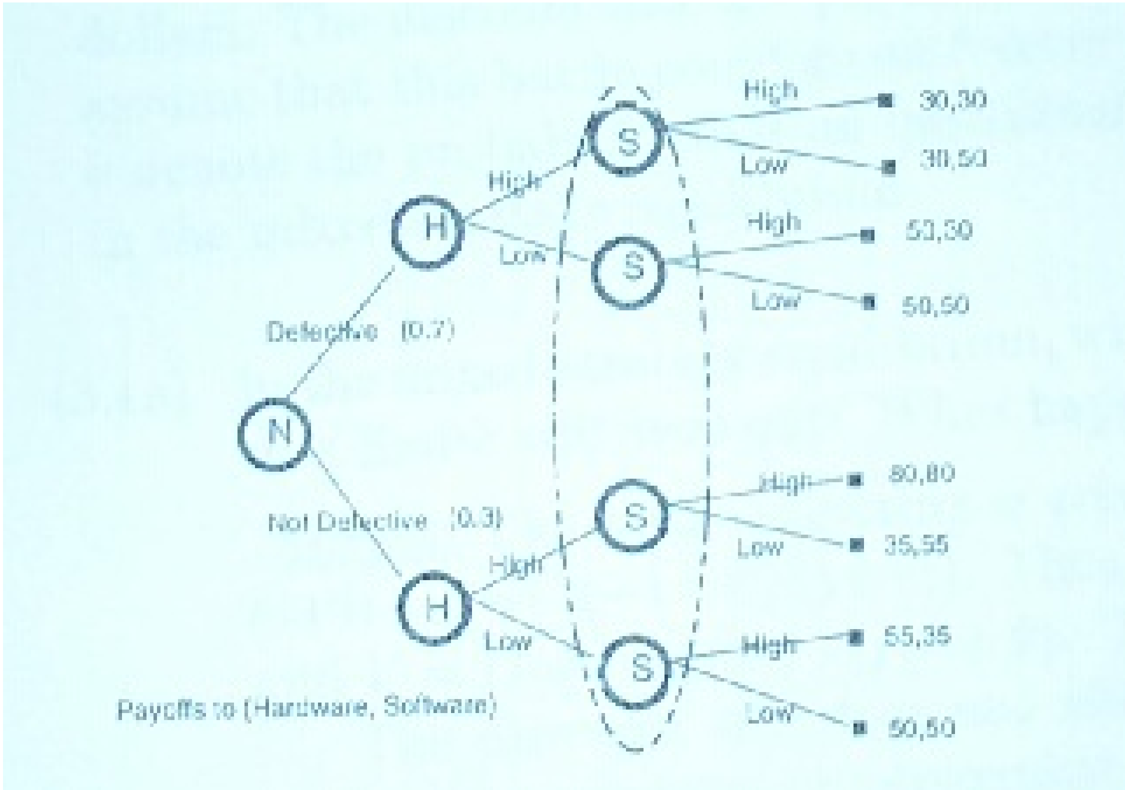

Figure A2.1: The Extensive Form for the Joint Ventures Game

(b) What is the Nash equilibrium?

<u>*Answer.*</u> (Hardware: *Low* if defective parts, *Low* if not defective parts; Software: *Low*).

$$\pi_{Hardware}(Low|Defective) = \frac{100}{2} = 50.$$

Deviating would yield Hardware a lower payoff:

$$\pi_{Hardware}(High|Defective) = \frac{100}{2} - 20 = 30.$$

$$\pi_{Hardware}(Low|Not\ Defective) = \frac{100}{2} = 50.$$

Deviating would yield Hardware a lower payoff:

$$\pi_{Hardware}(High|Not\ Defective) = 0.9\left(\frac{100}{2}\right) + 0.1\left(\frac{200}{2}\right) - 20 = 45 + 10 - 20 = 35.$$

$$\pi_{Software}(Low) = \frac{100}{2} = 50.$$

Deviating would yield Software a lower payoff:

$$\pi_{Software}(High) = 0.7\left(\frac{100}{2}\right) + .3\left[0.9\left(\frac{100}{2}\right) + 0.1\left(\frac{200}{2}\right)\right] - 20 = 35 + 0.3(45+10) - 20.$$

This equals $15 + 0.3(35) = 31.5$, less than the equilibrium payoff of 50.

Elaboration. A strategy combination that is *not* an equilibrium (because Software would deviate) is:

(Hardware: *Low* if defective parts, *High* if not defective parts; Software: *High*).

$$\pi_{Hardware}(Low|Defective) = \frac{100}{2} = 50.$$

Deviating would indeed yield Hardware a lower payoff:

$$\pi_{Hardware}(High|Defective) = \frac{100}{2} - 20 = 30.$$

$$\pi_{Hardware}(High|Not\ Defective) = \frac{200}{2} - 20 = 100 - 20 = 80.$$

Deviating would indeed yield Hardware a lower payoff:

$$\pi_{Hardware}(Low|Not\ Defective) = 0.9\left(\frac{100}{2}\right) + 0.1\left(\frac{200}{2}\right) = 55.$$

$$\pi_{Software}(High) = 0.7\left(\frac{100}{2}\right) + 0.3\left(\frac{200}{2}\right) - 20 = 35 + 30 - 20 = 45.$$

Deviating would yield Software a higher payoff, so the strategy combination we are testing is not a Nash equilibrium:

$$\pi_{Software}(Low) = 0.7\left(\frac{100}{2}\right) + 0.3\left[0.9\left(\frac{100}{2}\right) + 0.1\left(\frac{200}{2}\right)\right] = 35 + 0.3(45+10) = 35 + 16.5 = 51.5$$

More Elaboration. Suppose the probability of revenue of 100 if one player choose High and the other chooses Low were z instead of 0.9. If z is too low, the equilibrium described above breaks down because Hardware finds it profitable to deviate to $High|Not\ Defective$.

$$\pi_{Hardware}(Low|Not\ Defective) = \frac{100}{2} = 50.$$

Deviating would yield Hardware a lower payoff:

$$\pi_{Hardware}(High|Not\ Defective) = z\left(\frac{100}{2}\right) + (1-z)\left(\frac{200}{2}\right) - 20 = 50z + 100 - 100z - 20.$$

This comes to be $\pi_{Hardware}(High|Not\ Defective) = 80 - 50z$, so if $z < 0.6$ then the payoff from $(High|Not\ Defective)$ is greater than 50, and so Hardware would be willing to unilaterally supply High effort even though Software is providing Low effort.

You might wonder whether Software would deviate from the equilibrium for some value of z even greater than 0.6. To see that he would not, note that

$$\pi_{Software}(High) = 0.7\left(\frac{100}{2}\right) + 0.3\left[z\left(\frac{100}{2}\right) + (1-z)\left(\frac{200}{2}\right)\right] - 20.$$

This takes its greatest value at $z = 0$, but even then the payoff from $High$ is just $0.7(50) + 0.3(100) - 20 = 45$, less than the payoff of 50 from Low. The chances of non-defective parts are just too low for Software to want to take the risk of playing $High$ when Hardware is sure to play Low.

This situation is like that of two people trying to lift a heavy object. Maybe it is simply too heavy to lift. Otherwise, if both try hard they can lift it, but if only one does, his effort is wasted.

(c) What is Software's belief, in equilibrium, as to the probability that Hardware chooses low effort?

Answer. One. In equilibrium, Hardware always chooses Low.

(d) If Software sees that revenue is 100, what probability does he assign to defective parts if he himself exerted high effort and he believes that Hardware chose low effort?

Answer. 0.72 $(= (1)\frac{0.7}{(1)(0.7)+(0.9)(0.3)})$.

2.7. Smith's Energy Level (easy)

The boss is trying to decide whether Smith's energy level is high or low. He can only look in on Smith once during the day. He knows if Smith's energy is low, he will be yawning with a 50 percent probability, but if it is high, he will be yawning with a 10 percent probability. Before he looks in on him, the boss thinks that there is an 80 percent probability that Smith's energy is high, but then he sees him yawning. What probability of high energy should the boss now assess?

Answer. What we want to find is $Prob(High|Yawn)$. The information is that $Prob(High) = .80$, $Prob(Yawn|High) = .10$, and $Prob(Yawn|Low) = .50$. Using Bayes Rule,

$$Prob(High|Yawn) = \frac{Prob(High)Prob(Yawn|High)}{Prob(High)Prob(Yawn|High)+Prob(Low)Prob(Yawn|Low)}$$

$$= \frac{(0.8)(0.1)}{(0.8)(0.1)+(0.2)(0.5)} = 0.44.$$