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Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

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PROBLEMS FOR CHAPTER 4

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4.1. Repeated Entry Deterrence

Consider two repetitions without discounting of the game Entry Deterrence I from Section 4.2. Assume that there is one entrant, who sequentially decides whether to enter two markets that have the same incumbent.

(a) Draw the extensive form of this game.

Answer. See Figure A4.1. If the entrant does not enter, the incumbent's response to entry in that period is unimportant.

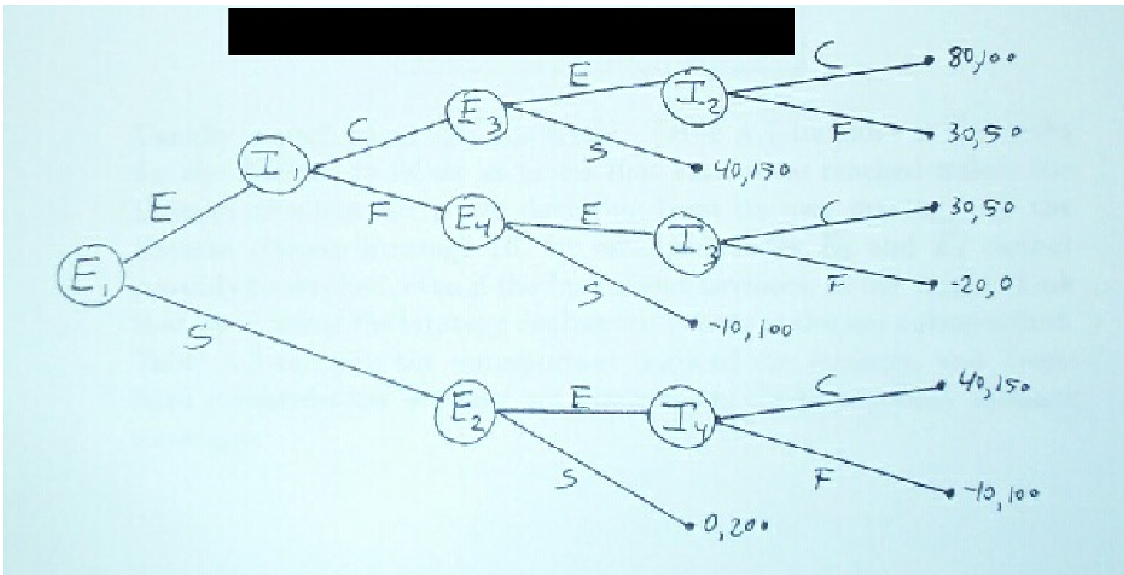

Figure A4.1: Repeated Entry Deterrence

(b) What are the 16 elements of the strategy sets of the entrant?

Answer. The entrant makes a binary decision at four nodes, so his strategy must have four components, strictly speaking, and the number of possible arrangements is $(2)(2)(2)(2) = 16$. Table A4.1 shows the strategy space, with E for *Enter* and S for *Stay out.*

Table A4.1: The Entrant's Strategy Set

Strategy	E_1	E_2	E_3	E_4
1	E	E	E	E
2	E	E	E	E
3	E	E	E	S
4	E	E	S	S
5	E	S	S	S
6	E	S	E	E
7	E	S	S	E
8	E	S	E	S
9	S	E	E	E
10	S	S	E	E
11	S	S	S	E
12	S	S	S	S
13	S	E	S	S
14	S	E	S	E
15	S	E	E	S
16	S	S	E	S

Usually modellers are not so careful. Table A4.1 includes action rules for the Entrant to follow at nodes that cannot be reached unless the Entrant trembles, somehow deviating from its own strategy. If the Entrant chooses Strategy 16, for example, nodes E_3 and E_4 cannot possibly be reached, even if the Incumbent deviates, so one might think that the parts of the strategy dealing with those nodes are unimportant. Table A4.2 removes the unimportant parts of the strategy, and Table A4.3 condenses the strategy set down to its six importantly distinct strategies.

Table A4.2: The Entrant's Strategy Set, Abridged Version I

Strategy	E_1	E_2	E_3	E_4
1	E	-	E	E
2	E	-	E	E
3	E	-	E	S
4	E	-	S	S
5	E	-	S	S
6	E	-	E	E
7	E	-	S	E
8	E	-	E	S
9	S	E	-	-
10	S	S	-	-
11	S	S	-	-
12	S	S	-	-
13	S	E	-	-
14	S	E	-	-
15	S	E	-	-
16	S	S	-	-

Table A4.3: The Entrant's Strategy Set, Abridged Version II

Strategy	E_1	E_2	E_3	E_4
1	E	-	E	E
3	E	-	E	S
4	E	-	S	S
7	E	-	S	E
9	S	E	-	-
10	S	S	-	-

(c) What is the subgame perfect equilibrium?

Answer. The entrant always enters and the incumbent always colludes.

(d) What is one of the nonperfect Nash equilibria?

Answer. The entrant stays out in the first period, and enters in the second period. The incumbent fights any entry that might occur in the first period, and colludes in the second period.

4.3. Heresthetics in Pliny and the Freedmens' Trial (Pliny, 1963, pp. 221-4, Riker, 1986, pp.78-88)
Afranius Dexter died mysteriously, perhaps dead by his own hand, perhaps killed by his freedmen (servants a step above slaves), or perhaps killed by his freedmen by his own orders. The freedmen went on trial before the Roman Senate. Assume that 45 percent of the senators favor acquittal, 35 percent favor banishment, and 20 percent favor execution,

and that the preference rankings in the three groups are $A \succ B \succ E$, $B \succ A \succ E$, and $E \succ B \succ A$. Also assume that each group has a leader and votes as a bloc.

(a) Modern legal procedure requires the court to decide guilt first and then assign a penalty if the accused is found guilty. Draw a tree to represent the sequence of events (this will not be a game tree, since it will represent the actions of groups of players, not of individuals) . What is the outcome in a perfect equilibrium?

Answer. Guilt would win in the first round by a vote of 55 to 45, and banishment would win in the second by 80 to 20. See Figure A4.2. Note that Figure A4.2 is not really an extensive form, since each node indicates a time when many players make their choices, not one, and the numbers at the end are not payoffs.

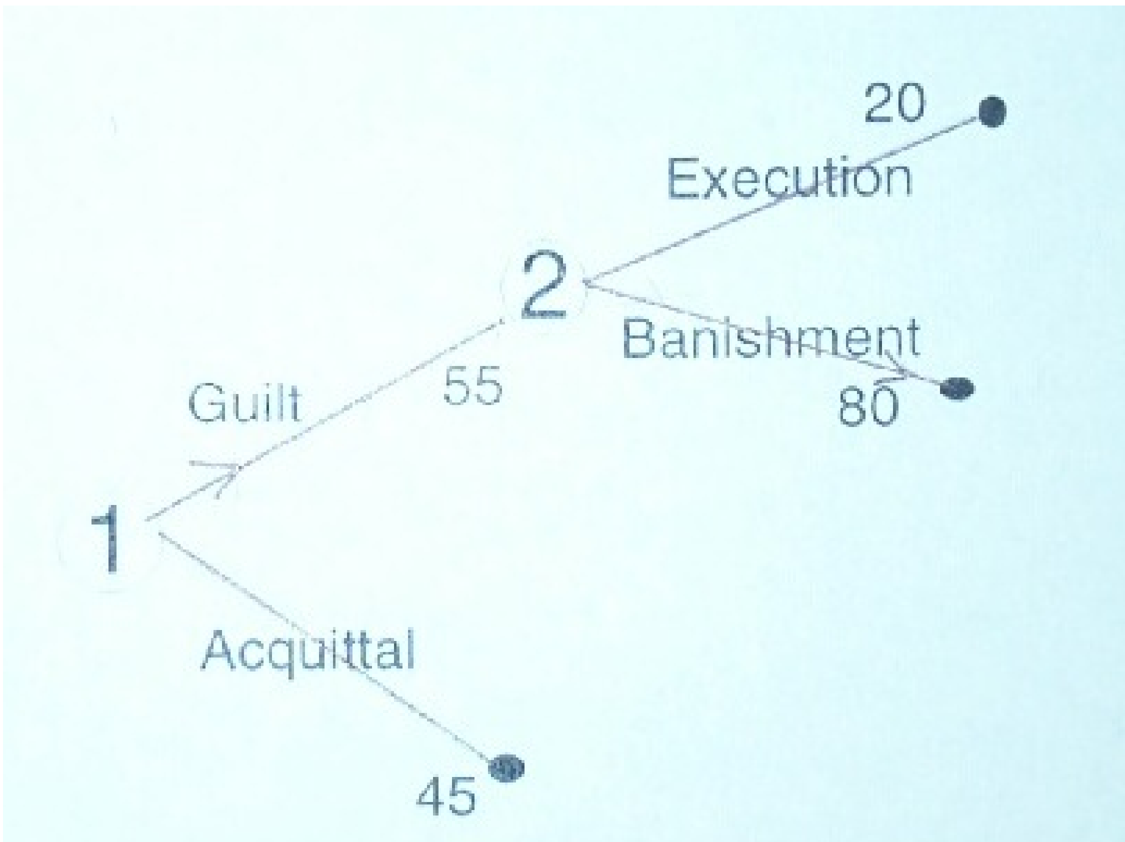

Figure A4.2: Modern Legal Procedure

(b) Suppose that the acquittal bloc can pre-commit to how they will vote in the second round if guilt wins in the first round. What will they do, and what will happen? What would the execution bloc do if they could control the second-period vote of the acquittal bloc?

Answer. The acquittal bloc would commit to execution, inducing the Banishment bloc to vote for Acquittal in the first round, and acquittal would win. The execution bloc would order the acquittal bloc to choose banishment in the second round to avoid making the banishment bloc switch to acquittal.

It is usually difficult to commit. If the acquittal bloc could sell or donate its voting rights contingent on Guilt to the execution bloc, this would work–but the execution bloc would turn down such an offer. Can you think of other ways to commit?

Preferences do not always work out this way. In Athens, six centuries before the Pliny episode, Socrates was found guilty in a first round of voting and then sentenced to death (instead of a lesser punishment like banishment) by a bigger margin in the second round. This would imply the ranking of the acquittal bloc there was AEB, except for the complicating factor that Socrates was a bit insulting in his sentencing speech.

(c) The normal Roman procedure began with a vote on execution versus no execution, and then voted on the alternatives in a second round if execution failed to gain a majority. Draw a tree to represent this. What would happen in this case?

<u>*Answer.*</u> Execution would fail by a vote of 20 to 80, and banishment would then win by 55 to 45. See Figure A4.3.

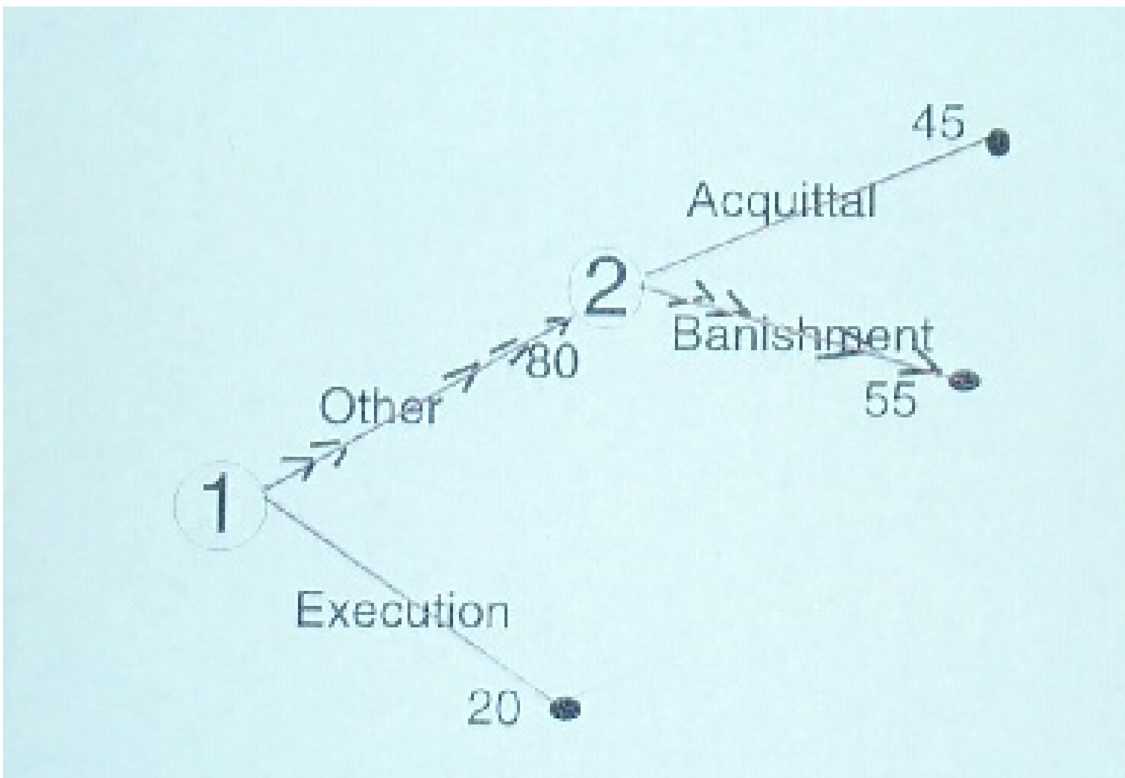

Figure A4.3: Roman Legal Procedure

(d) Pliny proposed that the Senators divide into three groups, depending on whether they supported acquittal, banishment, or execution, and that the outcome with the most votes should win. This proposal caused a roar of protest. Why did he propose it?

<u>*Answer.*</u> It must be that Pliny favored acquittal and hoped that every senator would vote for his preference. Acquittal would then win 45 to 35 to 25.

(e) Pliny did not get the result he wanted with his voting procedure. Why not?

<u>Answer.</u> Pliny wasn't very good at strategy, but he was good at making excuses. He said that his arguments were so convincing that the senator who made the motion for the death penalty changed his mind, along with his supporters, and voted for banishment, which won (by 55 to 45 in our hypothesized numbers). He didn't realize that people do not always vote for their first preference. The execution bloc saw that acquittal would win unless they switched to banishment.

(f) Suppose that personal considerations made it most important to a senator that he show his stand by his vote on acquittal vs. banishment vs. execution, even if he had to sacrifice his preference for a particular outcome. If there were a vote over whether to use the traditional Roman procedure or Pliny's procedure, who would vote with Pliny, and what would happen to the freedmen?

<u>Answer.</u> Traditional procedure would win by capturing the votes of the execution bloc and the banishment bloc, and the freedmen would be banished. In this case, the voting procedure would matter to the result, because each senator would vote for his preference.

You could depict this situation by adding a "Move 0" in which the Senators choose between Traditional and Pliny procedures. Under the actual rules, it seems that Pliny had the authority to pick the rule that he liked best. It is common for the leader of a group to have this authority, and it is a huge source of his power.

Sometimes people might have preferences even over the procedure used, though I did not suggest any in this example. Then, they might end up voting for a procedure even though it resulted in a loss for them on the substantive question. More often– at least when precedents for the future are not at stake–people are relatively indifferent about the procedure in itself, only caring about the result the procedure will attain.

4.5. Voting Cycles

Uno, Duo, and Tres are three people voting on whether the budget devoted to a project should be Increased, kept the Same, or Reduced. Their payoffs from the different outcomes, given in Table 3, are not monotonic in budget size. Uno thinks the project could be very profitable if its budget were increased, but will fail otherwise. Duo mildly wants a smaller budget. Tres likes the budget as it is now.

	Uno	Duo	Tres
Increase	100	2	4
Same	3	6	9
Reduce	9	8	1

Table 3: Payoffs from Different Policies

Each of the three voters writes down his first choice. If a policy gets a majority of the votes, it wins. Otherwise, *Same* is the chosen policy.

(a) Show that (*Same*, *Same*, *Same*) is a Nash equilibrium. Why does this equilibrium seem unreasonable to us?

Answer. The policy outcome is *Same* regardless of any one player's deviation. Thus, all three players are indifferent about their vote. This seems strange, though, because Uno is voting for his least-preferred alternative. Parts (c) and (d) formalize why this is implausible.

(b) Show that (*Increase*, *Same*, *Same*) is a Nash equilibrium.

Answer. The policy outcome is *Same*, but now only by a bare majority. If Uno deviates, his payoff remains 3, since he is not decisive. If Duo deviates to *Increase*,*Increase* wins and he reduces his payoff from 6 to 2; if Duo deviates to *Reduce*, each policy gets one vote and *Same* wins because of the tie, so his payoff remains 6. If Tres deviates to Increase, *Increase* wins and he reduces his payoff from 9 to 4; if Tres deviates to *Reduce*, each policy gets one vote and *Same* wins because of the tie, so his payoff remains 9.

(c) Show that if each player has an independent small probability ϵ of "trembling" and choosing each possible wrong action by mistake, (*Same*, *Same*, *Same*) and (*Increase*, *Same*, *Same*) are no longer equilibria.

Answer. Now there is positive probability that each player's vote is decisive. As a result, Uno deviates to *Increase*. Suppose Uno himself does not tremble. With positive probability Duo mistakenly chooses *Increase* while Tres chooses *Same*, in which case Uno's choice of *Increase* is decisive for *Increase* winning and will raise his payoff from 3 to 100. Similarly, it can happen that Tres mistakenly chooses *Increase* while Duo chooses *Same*. Again, Uno's choice of *Increase* is decisive for *Increase* winning. Thus, (*Same*, *Same*, *Same*) is no longer an equilibrium.

It is also possible that both Duo and Tres tremble and choose *Increase* by mistake, but in that case, Uno's vote is not decisive, because*Increase* wins even without his vote.

How about (*Increase*, *Same*, *Same*)? First, note that a player cannot benefit by deviating to his least-preferred policy.

Could Uno benefit by deviating to *Reduce*, his second-preferred policy? No, because he would rather be decisive for *Increase* than for *Reduce*, if a tremble might occur.

Could Duo benefit by deviating to *Reduce*, his most-preferred policy? If no other player trembles, that deviation would leave his payoff unchanged. If, however, one of the two other players trembles to *Reduce* and the other does not, then Duo's voting for *Reduce* would be decisive and *Reduce* would win, raising Duo's payoff from 6 to 8. Thus, (*Increase*, *Same*, *Same*) is no longer an equilibrium.

Just for completeness, think about Tres's possible deviations. He has no reason to deviate from *Same*, since that is his most preferred policy. *Reduce* is his least-preferred policy, and if he deviates to *Increase*, *Increase* will win, in the absence of a tremble, and his payoff will fall from 9 to 4– and since trembles have low probability, this reduction dominates any possibilities resulting from trembles.

(d) Show that (*Reduce*, *Reduce*, *Same*) is a Nash equilibrium that survives each player having an independent small probability ϵ of "trembling" and choosing each possible wrong action by mistake.

<u>*Answer.*</u> If Uno deviates to *Increase* or *Same*, the outcome will be *Same* and his payoff will fall from 9 to 3 If Duo deviates to *Increase* or *Same*, the outcome will be *Same* and his payoff will fall from 8 to 6. Tres's vote is not decisive, so his payoff will not change if he deviates. Thus, (*Reduce*, *Reduce*, *Same*) is a Nash equilibrium

How about trembles? The votes of both Uno and Duo are decisive in equilibrium, so if there are no trembles, each loses by deviating, and the probability of trembles is too small to make up for that. Only if a player's equilibrium strategy is weak could trembles make a difference.

Tres's equilibrium strategy is indeed weak, since he is not decisive unless there is a tremble.

With positive probability, however, just one of the other players trembles and chooses *Same*, in which case Duo's vote for *Same* would be decisive, and with the same probability just one of the other players trembles and chooses *Increase*, in which case Duo's vote for *Increase* would be decisive. Since Tres's payoff from *Same* is bigger than his payoff from *Increase*, he will choose *Same* in the hopes of that tremble.

(e) Part (d) showed that if Uno and Duo are expected to choose *Reduce*, then Tres would choose *Same* if he could hope they might tremble– not *Increase*. Suppose, instead, that Tres votes first, and publicly. Construct a subgame perfect equilibrium in which Tres chooses *Increase*. You need not worry about trembles now.

<u>*Answer.*</u> Tres's strategy is just an action, but Uno and Duo's strategies are actions conditional upon Tres's observed choice.

Tres: *Increase*.
Uno: *Increase*|*Increase*; *Reduce*|*Same*, *Reduce*|*Reduce*.
Duo: *Reduce*|*Increase*; *Reduce*|*Same*, *Reduce*|*Reduce*

Uno's equilibrium payoff is 100. If he deviated to *Same*|*Increase* and Tres chose *Increase*, his payoff would fall to 3; if he deviates to *Reduce*|*Increase* and Tres chose *Increase*, his payoff would fall to 9. Out of equilibrium, if Tres chose *Same*, Uno's payoff if he responds with *Reduce* is 9, but if he responds with *Same* it is 4. Out of equilibrium, if Tres chose *Reduce*, Uno's payoff is 9 regardless of his vote.

Duo's equilibrium payoff is 2. If Tres chooses *Increase*, Uno will choose *Increase* too and Duo's vote does not affect the outcome. If Tres chooses anything else, Uno will choose *Reduce* and Duo can achieve his most preferred outcome by choosing *Reduce*.

(f) Consider the following voting procedure. First, the three voters vote between *Increase* and *Same*. In the second round, they vote between the winning policy and *Reduce*. If, at that point, *Increase* is not the winning policy, the third vote is between *Increase* and whatever policy won in the second round.

What will happen? (watch out for the trick in this question!)

<u>Answer.</u> If the players are myopic, not looking ahead to future rounds, this is an illustration of the Condorcet paradox. In the first round, *Same* will beat *Increase*. In the second round, *Reduce* will beat *Same*. In the third round, *Increase* will beat *Reduce*. The paradox is that the votes have cycled, and if we kept on holding votes, the process would never end.

The trick is that this procedure does *not* keep on going– it only lasts three rounds. If the players look ahead, they will see that *Increase* will win if they behave myopically. That is fine with Uno, but Duo and Tres will look for a way out. They would both prefer *Same* to win. If the last round puts *Same* to a vote against *Increase*, *Same* will win. Thus, both Duo and Tres want *Same* to win the second round. In particular, Duo will *not* vote for *Reduce* in the second round, because he knows it would lose in the third round.

Rather, in the first round Duo and Tres will vote for *Same* against *Increase*; in the second round they will vote for *Same* against *Reduce*; and in the third round they will vote for *Same* against *Increase* again.

This is an example of how particular procedures make voting deterministic even if voting would cycle endlessly otherwise. It is a little bit like the T-period repeated game versus the infinitely repeated one; having a last round pins things down and lets the players find their optimal strategies by backwards induction.

Arrow's Impossibility Theorem says that social choice functions cannot be found that always reflect individual preferences and satisfy various other axioms. The axiom that fails in this example is that the procedure treat all policies symmetrically– our voting procedure here prescribes a particular order for voting, and the outcome would be different under other orderings.

(g) Speculate about what would happen if the payoffs are in terms of dollar willingness to pay by each player and the players could make binding agreements to buy and sell votes. What, if anything, can you say about which policy would win, and what votes would be bought at what price?

<u>Answer.</u>

Uno is willing to pay a lot more than the other two players to achieve his preferred outcome. Uno is willing to pay up to 97 (=100-3) to get *Increase* instead of *Same*.

Tres is willing to pay up to 5 (=9-4)to get *Same* instead of *Increase*. Duo would be willing to pay up to 4 (=6-2) to get *Same* instead of *Increase*. If *Increase* was not winning in equilibrium, Uno would be willing to pay 4 to Duo and 5 to Tres to get them to vote for *Increase*.

But this is a tricky situation, too ill-formed to give us an answer as to what the payments would actually be. The outcome would depend on the process by which payment offers are made.

Suppose Duo makes any offers first, then Tres, then Uno. A subgame perfect equilibrium is for Duo not to make any offer, Tres to offer 5 to Duo to vote *Same* and Duo to reject it, and Uno to offer 5 to Duo to vote *Same* and for Duo to accept. The same outcome would occur if Uno made his offer before Tres.

On the other hand, what if offers can be made at any time and continue until one is accepted? Suppose Uno pays x to Duo to vote for *Increase*, but but Tres offers y to Duo to vote for *Same*. Suppose $x = 3.9$ and $y = 0$. Duo would refuse Uno's offer and vote *Same*, to get the 4 extra in direct payoff. If $x \geq 4$, Duo would be willing to vote *Increase*, but unless $x > 5$, Tres would respond with an offer of up to $y = 5$. Thus we might expect that $x = y = 5$, with Duo accepting Uno's offer.

The problem is that we can then imagine Tres trying a new tactic. He could go to Uno just before Uno makes his offer and say, "I know I am going to lose anyway, so how about paying me .001 to vote *Same*, and not bothering to bribe Duo?" Uno would accept that offer. Someone whose vote does not matter to the result is willing, in fact, to accept 0 to change his vote, because he is indifferent. But of course Duo would go even earlier to Tres then, and offer to vote *Same* for .0002. There would be no equilibrium, just cycling.